तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN
PATNA REGION

CLASS-X

Based on the latest CBSE Exam pattern for the session-2023-24

STUDENT STUDY MATERIAL
(MATHEMATICS)

CHIEF PATRON

Sh. Anurag Bhatnagar

Deputy Commissioner

Kendriya Vidyalaya Sangathan

Patna Region

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Subject Convenor		
	Sh. Rajesh Srivastava	Principal PM Shri Kendriya Vidyalaya No.2 Gaya

PREPARED BY

S. No.	Topics	Name of teacher
1	REAL NUMBERS	SH. KAMAL NAYAN, TGT (MATHS), K. V. NO. 2 BAILEY ROAD FS
2	POLYNOMIALS	SH. KAMAL NAYAN, TGT (MATHS), K. V. NO. 2 BAILEY ROAD FS
3	PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	SH. KAMAL NAYAN, TGT (MATHS), K. V. NO. 2 BAILEY ROAD FS
4	QUADRATIC EQUATIONS	SH. KAMAL NAYAN, TGT (MATHS), K. V. NO. 2 BAILEY ROAD FS
5	ARITHMETIC PROGRESSIONS	MS MEGHA VERMA, TGT (MATHS), K. V. JHAPAHAN
6	TRIANGLES	SH. AMIT KUMAR, TGT (MATHS), K. V. NO. 2 GAYA
7	COORDINATE GEOMETRY	MS MEGHA VERMA, TGT (MATHS), K. V. JHAPAHAN
8	INTRODUCTION TO TRIGONOMETRY	MS MEGHA VERMA, TGT (MATHS), K. V. JHAPAHAN
9	SOME APPLICATIONS OF TRIGONOMETRY	MS MEGHA VERMA, TGT (MATHS), K. V. JHAPAHAN
10	CIRCLES	SH. SURAJ KUMAR, TGT (MATHS), K. V. KANKARBAGH SS
11	AREAS RELATED TO CIRCLES	SH. SURAJ KUMAR, TGT (MATHS), K. V. KANKARBAGH SS
12	SURFACE AREAS AND VOLUMES	SH. SURAJ KUMAR, TGT (MATHS), K. V. KANKARBAGH SS
13	STATISTICS	SH. SURAJ KUMAR, TGT (MATHS), K. V. KANKARBAGH SS
14	PROBABILITY	SH. SURAJ KUMAR, TGT (MATHS), K. V. KANKARBAGH SS
15	SAMPLE PAPER STANDARD (SOLVED)	SH. MANISHANKAR JHA, TGT (MATHS), K.V. MUZAFFARPUR FS
16	SOLUTION OF SAMPLE PAPER STANDARD (SOLVED)	SH. MANISHANKAR JHA, TGT (MATHS), K.V. MUZAFFARPUR FS
17	SAMPLE PAPER STANDARD (UNSOLVED)	SH. ANIL KUMAR, TGT (MATHS), K. V. PURNEA
18	SAMPLE PAPER BASIC (SOLVED)	SH. MANISHANKAR JHA, TGT (MATHS), K.V. MUZAFFARPUR FS
19	SOLUTION OF SAMPLE PAPER BASIC (SOLVED)	SH. MANISHANKAR JHA, TGT (MATHS), K.V. MUZAFFARPUR FS
20	SAMPLE PAPER BASIC (UNSOLVED)	SH. ANIL KUMAR, TGT (MATHS), K. V. PURNEA
21	COMPILED BY	SH. MITHLESH KUMAR, TGT (MATHS), K. V. NO. 2 GAYA SH. AMIT KUMAR, TGT (MATHS), K. V. NO. 2 GAYA

COURSE STRUCTURE CLASS -X

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}\ \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV: GEOMETRY

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0· and 90·. Values of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $sin^2A + cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression.

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	8	10
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

PRESCRIBED BOOKS:

1. Mathematics - Textbook for class IX - NCERT Publication
2. Mathematics - Textbook for class X - NCERT Publication
3. Guidelines for Mathematics Laboratory in Schools, class IX - CBSE Publication
4. Guidelines for Mathematics Laboratory in Schools, class X - CBSE Publication
5. Laboratory Manual - Mathematics, secondary stage - NCERT Publication
6. Mathematics exemplar problems for class IX, NCERT publication.
7. Mathematics exemplar problems for class X, NCERT publication.

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

MATHEMATICS STUDY MATERIAL FOR CLASS X (2023-24)

CONTENTS

REAL NUMBERS

GIST OF THE TOPIC

1. Euclid's division lemma: Given positive integers a and b there exist whole numbers q and r satisfying $a = bq + r, 0 \leq r < b$.

2. Euclid's division algorithm: In order to compute the HCF of two positive integers, say a and b, with a > b by using Euclid's algorithm we follow the following steps:
Step I :-Apply Euclid's division lemma to a and b and obtain whole numbers q, and r, such that
$a = bq_1 + r_1, 0 \leq r_1 < b$.
Step II: -If $r_1 = 0$, b is the HCF of a and b
Step III: -If $r_1 \neq 0$, apply Euclid's division lemma to b and r_1 and obtain two whole numbers q_1 and r_1 such that $b= q_1 r_1 + r_2$
Step IV: -If $r_2 \neq 0$, then, r_1 is the HCF of a and b.
Step V: - If $r_2 \neq 0$, then apply Euclid's division lemma to r_1, and r_2 and continue the above process till the remainder, r_n is zero. The divisor at this stage i.e. r_{n-1}, or the non-zero remainder at the previous stage, is the HCF of a and b.

3. The Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique except for the order in which the prime factors occur.

4. Every composite number can be uniquely expressed as the product of powers of primes in ascending or descending order.

5. Let a be a positive integer and p be a prime number such that p divides a^2, then p divides a.
6. There are infinitely many positive primes.

7. Every positive integer different from 1 can be expressed as a product of non-negative power or of 2 and an odd number.

8. A positive integer n is prime, if it is not divisible by any prime less than or equal to $\sqrt{n}$.

9. If p is a positive prime, then $\sqrt{p}$ is an irrational number. For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$ etc. are irrational numbers.

10. Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form p/q, where p and q are co-prime, and the prime factorization of q is of the form $2^m \times 5^n$, where m and n are non-negative integers.

11. Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^m \times 5^n$ where m, n are non-negative integers. Then, x has a terminating decimal expansion which terminates after k places of decimals, where k is the larger of m and n.

12. Let x = p/q be a rational number, such that the prime factorization of q is not of the for $2^m \times 5^n$ where m, n are non-negative integers. Then, x has non-terminating repeating decimal expansion.

MULTIPLE CHOICE QUESTIONS (1 Mark)

1. If p and q are two distinct prime numbers, then their HCF is **BASIC**
(a) 2 (b) 0 (c) either 1 or 2 (d) 1
Ans. (d)
Solution: - A prime number has no factor other than 1 and the number itself. Therefore, factors of p are: 1 and p only. Factors of q are: 1 and q only. Therefore, 1 is the only common factor of p and q. Hence, HCF (p, q) =1.

2. If p and q are two distinct prime numbers, then LCM (p, q) is **BASIC**
(a) 1 (b) p (c) q (d) pq
Ans. (d)
Solution: - Factors of p are: 1 and p. Factors of q are: 1 and q.
LCM (p, q) = 1x p x q = pq. LCM (p, q) x HCF (p, q)= pq LCM (p, q)x1 = pq
=> LCM (p, q) =pq

3. Let p be a prime number. The sum of its factors is **BA SIC**
(a) p (b) 1 (c) p+1 (d) p-1
Ans. (c)

4. Let n be a natural number. Then, the LCM (n, n + 1) is
(a) n `(b) n+1 (c) n (n+1) (d) 1
Ans. (c)

5. If P_1 and P_2 are odd prime numbers such that $P_1 > P_2$. Then $P_1^2 – P_2^2$ is
(a) an even number (b) an odd number (c) an odd prime number (d) a prime number
Ans. (a)

ASSERTION-REASON (1 Mark)

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1 (Assertion): If LCM (60, 72)= 360, then HCF (60,72) **BASIC**
Statement-2 (Reason): HCF (a, b) x LCM (a, b) = a + b.
Ans. (c)
Solution:- We have, $60 = 2^2$ x3x5 and $72 = 2^3 \times 3^2$. LCM (60, 72) = 2^3 x 3^2 x 5 = 360 and HCF (60,72) = 2^2 x 3 =12.So, statement-1 is true. We know that HCF (a, b) x LCM (a, b) = a x b. So, statement-2 is false. Hence, option (c) is correct.

2. Statement-1(Assertion): The square of an odd natural number leaves remainder 1 the divided by 4.
CB

Statement-2 (Reason): The square of an odd natural number is of the form 4k+1 for some natural number k.

Ans. (a)

SHORT ANSWER TYPE (2 MARKS)

1. Find the HCF and LCM of 12, 14 and 16 using prime factorization method.
Ans. HCF = 2 and LCM = 336.
BASIC
Solution: - $12 = 2 \times 2 \times 3 = 2^2 \times 3^1$, $14 = 2^1 \times 7^1$, $16 = 2 \times 2 \times 2 \times 2 = 2^4$,
H.C.F (12, 14, 16) = 2^1, LCM (12, 14, 16) = $2^4 \times 3^1 \times 7^1 = 16 \times 21 = 336$

2. Find the prime factorization of the denominator of the rational number 2.345.
Ans. $2^3 \times 5^3$
BASIC
Solution: - 2.345 = 2345/1000, D=1000= 10x10x10 = $(2 \times 5)^3$.

3. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is ………
Ans. 2 **BASIC**

4. Two numbers are in the ratio 21:17. If their HCF is 5, find the numbers.
Ans. 105 and 85.

5. Can we have any $n \in N$, where 7^n end with the digits zero? **CB**
Ans. No.

SHORT ANSWER TYPE (3 MARKS)

1. Prove that $5-\sqrt{3}$ is an irrational number.
BASIC
Solution:- Let the given number be a rational and we will write the given number in p/q form.
$5-\sqrt{3} = p/q$
$5-p/q = \sqrt{3}$
Since p and q are integers, we get 5- p/q is rational, and so, $\sqrt{3}$ is rational. This contradicts the fact that $\sqrt{3}$ is irrational. Hence, our assumption that given number is rational is false.
=>$5-\sqrt{3}$ is irrational.

2. Prove that $(1 / 7 + \sqrt{7} / 5)$ is irrational.
BASIC
Solution:-Let $1 / 7 + \sqrt{7} / 5$ be rational.
$1 / 7 + \sqrt{7} / 5 = p/q$, where p and q co-primes and $q \neq 0$.
$\sqrt{7} / 5 = p/q - 1/7$
$\sqrt{7} = 5(p/q - 1/7)$.
Since p/q is rational, so $(p/q - 1/7)$ and $5(p/q - 1/7)$ are rational.
But it is a contradiction, as $\sqrt{7}$ is irrational. Hence, $(1 / 7 + \sqrt{7} / 5)$ is irrational.

3. Give an example each of two irrational numbers, whose
BASIC
i. Product is an irrational number.
ii. Product is a rational number.
iii. Quotient is a rational number.
Answer: - i) $\sqrt{2}$ and $\sqrt{3}$. ii) $2+\sqrt{3}$ and $2-\sqrt{3}$. iii) $3+3\sqrt{2}$ and $1+\sqrt{2}$.

4. LCM of two numbers is 10 times their HCF. Sum of HCF and LCM is 495. If one number is 90, then find the other number.
Answer: - Other number is 225.

5. Prove that $\sqrt{7}$ is an irrational number.
Answer: - Proof.

CASE STUDY BASED QUESTIONS (4 MARKS)

QUESTION 1:- There are two sections in class 10th in a school- Section A and Section B. To enhance the reading skills of the students, on each day, the school nominates 2 students from each section to set up a class library. There are 42 students in section A and 40 students in section B.

Read the above information and answer the following questions:

BASIC

(i) If the product of two positive integers is equal to the product of their HCF and LCM is true then, find the HCF (42, 40).

(ii) Find the LCM of 42 and 40.

(iii) Express 40 as a product of its prime numbers. **OR**

Find the HFC of first prime number and first composite number.

Solution: - (i) $42 = 2\times3\times7$ and $40 = 2\times2\times2\times5$. HCF (42, 40) = 2.

(ii) LCM (42, 40) = $2\times2\times2\times3\times5\times7 = 840$.

(iii) $40 = 2\times2\times2\times5 = 2^3\times5$. **OR**

First prime number =2. First composite number = 4. Their HCF = 2.

QUESTION 2: - To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections-section A and section B of grade X. There are 32 students in section A and 36 students in section B.

Read the above information and answer the following questions:

BASIC

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of section A or section B?

(ii) (7 x 11 x 13 x 15) + 15 is a ………

(a) Prime number (b) Composite number

(c) Neither prime nor composite (d) None of the these.

(iii) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers then find the LCM (p, q).

OR, Express 36 as a product of its primes .

Solution:-

(i) Since Books are to be distributed equally among students of section A or section B. So, the number of books is a multiple of 32 as well as 36 and it is the least such number. Therefore, required number of books is the LCM of 32 and 36.

Now, $32=2^5$ and $36 = 2^2 \times 3^2$. LCM (32, 36)= $2^5 \times 3^2 = 32 \times 9 = 288$. Hence, required number of books = 288.

(ii) Ans. (b): $7\text{x}11\text{x}13\text{x}15+ 15 = (7 \times 11 \times 13+1) \times 15$

7x11x13x15+15 is a composite number.

(iii) It is given that: $p = ab^2$ and $q = a^2b$. Therefore, LCM (p, q) = $a^2 b^2$. **OR,** $36 = 2^2 \times 3^2$.

QUESTION 3: - A seminar is being conducted by an educational organization, where the participants will educators of different subjects. The numbers of participants in Hindi, English and Mathematics are 60,84 108 respectively.

CB

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence the maximum number of participants that can be accommodated in each room is ………… .

(ii) The minimum number of rooms required during the event?

(iii) The LCM of 60, 84 and 108 is …….. . **OR**

The product of HCF and LCM of 60, 84 and 108 is …………. .

Answer: - (i) Ans. 12 (ii) Ans. 5 (iii) Ans. 3780 **OR,** Ans. 45360.

POLYNOMIALS

GIST OF THE TOPIC

1. Let x be a variable, n be a positive integer and $a_0, a_1, \ldots a_n$ be constants (real numbers). Then, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$, is called a polynomial in variable x.
2. The exponent of the highest degree term in a polynomial is known as its degree. A polynomial of degree 0 is called a constant polynomial. A polynomial of degree 1, 2 or 3 is called a linear polynomial, a quadratic polynomial or a cubic polynomial respectively.
3. If f(x) is a polynomial and α is any real number, then the real number obtained by replacing x by α in f(x) is known as the value of f(x) at x = α and is denoted by f(α).
4. A real number a is a zero of a polynomial f(x), if f(α) = 0.
5. A polynomial of degree n can have at most n real zeros.
6. Geometrically the zeros of a polynomial f(x) are the x-coordinates of the points where the graph y= f(x) intersects x-axis.
7. If α and β are the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$, then $\alpha + \beta = -b/a = -$ Coefficient of x / Coefficient of x^2, $\alpha\beta = c/a =$ Constant term / Coefficient of x^2
8. If α, β, γ are the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\alpha+\beta+\gamma = -b/a$, $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$, $\alpha\beta\gamma = -d/a$.
9. If f(x) is a polynomial and g(x) is a non-zero polynomial, then there exist two polynomials q(x) and r(x) such that f(x)= g(x) x q(x) + r(x), where r(x) = 0 or degree r(x) < degree g(x). This is known as the division algorithm.
10. A quadratic polynomial whose zeros are reciprocal of the zeros of a given quadratic polynomial can be obtain by interchanging the coefficient of x^2 and constant term.
11. A quadratic polynomial whose zeros are negative of the zeros of a given quadratic polynomial can be obtained by changing the sign of coefficient of first degree term.

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. Which of the following is a polynomial?

   BASIC

   (a) $2x^2 + 3/x - 5$ (b) $-3x^2 + \sqrt{2x} + 4$

   (c) $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$ (d) $5/x^3 + 2x^2 - 3x + 1/7$

   Ans. (c)

   Solution: - An expression of the form $f(x)=a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0,\ a_0 \neq 0$ is a polynomial of degree n (a natural number).Clearly, in expression given in option (a) has a term containing x^{-1}. So, it is not a polynomial. Expression in option (b) has a term containing $x^{-1/2}$, so it is not a polynomial. The expression in option (d) has a term containing x^{-3}, so it is not a polynomial. Expression in option (c) satisfies the definition of a polynomial.

2. If one zero of the quadratic polynomial kx^2+3x+k is 2, then the value of k.

   (a) 5/6 (b) -5/6

   (c) 6/5 (d) -6/5

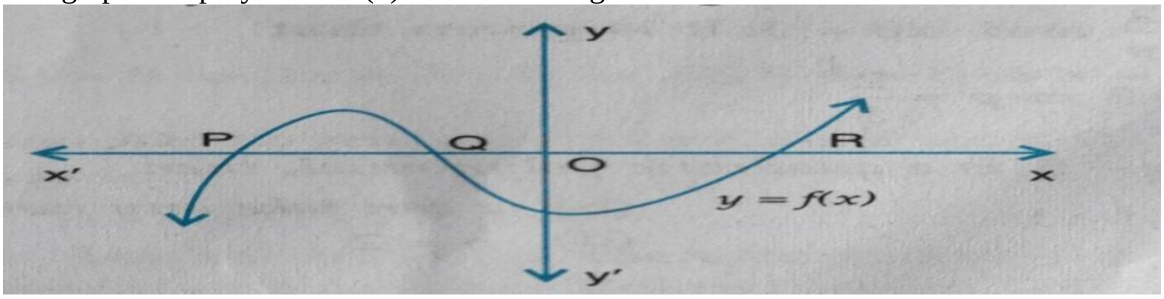
   BASIC

   Ans. (d)

   Solution: - It is given that 2 is a zero of $f(x)=kx^2+3x+k$.

   Therefore, $f(2) = 0 \Rightarrow k \times 2^2 + 3 \times 2 + k = 0 \Rightarrow 5k+6 = 0 \Rightarrow k = -6/5$

3. The graph of a polynomial f(x) is shown in Fig



The number of zeroes of f(x) is

(a) 3 (b) 2 (c) 1 (d) 4 **BASIC**

Ans. (a)

4. If α and β are the zeros of the polynomial $f(x) = px^2-2x+3p$ and $\alpha + \beta = \alpha\beta$, then the value of p is

(a) -2/3 (b) 2/3 (c) 1/3 (d) -1/3 **CB**

Ans. (b)

5. If zeroes of the quadratic polynomial $f(x) = (k^2+4) x^2 + 7x + 4k$ are reciprocal of each other, then the value (s) of k is (are)

(a) 1 (b) -1 (c) 2 (d) -2

Ans. (c)

ASSERTION-REASON BASED QUESTIONS (1 Mark)

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1 (Assertion): 5x + 2 is a linear polynomial. **BASIC**

Statement-2 (Reason): A polynomial of degree 1 is a linear polynomial.

Ans. (a)

Solution: -we know that definition of degree of a polynomial is same as given in reason.

2. Statement-1(Assertion): A quadratic polynomial having $2+ \sqrt{3}$ and $2-\sqrt{3}$ as its zeroes is given by $f(x) = x^2-4x+1$.

Statement-2 (Reason): Quadratic polynomials whose two zeroes are a and B are given by $f(x) = k (x^2-x (\alpha +\beta) + \alpha\beta)$, where k is any non-zero real number

Ans. (a)

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. The sum of the zeroes of the given quadratic polynomial $-3x^2 + k$ is

Ans. 0

BASIC

Solution: - Since polynomial is $-3x^2+0x+k$

$a = -3, b = 0, c = k$ and sum of zeroes $= -b/a = 0 / -3 = 0$

2. If one zero of the polynomial $x^2 – 4x +1$ is $2+ \sqrt{3}$, then the other zero is

Ans. 2- √3
BASIC
Solution: - Let other zero be α, (2+√3) + α = -b / a = -(-4) / 1 = 4
α = 4 – (2 + √3) = 2 - √3.

3. Find a quadratic polynomial whose zeroes are - 9 and -1/9.
Ans. $9x^2 + 82x + 9$.
BASIC
4. If zeroes of $p(x) = ax^2+bx+c$ are negative reciprocal of each other, find the relationship between a and c.
Ans. a + c = 0
5. The zeroes of the polynomial $(x-2)^2 + 4$ is
Ans. No zero.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. If α and β are the zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are 2α and 2β.
BASIC
Solution: - As α and β are zeroes of the polynomial $x^2 - 2x - 15$. α + β = 2 and α x β = -15
Now, 2α + 2β = 2 x 2 = 4 and 2α x 2β = 4 x (-15) = -60
So required quadratic polynomial = $x^2 - 4x - 60$.
2. If one zero of the polynomial $(a^2 + 9) x^2 +13x + 6a$ is reciprocal of the other, then find the value of a.
BASIC
Solution: - Let one of the zeroes of the polynomial be α. Then the other zero will be 1/α.
Now, product of zeroes = α x 1/α = 1.
=> $6a / (a^2 +9) = 1$ => $a^2 – 6a +9 = 0$ => $(a – 3)^2 = 0$ => a = 3.
3. Find the quadratic polynomial if sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained. **BASIC**
Answer: - Required polynomial is $x^2 + x – 20$. α = 4 and β = - 5.
4. If (x-2) is a factor of $x^3 + ax^2 +bx +16$. And b= 4a, find the value of a and b.
Answer: - a = - 2 and b = - 8
5. If the sum and product of zeroes of the polynomial $p(x) = kx^2 + 2x + 3k$ is equal, then find the value of k. **CB**
Answer: - k = -2/3.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. If the square of the difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px +15$ is equal to 196, then find the value of p.
BASIC
Solution:- Let α and β be the zeroes of the polynomial $f(x) = x^2 + px +15$.
Then α+β = -p and αβ = 15. $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-p)^2 – 4 \times 15 = p^2 -60$.
=> $196 = p^2 -60$ => p = √256 = 16 or -16.
2. If α and β be the zeroes of the polynomial $f(x) = kx^2 + 2x -15$, such that $\alpha^2 +\beta^2 = 34$. Then find the value of k. **BASIC**
Solution:- $\alpha^2 +\beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = (\alpha+\beta)^2 - 2\alpha\beta = 34$
αβ = -2 / k and α+β = -15 / k
$(-2 / k)^2 – 2(-15 / k) =34$ => (17k +2)(k -1) = 0 => k = -2/17 or 1.

3. Find the zeroes of the quadratic polynomial $f(x) = x^2 - 2\sqrt{a}\ x + (a - b)$. Verify relationship between zeroes and coefficients. **BASIC**

   Answer:- $\alpha = (\sqrt{a} + \sqrt{b})$ and $\beta = (\sqrt{a} - \sqrt{b})$.

   $\alpha\beta = 2\sqrt{a}$ and $\alpha+\beta = a - b$.

4. If α, β, γ be zeros of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

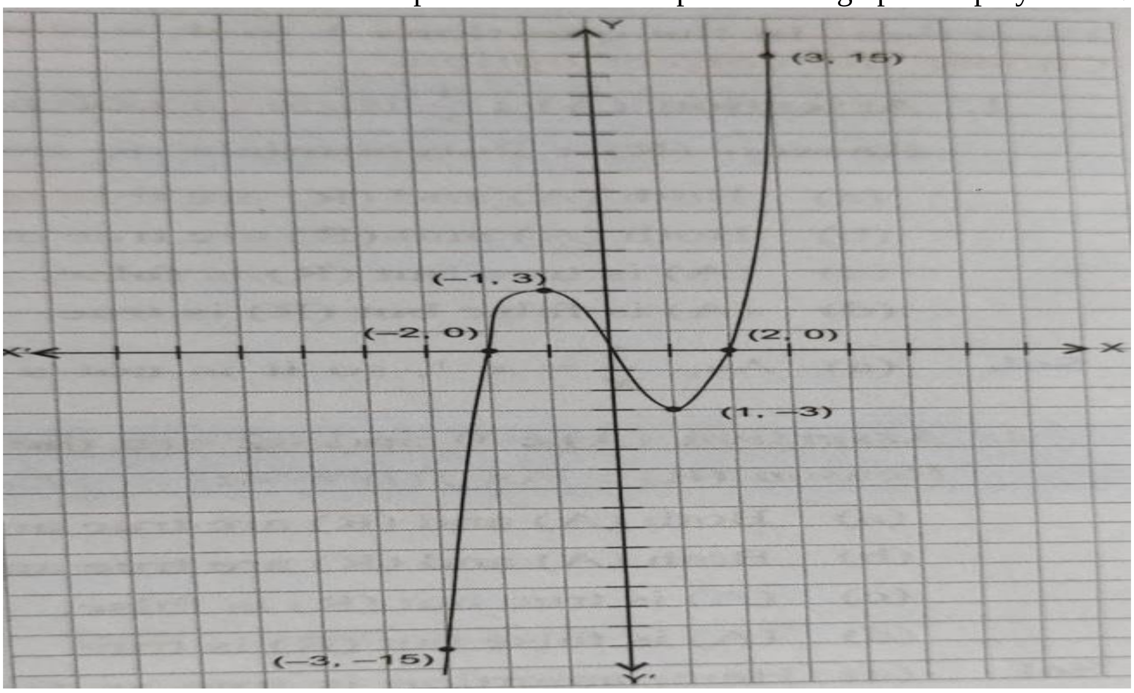
   Answer:- $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$. **CB**

5. If one zero of the quadratic polynomial $4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of the polynomial $kx^2 + 3kx + 2$.

   Answer:- $k = 1$. $P(x) = x^2 + 3x + 2$, then the value of $x = -2$ or $- 1$.

CASE STUDY BASED QUESTIONS (4 MARKS)

QUESTION 1:- One day due to heavy storm an electric wire got bent as shown in the figure. It followed some mathematical shape of curve which represents the graph of a polynomial.



Answer the following questions:

BASIC

(i) How many zeroes are there for the polynomial?

(ii) Find the zeroes of the polynomial.

(iii) Write the degree of the given polynomial.

OR, Write the corresponding cubic polynomial shown in the graph.

Solution: -

(i) Since the graph cuts the X-axis at three distinct points, therefore it has three zeroes.

(ii) From the graph, the zeroes are 2, -2 and 0.

(iii) It is a cubic polynomial. Therefore, its degree is 3. OR

Cubic polynomial

$= x^3 - (\alpha+\beta+\gamma)\ x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\ x - \alpha\beta\gamma$

$= x^3 - (2 - 2 + 0)\ x^2 + [(2 \times (-2) + (-2 \times 0) + (0 \times 2)]\ x - 2 \times (-2) \times 0 = x^3 - 4x$.

QUESTION 2:- **BASIC**


In Fig. various structures are parabolic in shape. A parabola is a curve represented by a quadratic polynomial $p(x) = ax^2 + bx + c$.

(i) In the standard form of quadratic polynomial $ax^2 + bx + c$, what is the characteristics of a, b, c?

(ii) If the roots of the quadratic polynomial $ax + bx + c$ are equal, then find the nature of discriminant D.

(iii) If α and $1/\alpha$ are zeroes of the quadratic polynomial $2x^2 - x + 8k$, then find the value of k.

OR

Is the graph of the polynomial $p(x)= x^2+1$ intersects any axis (by using the concept of D).

Solution: -

(i) The general form of a quadratic polynomial is $ax^2 + bx + c$, where a , b and c are real numbers such that $a \neq 0$.

(ii) If the roots of a quadratic equation are equal, then its discriminant D is equal to zero.

(iii) If α and $1/\alpha$ are zeros of the quadratic polynomial $2x^2 -x+8k$,

then $\alpha\beta = 8k/2 = \alpha \times 1/\alpha = 1 \Rightarrow k = 1 / 4$.

OR, The discriminant D of the polynomial $p(x) = x^2+0x+1$ is given by

$D = 0 - 4 = - 4 < 0$. So, it has no real zeroes.

Consequently, its graph neither touches nor intersects any axis.

QUESTION 3: - Basketball is played with a spherical ball. Even through an athlete dibbles the ball in sports, a basketball player uses his hand. Usually, the basketball is played indoor on a court made out of wood. The projectile (path traced) of basketball in the form of parabola representing a quadrilateral polynomial. **CB**

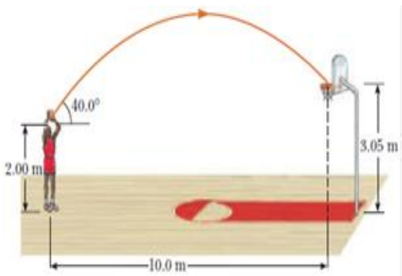


Fig 1

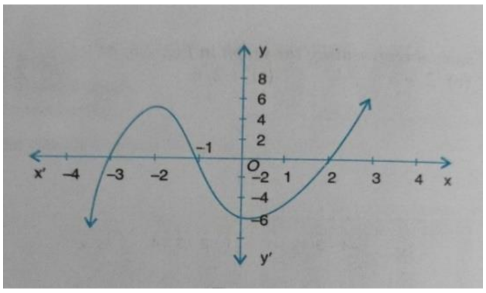


Fig 2

(i) The shape of the path traced shown in fig1, is a ……..

(ii) The polynomial representing the graph shown in fig2 has …… zeroes.

(iii) write all zeroes of the polynomial representing in the fig2 .

OR, The graph shown in fig2 is represented by the polynomial ….

(a) $x^3 + 2x^2 - 5x - 6$ (b) $x^3 + 2x^2 - 5x + 6$

(c) $x^3 + 2x^2 + 5x - 6$ (d) $x^3 + 2x^2 + 5x +6$

Answer :- (i) Ans. parabola (ii) Ans. 3 (iii) Ans. -3,- 1, 2. **OR,** Ans. (a).

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

GIST OF THE TOPIC

1. A pair of linear equations in two variables x and y can be represented algebraically as follows
   $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that $a_1^2+b_1^2 \neq 0$, $a_2^2+b_2^2 \neq 0$.
2. Graphically or geometrically a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2= 0$ in two variables represents a pair of straight lines which are
   (i) intersecting, if $a_1 / a_2 \neq b_1 / b_2$ (ii) parallel, if $a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$
   (i) coincident, if $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$.
3. To solve a pair of linear equations in two variables by Graphical method, we first draw the lines represented by them.
   (i) If the pair of lines intersect at a point, then we say that the pair is consistent and the co-ordinates of the point provide us the unique solution.
   (ii) If the pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations.
   (iii) If the pair of lines are coincident, then it has infinitely many solutions - each point on the line being a solution. In this case, we say that the pair of linear equations is consistent with infinitely many solutions.
4. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a pair of linear equations in two variables x and y such that
   (i) If $a_1 / a_2 \neq b_1 / b_2$, then the pair of linear equations is consistent with unique solution.
   (ii) If $a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$, then the pair of linear equations is inconsistent with no solution.
   (iii) If $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$, then the pair of linear equations is consistent with infinite many solutions.

MULTIPLE CHOICE QUESTIONS (1 Mark)

1. If pair of equations is consistent, then the lines representing them are
   (a) parallel (b) always consistent (c) intersecting or coincident (d) always intersecting
   Ans. (c)
   BASIC
   Solution: - If a pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is consistent then either it has unique solution or infinite many solutions. Consequently, lines representing the two equations either intersecting or coincident.
2. The value of k for which the system of linear equations x + 2y =3 and 5x + ky + 7=0 are inconsistent.
   BASIC
   (a) - 14/ 3 (b) 2 / 5 (c) 5 (d) 10
   Ans. (d)
   Solution:- Given system of equations will be inconsistent, if $a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$
   $1 / 5 = 2 / k \neq -3 / 7 \Rightarrow 1 / 5 = 2 / k \Rightarrow k = 10$.
3. The system of equations x = 0, y = 3 has **BASIC**
   (a) a unique solution (b) no solution (c) two solutions (d) infinite many solutions
   Ans. (a)
4. If the system of equations 2x + 3y = 7 and 2ax + (a + b) y = 8 has infinite many solutions, then
   (a) a = 2b (b) a + 2b = 0 (c) b = 2a (d) 2a + b = 0
   Answer:- (c)

5. The pair of equations x = 4 and y = -3 graphically represent lines which are
   (a) coincident (b) parallel (c) intersecting at (4, -3) (d) intersecting at (-3, 4)
   Answer:- (c)

ASSERTION-REASON BASED QUESTIONS (1 Mark)

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1 (Assertion): The system of linear equations 9x + 3y +12 = 0 and 18x+ 6y + 24=0 have infinitely many solutions.
   Statement-2 (Reason): The system of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x+b_2y + c_2 = 0$ have infinitely many solutions, if $a1 / a_2 = b_1 / b_2 = c_1 / c_2$.
   BASIC
   Ans. (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
   Solution: -we have 9/8 = 3/6 = 12/24 as $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$, so the system of equations in statement-1 is true and statement-2 is the correct explanation of statement-1.
2. Statement-1(Assertion): The system of linear equations 2x + y + 9 = 0 and x + 3y + 7 = 0 is consistent having unique solution.
   Statement-2 (Reason): The system of linear equations ax + by + c = 0 and px + qy + r = 0 is always consistent, if $a\,q \neq b\,p$.
   Answer: - (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. One equation of a pair of dependent linear equations is -5x + 7y – 2 = 0, the second equation can be …..
   BASIC
   Ans. - 4
   Solution: - Two equations ax + by + c = 0 and ax + by + c_2 = 0 represent a pair of dependent linear equations if $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$ i.e., one equation is constant times the other. For the equation in option (d), we find that 10x - 14y + 4 = -2(-5x+7y-2).
   i.e. 10 / -5 = 14 / 7 = 4 / -2. So, the second equation can be 10x - 14y = - 4.
2. If the pair of linear equations (3k+1) x + 3y – 5 = 0 and 2x - 3y + 5 = 0 have infinite number of solutions, then the value of k is ……
   BASIC
   Ans. -1
   Solution: - For infinitely many solutions, we must have
   (3k+1) / 2 = 3 / - 3 = -5 / 5 => 3k + 1= - 2 => k = -1.
3. If the pair of linear equations 2x - 3y = 0 and kx + 6y = 0 has non-zero solutions, then the value of k is ……
   BASIC
   Answer: - - 4

4. If $x = a$ and $y = b$ is the solution of the systems of equations $x – y = 2$ and $x + y = 4$, then the values of a and b are, respectively….
   Answer: - $a = 3$ and $b = 1$
5. If the sum of the age of a father and his son in years is 65 and twice the difference of their ages in years is 50, then the age of father is …
   CB
   Answer: - 45 years

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. I am three times as old as my son. Fifteen years later, I shall be two times as old as my How old am I and how old is my son?
   BASIC
   Solution: - Let my age be x years and my son's age be y years,
   then $x=3y$ (i)
   Fifteen years later, $x + 15 = 2(x+15)$
   $\Rightarrow x-2y = 15$ (ii)
   Put $x = 3y$ in eqn. (ii), we get $y = 15$. Then $x = 45$.
2. Solve the following systems of equations by using the method of substitution:
   $(2x / a) + (y / b) = 2$ and $(x / a) - (y / b) = 4$
   BASIC
   Solution: - The given system of equation is
   $(2x / a) + (y / b) = 2$ …..(i) and $(x / a) - (y / b) = 4$ …..(ii)
   From equation (i), we get: $y/b = 2 – (2x / a) \Rightarrow y = b (2 – (2x / a))$.
   Substituting $y = b (2 – (2x / a))$ in equation (ii), we get
   $x/a – (b/b)(2 – (2x / a)) = 4 \Rightarrow x/a – 2 + 2x /a = 4$
   $\Rightarrow 3x/a = 6 \Rightarrow 3x = 6a \Rightarrow x = 2a$
   Putting $x = 2a$ in equation (i), we get: $4 + y/b = 2 \Rightarrow y = -2b$
   Hence, the solution of the given system of equations is $x = 2a$ and $y = - 2b$.
3. Aruna has only Rs.1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is 75, then the number of Rs.1 and Rs.2 coins are,……. respectively.
   BASIC
   Answer: - 25 and 25.
4. If 8 chairs and 5 tables cost Rs.10500, while 5 chairs and 3 tables cost Rs 6450. The cost of each chair will be … **CB**
   Answer: - Rs. 750.
5. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is
   Answer: - 36.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

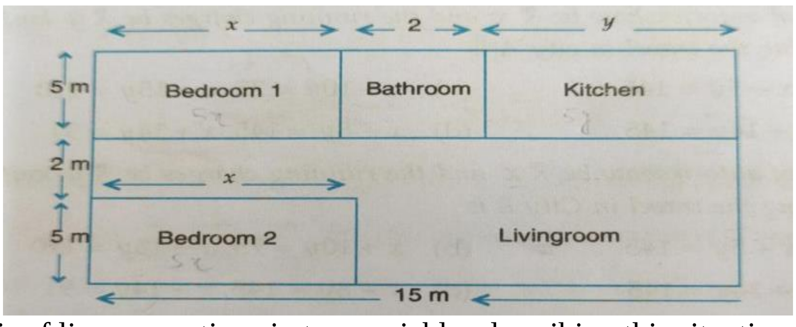
1. Consider a fraction such that when the numerator is increased by 1 and the denominator a increased by 3, we get 2 / 3. If only the denominator is increased by 4, we get $\frac{1}{2}$. Find the fraction.
   BASIC
   Solution: - Let numerator be x and denominator be y. As per the given condition.
   $(x+1) / (y+3) = 2 / 3$ ....(i) and $x / (y + 4) = 1 / 2$ ...(ii)
   From (i), $3(x+1) = 2(x+3) \Rightarrow 3x+3 = 2y+6 \Rightarrow x = (2 / 3)y + 1$ …(iii)
   Using equation (iii) to write equation (ii), we have
   $((2 / 3)y + 1) / (y + 4) = 1 / 2 \Rightarrow (4 / 3)y + 2 = y + 4 \Rightarrow y / 3 = 2 \Rightarrow y = 6$.
   Then $x = 5$. Thus, the required fraction is 5 / 6.

2. A father has 3 children with a gap of 2 years in every two consecutive children The sum of present ages of children is half the present age of the father 4 years ago the sum of ages of children war 1 year more than the one-fourth of age of the father Find the present ages of children and the father.
BASIC
Solution:- Let the present ages of three children be x, x+2, x+ 4 years, respectively, and the present age of father be y years. According to first condition, x + x +2 + x + 4 = y / 2.
=> 3x+6 = y / 2 => 6x – y + 12 = 0 …..(i)
According to second condition, ages of children were x - 4, (x+2-4) and (x + 4 - 4) years and age of father was (y - 4) years.
(x - 4) + (x - 2) + x = (y – 4) / 4 + 1
=> 3x – 6 = y / 4 => 12x – y – 24 = 0 …(ii)
From equation (i), we get y = 6x +12 Substituting value of y in equation (ii), we get,
12x - 6x – 12 – 24 = 0 => 6x -36 = 0 => x = 6 years
Substituting the value of x in equation (iii), we get y = 6(6) + 12 = 48 years
So, the present ages of children are 6, 8 and 10 years and the age of the father is 48 years
3. Consider a 2-digit number such that when its digits are reversed, the new number obtained is 6 more than 3 times the original number Also, one's digit is 5 times that of the other. Find the original number.
BASIC **Answer:-** The original number be 15
4. Solve the following system of linear equations by substitution:
bx + cy = a + b
and ax(1 /(a – b) – 1 / (a + b)) + cy(1 /(b - a) – 1 / (b + a)) = 2a / (a + b).
Answer:- x = a / b and y = b / c.
5. Solve the following system of linear equations by elimination method:
$(a – b)x + (a + b)y = a^2 – 2ab – b^2$
$(a + b) (x + y) = a^2 + b^2$
Answer: - x = a + b and y = -2ab / (a + b).

CASE STUDY BASED QUESTIONS (4 MARKS)

QUESTION 1:- Ravish is planning to buy a house whose layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 m^2.

BASIC



(i) Write a pair of linear equations in two variables describing this situation.
(ii) Find perimeter and area of the house.
(iii) Find value of x y. **OR,**
Find the value of x - y.

SOLUTION: -
(i) We observe that x + 2 + y = 15 and 5x + 5x + 5y = 95

$x + y = 13$ and $2x + y = 19$

(ii) Perimeter = 2(15+12) - 54 m, Area = 15×12 m² = 180 m²

(iii) Solving $x + y = 13$ and $2x + y = 19$, we obtain: $x = 6$, $y = 7$. Therefore, $xy = 42$.

OR, Ans. We have, $x = 6$, $y = 7$. Therefore, $x – y = 6 – 7 = - 1$.

QUESTION 2:- It is common that governments revise travel fares from time to time based on various factors such as inflation (a general increase in prices and fall in the purchasing value of money) on different types of vehicles like auto rickshaws, taxis, radio cabs etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations:
BASIC

Name of the City	*Distance travelled (km)*	*(Amount paid (₹))*
City *A*	10	75
	15	110
City *B*	8	91
	14	145

(i) If the fixed charges of autorickshaw be Rs. x and the running charges be Rs. y km/hr, the pair of linear equations representing the travel in city A will be ……

(ii) If the fixed charges of auto rickshaw be Rs. x and the running charges be Rs. y km/hr, the pair of linear equations representing the travel in City B will be

(iii) Find the amount paid by a person travelling 100 km in city A.

OR, Find the amount paid by a person travelling 60 km in city B.

SOLUTION: -

(i) In travelling 10 km in City A, the amount paid is 75.

Therefore, $x+10y = 75$. Similarly, we obtain $x+15y = 110$.

(ii) In travelling 8 km in City B, the amount paid is 91. Therefore, $x+8y = 91$.

Similarly, we obtain $x+14y = 145$.

(iii) Two equations describing the travel in City A are: $x+10y = 75$ and $x+15y = 110$. Solving these two equations, we obtain $x=5$, $y=7$.

Amount paid in travelling 100 km in City A = $(x+100y) = (5+700) = 705$

OR, Ans. Two equations describing the travel in City B are: $x+8y = 91$ and $x+14y = 145$. Solving these equations, we obtain: $x = 19$, $y = 9$.

Amount paid in travelling 60 km in City B $(x+60y) = 7(19+60×9) = 559$

QUESTION 3: -A test consists of True' or 'False' questions. One mark awarded for every correct answer while 1 4 mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and scored 95 marks.
CB

(i) If answer to all questions he attempted by guessing were wrong, then the number of questions he answered correctly is …..

(ii) The number of questions he guessed, is……

(iii) If answer to all question he attempted by guessing were wrong and answered 80 correctly, then how many marks he got? **OR,**

If answer to all questions he attempted by guessing were wrong, then the number of questions answered correctly to score 95 marks is

Answer: - (i) Ans. 96 (ii) Ans. 24 (iii) Ans. 70 **OR,** Ans. 100.

QUADRATIC EQUATIONS

GIST OF THE TOPIC

1. A polynomial of degree 2 is called a quadratic polynomial. The general form of a quadratic polynomial is $ax^2 + bx + c$, where a, b, c are real numbers such that $a \neq 0$ and x is a real variable.
2. If $p(x) = ax^2 + bx + c$, $a \neq 0$ is a quadratic polynomial and a is a real number, then $p(\alpha) = a\alpha^2 + b\alpha + c$ is known as the value of the quadratic polynomial p (x).
3. A real number a is said to be a zero of the quadratic polynomial $p(x) = ax^2 + bx + c$, if $p(a)= 0$.
4. If $p(x) = ax^2+ bx + c$ is a quadratic polynomial, then $p(x) = 0$ i.e., $ax^2 + bx + c = 0$; $a \neq 0$ is called a quadratic equation.
5. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. In other words, α is a root of $ax^2 + by + c = 0$ if and only if α is a zero of the polynomial $p(x) = x^2 + bx + c$.
6. If $ax^2 + bx + c$, $a \neq 0$ is factorizable into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
7. The real roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ can be found by using the quadratic formula $(- b \pm \sqrt{(b^2 - 4ac)}) / 2a$ provided that $b^2 - 4ac \geq 0$.
8. If α, β are roots of the quadratic equation $ax^2 + bx + c = 0$, then $\alpha + \beta= - b / a$ and $\alpha\beta = c/a$.
9. Nature of the roots of quadratic equation $ax^2 + bx + c = 0, a \neq 0$ depends upon the value of
   $D = b^2-4ac$, which is known as the discriminate of the quadratic equation.
10. The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has:
    (i) two distinct real roots, if $D = b^2- 4ac > 0$
    (ii) two equal roots i.e., coincident real roots, if $D = b^2- 4ac = 0$
    (iii) no real roots, if $D = b^2- 4ac < 0$.

MULTIPLE CHOICE QUESTIONS (1 Mark)

1. The equation $ax^2 + bx + c = 0$ is a quadratic equation for
   BASIC
   (a) all values of a (b) all non-zero values of a
   (c) all non-zero values of b (d) all non-zero values of c
   Ans. (b)
   Solution: - The equation $ax^2 + bx + c = 0$, $a \neq 0$ is defined as a quadratic equation for all values of b and c. Hence, option (b) is correct.
2. A quadratic equation has
   BASIC
   (a) at most two roots (b) at least two roots
   (c) exactly two roots (d) at least one root
   Ans. (c)
   Solution: - A quadratic equation has exactly two roots, say α and β, such that $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.
3. If $x = 0.2$ is a root of the equation $x^2 - 0.4k = 0$, then k =?
   BASIC
   (a) 1 (b) 10 (c) 0.1 (d) 100.
   Answer: - (c) 0.1
4. Check whether the equation $x^2 + 3x + 1 = (x-2)^2$ is quadratic equation or not?

Answer: - This is not a quadratic equation.

5. Which of the following is the quadratic equation for the given situation? The sum of the squares of two consecutive even integers is 340.
   (a) $x^2 + 4x + 8 = 0$ (b) $x^2 - 169 = 0$
   (c) $x^2 + 2x – 168 = 0$ (d) $x^2 + 8x – 154 = 0$
   Answer: - (c) $x^2 + 2x – 168 = 0$.

ASSERTION-REASON BASED QUESTIONS (1 Mark)

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. Statement-1 (Assertion): If the equation $x^2 - ax + b = 0$ and $x^2 + bx - a = 0$ have a common root and $a+b = 0$, then $a-b=1$
   Statement-2 (Reason): A common root of two equations satisfies both the equations.
   BASIC
   Ans. (a)
   Solution: -Clearly, statement-2 is always true. Let α be a common root of the given equations. Then,
   $\alpha^2 - a\alpha+b=0$ and $\alpha^2 + b\alpha - a = 0$
   $(\alpha^2 - a\alpha+b)-(\alpha^2 + b\alpha - a) = 0 \Rightarrow -\alpha (a + b) = - (a + b) \Rightarrow \alpha = 1$
   Putting $\alpha = 1$ in $\alpha^2 - a\alpha + b = 0$, we obtain $a - b = 1$. So, statement-1 is true. Clearly, Statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.
2. Statement-1 (Assertion): If the difference of roots of the equation $x^2 - 2px + q = 0$ is same as the difference of the roots of the equation $x^2 - 2rx + 5 = 0$, then $s -q = r^2 - p^2$.
   Statement-2 (Reason): The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = (-b\pm\sqrt{D}) / 2a$, where D is the discriminant.
   Answer: - (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. A quadratic equation whose one root is $1+ \sqrt{2}$ and the sum of its roots is 2, is
   (a) $x^2 -2x + 1 = 0$ (b) $x^2 - 2x - 1 = 0$ (c) $x^2 + 2x + 1 = 0$ (d) $x^2 + 2x - 1 = 0$
   Ans. (b)
   BASIC
   Solution: - Let α , β be the roots of the desired equation and let $\alpha = 1+ \sqrt{2}$ and $\alpha + \beta = 2$
   Then $\alpha = \sqrt{2} + 1$ and $\beta = 1-\sqrt{2}$. So, the required equation is
   $x^2 - (\alpha+\beta)x + \alpha\beta = 0$ or, $x^2 - 2x - 1 = 0$.
2. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then
   (a) $b^2 = 16ac$ (b) $b^2 = 3ac$ (c) $3b^2 = 16ac$ (d) $16b^2 = 3ac$
   Ans. (c)
   BASIC
   Solution: - Let the roots be α and 3α. Then,
   $\alpha + 3\alpha = - b / a$ and $\alpha \times 3\alpha = c / a \Rightarrow \alpha = - b /4a$ and $3\alpha^2 = c / a \Rightarrow 3 \times (- b / 4a)^2 = c / a$

$\Rightarrow 3b^2 = 16ac$.

3. The values of k for which the quadratic equation $16x^2 + 4kx + 9 = 0$ has real and equal roots are

   Answer: - 6 and - 6.
4. If one root of the equation $x^2+ax+3=0$ is 1, then its other root is

   Answer: - 3
5. If x = 1 is a common root of the equations $ax^2+ax+3=0$ and $x^2+x+b=0$, then ab =?

   Answer: - 3.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. The sum of two numbers is 18 and their product is 56. Find the numbers.

   Ans. 4 and 14.

   BASIC **solution: -** Let the smaller number be x than the other number will be (18-x)

   From the given condition, $x(18-x) = 56 \Rightarrow 18x - x^2 = 56$

   $x^2 - 18x + 56 = 0 \Rightarrow x^2 - 14x - 4x + 56 = 0 \Rightarrow (x-14)(x-4) = 0$

   x = 4 or 14. The required numbers are 4 and 14.
2. A natural number is greater than twice its square root by 3. Find the number.

   Ans. 1, 9.

   BASIC

   Solution: - Let the number be x

   As per given condition:

   $x - 2\sqrt{x} = 3 \Rightarrow x-3 = 2\sqrt{x}$

   Squaring both sides, $(x-3)^2 = (2\sqrt{x})^2 \Rightarrow x^2 + 9 - 6x = 4x \Rightarrow x^2 - 10x + 9 = 0$

   $\Rightarrow x^2 - 9x - x + 9 = 0 \Rightarrow (x - 9)(x - 1) = 0 \Rightarrow x = 1, 9$.
3. A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.

   BASIC

   Answer: - Required number be 36.
4. A two digit number is such that the product of its digit is 15. If 18 is added to the number, then the digits interchange their places. Find the number.

   Answer: - Required number be 35.
5. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

   Answer: - The required parts are 10 and 6.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. One side of a rectangle exceeds its other side by 2 cm. If its area is 195 cm², determine the sides of the rectangle.

   BASIC

   Ans. 13 cm and 15 cm.

   Solution: - Let one side be x cm and the other side will be (x+2) cm.

   Area of rectangle = x(x+2)

   As per the given condition, $x(x+2) = 195 \Rightarrow x^2 + 2x - 195 = 0 \Rightarrow x^2 + 15x - 13x - 195 = 0$

   $(x+15)(x-13) = 0, \Rightarrow x = -15$ or 13.

   Since the sides of the rectangle cannot be negative. So, sides are 13 cm and (x+2) = 15 cm.

2. The hypotenuse of a right angled triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 17 cm. Find the lengths of these sides. **BASIC**
   Ans. 7cm and 24cm.
   Solution. Let the length of the shorter side be x cm. Then, the length of the longer side = (x + 17) cm
   By Pythagoras theorem,
   $(25)^2 = x^2 + (x+17) \Rightarrow 625 = x^2 + x + 289 + 34x$
   $2x^2 + 34x - 336 = 0 \Rightarrow x^2 + 17x - 168 = 0 \Rightarrow x^2 + 24x - 7x - 168 = 0$
   $(x + 24)(x - 7) = 0 \Rightarrow x = -24$ or 7
   $\Rightarrow x = 7$, as length cannot be negative.
   Length of the shorter side x =7 cm. The length of longer side = x+17 = 7+17 = 24 cm.
3. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B alone to finish the work. Sol. Let B take x days to complete the work. Then A takes (x-6) days to do the same work. **BASIC**
   Answer: - The time taken by B alone be 12 days.
4. Father's age is equal to the square of his son's age. One year ago, he was 8 times as old as his son. Find their present ages.
   Answer: - present age of son = 7 years and present age of father= 49 years.
5. A diplomat has gone on a tour with 2500 for his expenses. Now if he reduces his tour by 15 days, then he can spend 30 more per day during his tour. Find the original duration of his tour.

   CB
   Answer: - original duration of the tour is 25 days.

CASE STUDY BASED QUESTIONS (4 MARKS)

QUESTION 1:-Johan and Jayant are very close friends. They decided to go to Ranikhet with their families in separate cars. Johan's car travels at a speed of x km/hr while Jayant's car travels 5 km/hr faster than Johan's car. Johan took 4 hours more than Jayant to complete the Journey of 400 km.

BASIC

(i) What is the distance covered by Jayant's car in two hours?
(ii) The quadratic equation describing the speed of Johan's car is
(iii) What is the speed of Johan's car (in km/hour)? **OR,**
What is the speed of Jayant's car (in km/hour)?

SOLUTION: - (i) Speed of Jayant's car is (x+5) km/hr.
Distance covered in 2 hours =2(x+5) km
(ii) Time taken by Johan's car to cover 400 km is 400 / x hours.
Time taken by Jayant's car to cover 400 km is 400 / (x+5) hours.
Johan takes 4 hours more than Jayant to cover 400 km
$(400 / x) - (400 / (x + 5)) = 4 \Rightarrow 100 (x+5-x) / x(x+5) \Rightarrow x(x+5) = 500$
$\Rightarrow x^2 + 5x - 500 = 0$.
(iii) The speed of Johan's car is given by
$x^2 + 5x - 500 = 0 \Rightarrow x^2 + 25x - 20x - 500 = 0 \Rightarrow (x + 25)(x - 20) = 0 \Rightarrow x = 20$
OR, Ans. we have, x = 20. Speed of Jayant's car (x+5) km/hr = 25 km/hr

QUESTION 2: - The speed of a motorboat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream then downstream.
BASIC

(ii) If the speed of stream is x km/hr, then find the speed of the motorboat in upstream.

(iii) If the speed of stream is x km/hr, then find the speed of the motorboat in downstream.

(iii) The quadratic equation giving the speed of the current is

(a) $x^2 + 30x - 200 = 0$ (b) $x^2 + 20x - 400 = 0$

(c) $x^2 + 30x - 400 = 0$ (d) $x^2 - 20x - 400 = 0$

OR, what is the speed of the current?

SOLUTION: -

(i) Speed of motorboat = 20 km/hr and, Speed of stream = x km/hr Speed upstream of the motorboat = (20-x) km/hr

(ii) Speed of motorboat = 20 km/hr and, Speed of stream = x km/hr Speed downstream of the motorboat = (20 + x) km/hr

(iii) Ans. (c): Time taken by the motorboat to cover 15 km upstream = 15 / (20-x) hours
Time taken by the motorboat to cover 15 km downstream = 15 / (20+x) hours.

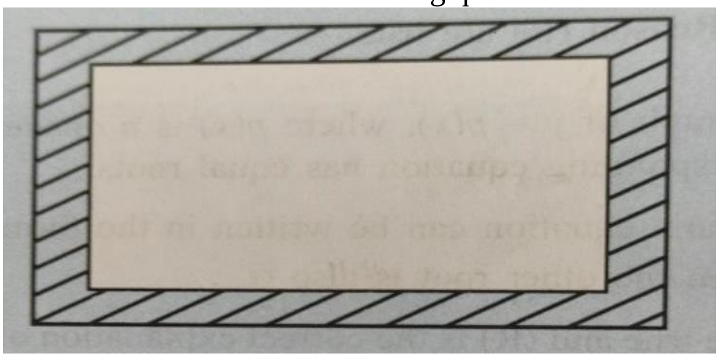
$\Rightarrow (15 / (20-x)) - (15 / (20+x)) = 1 \Rightarrow x^2 + 30x - 400 = 0.$

OR, The speed of the current is given by

$x^2 + 30x - 400 = 0 \Rightarrow x^2 + 40x - 10x - 400 = 0$

$\Rightarrow (x+40)(x-10) = 0 \Rightarrow x=10$

QUESTION 3:-A company is going to make frames as part of a new product they are launching. The frame will be cut out from a piece of steel, and to keep the weight down, the final area of the frame should be ‘k' cm. The inside of the frame has to be 11 cm by 6 cm. Let x be the width of the frame then answer the following questions.



(i) Write the quadratic equation to find the width of frame if the area of piece of steel before cutting be 150 cm^2.

(ii) If the area of frame be 84 cm^2, then find the width of the frame.

(iii) Calculate the final area of the frame, if the width of the frame would be 4 cm.

Answer:- (i) $2x^2 + 17x - 42 = 0$.

(iv) width of the frame = 2 cm.

(iii) area of the frame = 200 cm^2 .

ARITHMETIC PROGRESSIONS

Gist of the topic

1. An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference. The general form of an AP is a, a + d, a + 2d, a + 3d, ……..

2. A given list of numbers $a_1, a_{2,} a_{3,} a_{4,} a_5$……… is an AP, if the differences $a_{2,} - a_1, a_3 - a_{2,}$ . . ., give the same value.

3. In an AP with first term a and common difference d, the nth term (or the general term) is given by a_n = a + (n – 1) d.

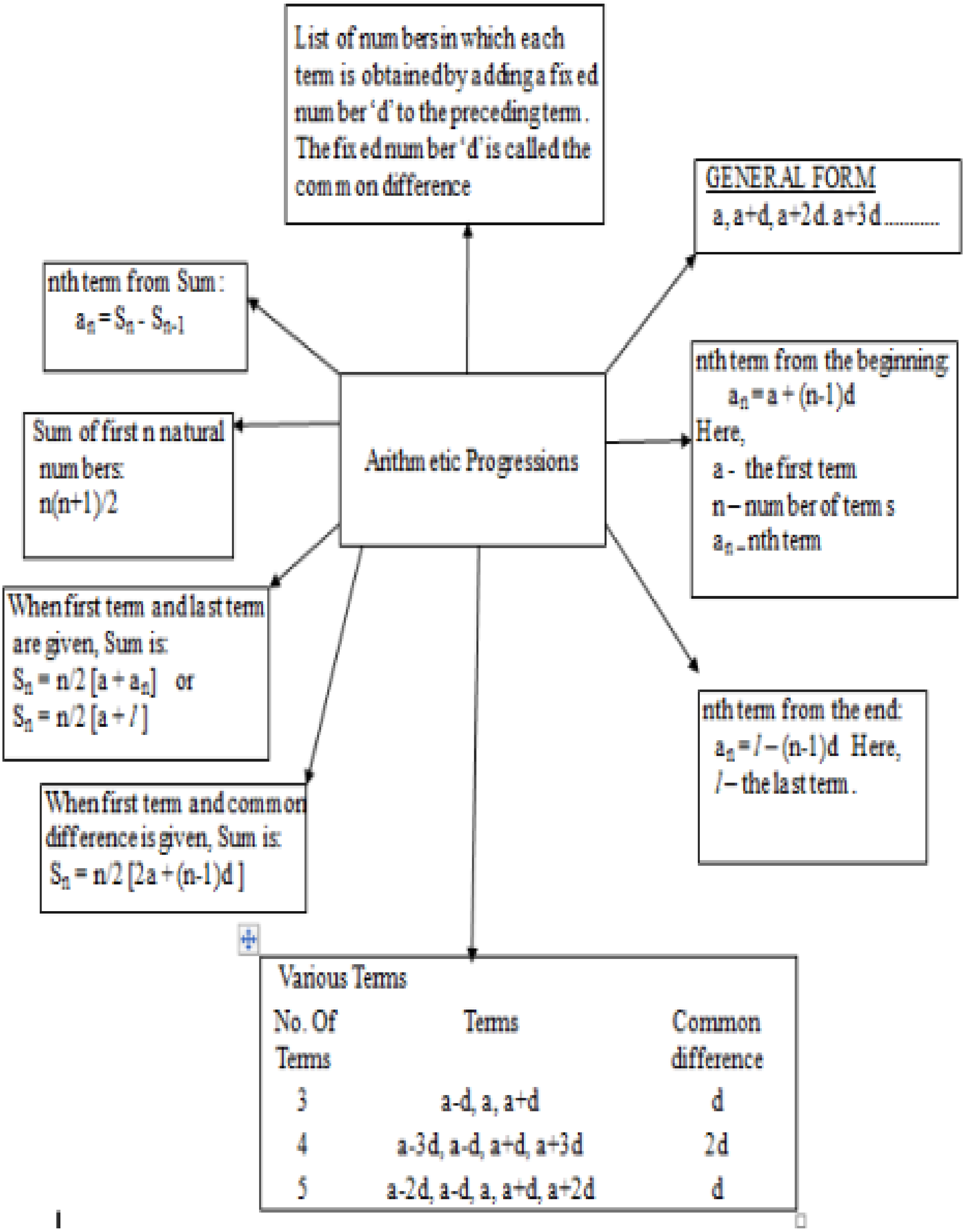
4. The sum of the first n terms of an AP is given by :

$S_n = \frac{n}{2}\{2a + (n - 1)\ d\}$

5. If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by : $S_n = \frac{n}{2}(a + l)$

6. If a, b, c are in AP, then b = $\frac{(a+b)}{2}$ {and b is called the arithmetic mean of a and c}

CONCEPT MAP



MULTIPLE CHOICE QUESTIONS (1 MARK)

1. The nth term of an A.P. 5, 2, -1, -4, -7 … is
   (Basic)

(a) 2n + 5 (b) 2n – 5 (c) 8 – 3n (d) 3n – 8

Answer: (c)

Explanation: Here a = 5, d = 2 – 5 = -3

a_n = a + (n – 1)d

= 5 + (n – 1) (-3)

= 5 – 3n + 3 = 8 – 3n

2. The sum of the first five multiples of 3 is:

(Basic)

(a) 45 (b) 55 (c) 65 (d) 75

Answer: (a) 45

Explanation: The first five multiples of 3 is 3, 6, 9, 12 and 15

a=3 and d=3n=5

Sum, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_5 = \frac{5}{2}[2(3)+(5-1)3]$

$= \frac{5}{2}[6+12]$

$= \frac{5}{2}[18]=5 \times 9= 45$

3. The 10th term from the end of the A.P. 4, 9,14, …, 254 is

(a) 209 (b) 205 (c) 214 (d) 213

(Basic)

Answer: (a)

Explanation: Here l = 254, d = 9 - 4 = 5

∴ 10th term from the end = l – (10 – 1)d

= 254 -9d = 254 - 9(5) = 254 – 45 = 209

4. If the common difference of an AP is 7 then a_{25}-a_{21} is equal to

a) 14 b) 20 c) 28 d) 35

Answer: (d)

5. The value of x for which $2x,\ x + 10,\ and\ 3x + 2$ are the three consecutive terms of an AP

(a) -6 (b)18 (c)6 (d) -18

Answer: (c)

ASSERTION AND REASON BASED QUESTIONS (1MARK)

1. Assertion: Sum of natural number from 1 to 100 is 5050

Reason: Sum of n natural number is = $\frac{\mathrm{nx(n+1)}}{2}$

a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c.) Assertion is true but the reason is false.

d.) Both assertion and reason are false.

Answer: (a)

2. Assertion: Sum of first 10 terms of the arithmetic progression −0.5, −1.0, −1. 5,……… is 31 .

Reason : Sum of n terms of an AP is given aa Sn=n/2[2a+(n−1)d] where a is first term and d common difference.

(Basic)

a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c.) Assertion is true but the reason is false.

d.) Both assertion and reason are false.

Solution: The correct option is A (d) Assertion (A) is false but reason (R) is true.
Assertion, $S_{10} = \frac{10}{2}$ [2(−0.5)+(10−1)(−0.5)] =5[−1−4.5] =5(−5.5)=27.5
Assertion (A) is false but reason (R) is true.
Thus (d) is correct option

SHORT ANSWER TYPE QUESTION (2 Marks)

1. If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.
   (Basic)
   Solution: a_n = a+(n-1)d
   Given, $a_{17} - a_{10} = 7$
   ⟹ (a +16d) − (a+9d) = 7
   ⟹7d = 7
   ⟹d = 1
   Therefore, the common difference is 1.
2. Find the sum: 34+ 32+30+……+10
   (Basic)
   Solution: Given, 34 + 32 + 30 + … + 10, first term, a = 34, d = a2−a1 = 32−34 = −2,
   Let 10 be the n th term of this A.P.,
   a_n = a +(n−1) d,
   10 = 34+(n−1) (−2),
   −24 = (n −1) (−2),
   12 = n −1,
   n = 13,
   Sn = $\frac{n}{2}$ (a + l),
   l = 10,
   Sn = $\frac{n}{2}$ (34 + 10)
   = $\frac{13}{2}$ x 44 = 286
3. The fourth term of an AP is 11 and the eleventh term is 25. Determine the first term and common difference
   (Basic)
   Solution: a + 3d = 11….(1)
   a + 10d = 25…..(2)
   Subtracting equation (1) from equation (2)
   a + 10d – (a + 3d) = 25 – 11,
   7d = 14,
   d = 2,
   Putting value of d=2 in the equation 2,
   a + 10x2 = 25,
   a + 20 = 25,
   a = 25 – 20, a = 5
4. How many numbers of multiples of 4 lie between 10 and 250?
   Answer: n = 60
5. What is the 20th term from the last term of the A.P. 3, 8, 13, …, 253?
   Answer: a_{20} = 158

SHORT ANSWER TYPE QUESTION (3 Marks)

1. Check whether – 150 is a term of the AP: 11, 8, 5, 2 . . .
   (Basic)
   Solution: Given AP: 11, 8, 5, 2, …
   First term, a = 11
   Common difference, d = $a_2 - a_1$ = 8 − 11 = −3
   Let −150 be the nth term of this AP.
   As we know that
   a_n = a + (n − 1) d
   ⇒ -150 = 11 + (n – 1)(-3)
   ⇒ -150 = 11 – 3n + 3
   ⇒ -164 = -3n
   ⇒ n = $\frac{164}{3}$
   Clearly, n is not an integer but a fraction. Therefore, – 150 is not a term of the given AP.
2. How many terms of the AP: 9, 17, 25…. must be taken to get a sum of 636? (Basic)
   Solution: Given that first term, a = 9,
   Common difference, d = 17 - 9 = 8,
   Sum up to nth terms, Sn = 636
   Where, Sn = $\frac{n}{2}$ [2a + (n - 1) d],
   636 =$\frac{n}{2}$ [2 × 9 + (n - 1) 8]
   636 = $\frac{n}{2}$ [18 + 8n - 8],
   636 = $\frac{n}{2}$ [10 + 8n],
   636 = n [5 + 4n],
   636 = 5n + $4n^2$,
   $4n^2$ + 5n - 636 = 0,
   $4n^2$ + 53n - 48n - 636 = 0,
   n (4n + 53) - 12 (4n + 53) = 0
   (4n + 53)(n - 12) = 0
   Either 4n + 53 = 0 or n - 12 = 0
   n = - $\frac{53}{4}$ or n =12
   n cannot be $-\frac{53}{4}$ because the number of terms can neither be negative nor fractional, therefore, n = 12
3. The sum of the first 9 terms of an AP is 171 and the sum of of its first 24 terms is 996.Find the first term and the common difference?
   (Basic)
   Solution: S_9=171, S_{24}= 996,
   $\frac{9}{2}$ [2a + (9- 1) d]=171….(.1),
   $\frac{24}{2}$ [2a + (24- 1) d]= 996……(.2)
   2a+8d= (171x2)/9 = 2a+ 8d= 38…..(3)
   2a+23d= (996x2)/24 = 2a+23d=83……..(4)
   Solving (3) and (4)
   23d-8d= 83-38,
   15d= 45,
   d=3.
   Put d=3 in equation (3)

2a+ 8x3= 38,
2a= 38-24= 14,
a= 7.

4. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, then show that the sum of its first (m + n) terms is zero.
   Answer: Proof
5. In an AP, ratio of 4th term and 9th term is 1:3, find the ratio of 12th and 5th term. .
   Answer: 3:1

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. If the 9th term of an A.P. is zero, prove its 29th term is double the 19th term. (Basic)
   Solution: Given, a_9 = 0
   We know that, the nth term a_n = a + (n – 1)d
   So, a + (9 – 1)d = 0 ⇒ a + 8d = 0 ……(i)
   Now, The 29th term
   a_{29} = a + (29 – 1)d ⇒ a_{29} = a + 28d
   And, a_{29} = (a + 8d) + 20d
   [using (i)] ⇒ a_{29} = 20d ….. (ii)
   Similarly, the 19th term a_{19} = a + (19 – 1)d
   ⇒ a_{19} = a + 18d
   And, a_{19} = (a + 8d) + 10d
   [using (i)] ⇒ a_{19} = 10d …..(iii)
   On comparing (ii) and (iii), a_{29} = 2(a_{19})
   Therefore, the 29th term is double the 19th term.
2. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that the 25th term of the A.P. is zero.
   (Basic)
   Solution: Given,10 times the 10th term of an A.P. is equal to 15 times the 15th term.
   We know that, the nth term an = a + (n – 1)d
   ⇒ 10(a_{10}) = 15(a_{15})
   ⇒10(a + (10 – 1)d) = 15(a + (15 – 1)d)
   ⇒10(a + 9d) = 15(a + 14d)
   ⇒10a + 90d = 15a + 210d
   ⇒5a + 120d = 0
   ⇒5(a + 24d) = 0
   ⇒a + 24d = 0
   ⇒a + (25 – 1)d = 0
   ⇒ a_{25} = 0
   Therefore, the 25th term of the A.P. is zero.
3. Three numbers are in A.P. If the sum of these numbers is 27 and the product 648, find the numbers.
   (Basic)
   Solution: Let the three numbers of the A.P. be a – d, a, a + d
   From the question, Sum of these numbers = 27
   a – d + a + a + d = 27
   ⇒ 3a = 27
   ⇒a = $\frac{27}{3}$ = 9
   Now, the product of these numbers = 648

$\Rightarrow$ (a – d)(a)(a + d) = 648
$\Rightarrow$ a($a^2 - d^2$) = 648
$\Rightarrow$ 9(9^2 – d^2) = 648
$\Rightarrow 9^3 - 9d^2$ =648
$\Rightarrow$729 – 648 = $9d^2$
$\Rightarrow$81 = $9d^2$
$\Rightarrow d^2$= 9
$\Rightarrow$d = 3 or -3 Hence, the terms are 9-3, 9 and 9+3
$\Rightarrow$ 6, 9, 12 or 12, 9, 6 (for d = -3).

4. The sum of the 4th and 8th terms of an A.P. is 24, and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.
   Answer: a= -1/2, d=5/2
5. Find the four numbers in A.P. whose sum is 50 and in which the greatest number is 4 times the least.
   Answer: 5,10,15,20

CASE STUDY BASED QUESTIONS (4 marks)

CASE STUDY QUESTION 1:

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. Based on the above information, answer the following questions:

i. Find the production during the first year.
ii. In which year, the production is 29,200.
(Basic)

Solution: i) a_9 = a +8d =22600…(1),
a_6 = a +5d = 16000….(2) ,
by solving the two equations we get d= 2200
Put the value of d in second equation,
a+ 5x2200 = 16000 ,
a+ 11000 = 16000,
a = Rs 5000
ii) a_n = 29200, a+ (n-1)d = 29200,
5000 + (n-1)2200 = 29200
2200n -2200= 29200 -5000
2200n = 24200 + 2200
n = $\frac{26400}{2200}$= 12

CASE STUDY QUESTION 2: Anuj gets pocket money from his father every day. Out of the pocket money, he saves Rs. 2.75 on first day, Rs. 3 on second day, Rs. 3.25 on third day and so on. On the above information, answer the following questions.

I. what is the amount saved by Anuj on 11th day?
II. what is the amount saved by Anuj in 4 days?
III. what is the amount saved by Anuj on 30th day?
OR
What is the total amount saved by him in the month of June?
(Basic)

Answer: a=2.75d=0.25

(i) a_n=2.75+10x0.25=5.25

(ii) $S_4=\frac{4}{2}$(2x2.75+3x0.25)=12.50

(iii) a_{30}=2.75+29x0.25=10

OR, $S_{30}=\frac{30}{2}$(2x2.75+29x0.25)=191.25

CASE STUDY QUESTION 3: The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(i) If the first circular row has 30 seats, how many seats will be there in the 10th row?

(ii) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

(iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?

Answer: i. 120

ii. 15 OR 750

iii. 110 seats

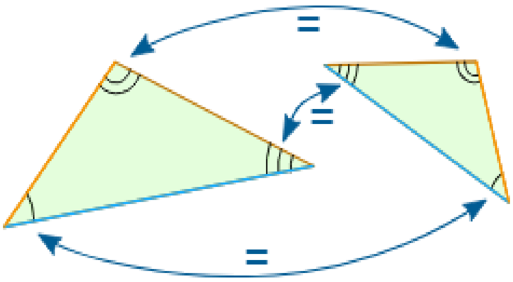
TRIANGLES

GIST OF THE TOPIC

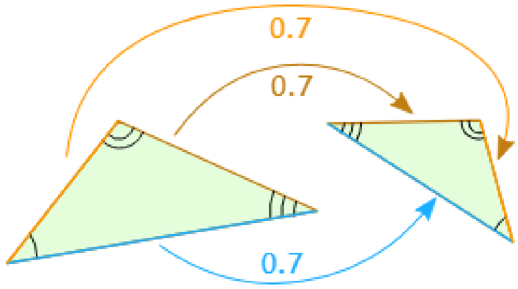
Two triangles are Similar if the only difference is size (and possibly the need to turn or flip one around).

For similar triangles:

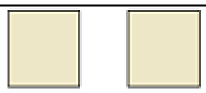
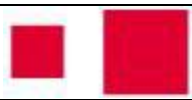
All corresponding angles are equal



All corresponding sides have the same ratio.



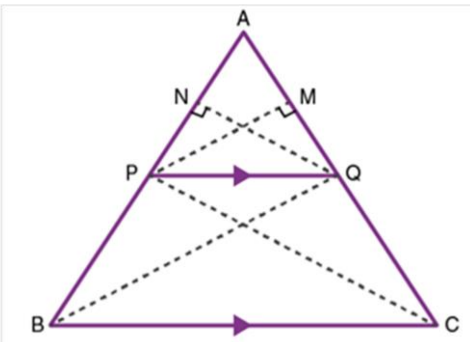
Congruent vs. Similar figures

	Congruent	*Similar*
Angles	Corresponding angles are same.	Corresponding angles are same.
Sides	Corresponding sides are same.	Corresponding sides are proportional.
Example		
Explanation	Both the square have the same angles and same side.	Both the squares have same angles but not the same sides.
Symbols	$\cong$	$\sim$

Basic Proportionality Theorem (Thales Theorem)

According to **Thales theorem: -**

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given, ΔABC in which PQ //BC

To prove: $\frac{AP}{PB} = \frac{AQ}{QC}$

Construction: Join BQ and CP

And draw QN⊥AB and also draw PM⊥AC.

Proof

Now the area of ΔAPQ = $\frac{1}{2} \times AP \times QN$

(Since, area of a triangle= $\frac{1}{2}$× Base × Height)

Similarly, area of ΔPBQ= $\frac{1}{2} \times PB \times QN$

area of ΔAPQ = $\frac{1}{2} \times AQ \times PM$

Also, area of ΔQCP = $\frac{1}{2} \times QC \times PM$

Now, if we find the ratio of the area of triangles ΔAPQ and ΔPBQ, we have

$$\frac{Area\ of\ \Delta APQ}{Area\ of\ \Delta PBQ} = \frac{\frac{1}{2} \times AP \times QN}{\frac{1}{2} \times PB \times QN} = \frac{AP}{PB} - - - - - - - - - -(i)$$

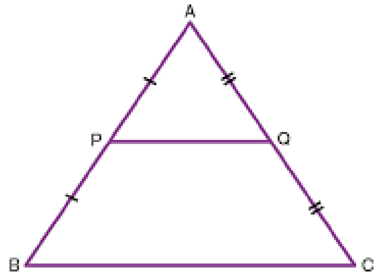
$$\frac{Area\ of\ \Delta APQ}{Area\ of\ \Delta QCB} = \frac{\frac{1}{2} \times AQ \times PM}{\frac{1}{2} \times QC \times PM} = \frac{AQ}{QC} - - - - - - - - - (ii)$$

According to the property of triangles, the triangles drawn between the same parallel lines and on the same base have equal areas.
Therefore, we can say that ΔPBQ and QCP have the same area.
Area of ΔPBQ = area of ΔQCP ………….. (iii)
Therefore, from the equations (i), (ii) and (iii) we can say that,

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Converse of Thales theorem: If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

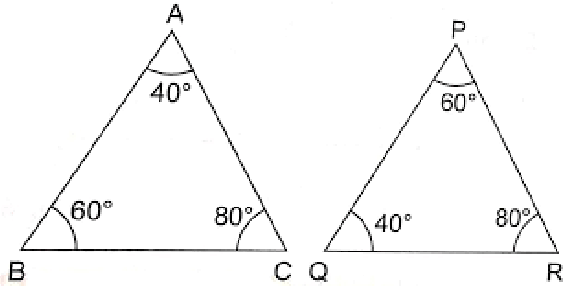
$IF\ \frac{AP}{PB} = \frac{AQ}{QC}, then\ PQIIBC$



(AAA **similarity criterion**) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
IF ∠A = ∠Q =40^O
∠B = ∠P =60^O
∠C = ∠R =80^O
Then,

$$\frac{BC}{PR} = \frac{AC}{QR} = \frac{AB}{PQ}$$

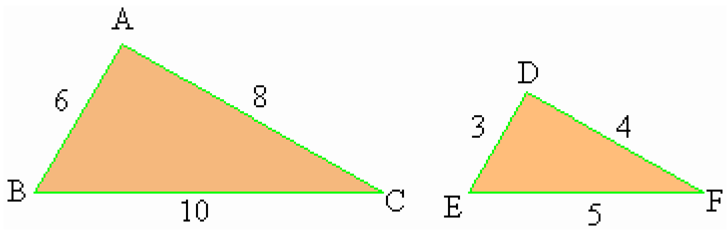
and ΔABC ~ ΔQPR



(SSS **similarity criterion**) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

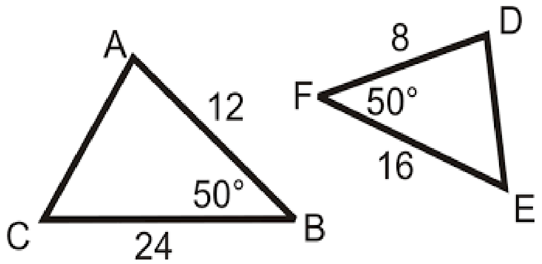
$$IF\ \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{2}{1}$$

Then, ∠A = ∠D
∠B = ∠E
∠C = ∠F
and ΔABC ~ ΔDEF



(SAS similarity criterion)

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.



$\angle B = \angle F = 50^0$

And

$\frac{AB}{DF} = \frac{12}{8} = \frac{3}{2}$ and $\frac{BC}{EF} = \frac{24}{16} = \frac{3}{2}$

And $\Delta ABC \sim \Delta DFE$

Assertion and reason questions(1Mark)

1. Assertion (A): In ABC, D and E intersects AB and AC respectively, such that DE || BC. If AE = 4cm, EC = 3 cm, BD = 6 cm, then DA = 8 cm
   Reason(R): If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.
   (Basic)
   (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
   (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (c) Assertions (A) is true but reason (R) is false.
   (d) Assertions (A) is false but reason (R) is true.
   Ans:- (a) $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{6} = \frac{4}{3} \Rightarrow AD = \frac{24}{3}$ = 8 cm

2. Assertion (A): In ABC, D and E intersects AB and AC respectively, such that DE || BC. Then the value of y is 3, when AD = y cm, DB = (y-2) cm, AE = (y + 3) cm and EC = (y – 1) cm.
   Reason(R): If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

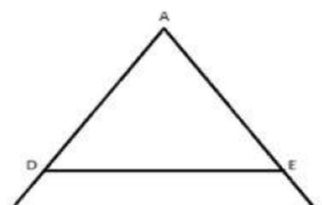
   (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
   (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (c) Assertions (A) is true but reason (R) is false.
   (d) Assertions (A) is false but reason (R) is true.
   Ans: - (a)

MULTIPLE CHOICE QUESTIONS (1 mark)

1. If in △ CAB and △ FED, $\frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$, then:
   (BASIC)
   (A) △ ABC~△ DEF (B) △ CAB~△ DEF
   (C) △ ABC~△ EFD (D) △ CAB~△ EFD
   Answer: (C) △ ABC~△ EFD

2. If in triangles △ABC and △DEF, $\frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{BC}}{\mathrm{FD}}$, then they will be similar, when
(BASIC)
   (A) ∠B = ∠E (B) ∠A = ∠D (C) ∠B = ∠D (D) ∠A = ∠F
   Answer: (C) ∠B = ∠D

3. △ ABC is an acute angled triangle. DE is drawn parallel to BC as shown. Which of the following are always true? (**BASIC**)



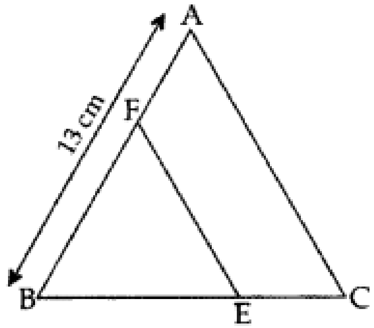
i) △ABC ~ △ ADE
ii) AD/BD= AE/EC
iii) DE= BC/2
(A) Only (i)
(B) (i) and (ii) only
(C) (i), (ii) and (iii)
(D) (ii) and (iii) only
Answer: (B) (i) and (ii) only

4. If Δ ABC and Δ DEF are similar such that 2AB = DE and BC = 8 cm, then Find EF.
(A) 16 cm (B) 12 cm (C) 8 cm (D) 4 cm
Answer: (A) 16 cm

5. If the ratio of the corresponding sides of two similar triangles is 4:5, then the ratio of corresponding altitudes is:
(A) 5:4 (B) 4:5 (C) 8:10 (D) 16:25
Answer: (B) 4:5

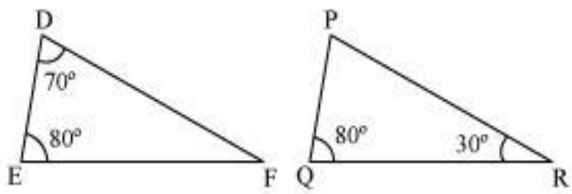
SHORT ANSWER TYPE QUESTIONS (2 marks)

1. In the figure, EF || AC, BC = 12 cm, AB = 13 cm and EC = 3 cm, find AF.
(BASIC)

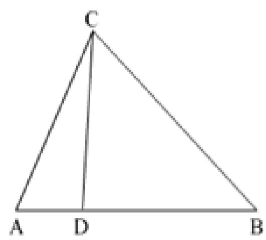


Answer:
BE = BC – EC = 12 – 3 = 9 cm
Let AF = x cm, then BF = (13 – x) cm
In ΔABC, EF || AC … [Given
$\frac{BF}{FA} = \frac{BE}{EC}$
$\frac{13 - x}{x} = \frac{9}{3} \Rightarrow 3x = 13 - x$ (Thales' theorem)
⇒ 4x = 13
⇒ x = 13/4 = 3.25 cm

2. Write the pair of similar triangles in the symbolic form and also write the similarity criterion used by you for answering the question.
(BASIC)
Answer: △DEF ~ △ PQR(ASA)

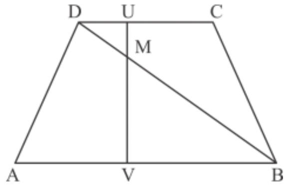


3. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why? **(BASIC)**
Answer: Let the sides of first triangle be *x*, *y* and *z*. As per given information, sides of second triangle will be 3*x*, 3*y* and *z*'.
Since, the perimeter of other triangle is three times the perimeter of first triangle.
⇒$3x + 3y + z' = 3(x + y + z)$
⇒$3x + 3y + z' = 3x + 3y + 3z$
⇒$z' = 3z$
Here, the corresponding three sides of triangle are in proportion.



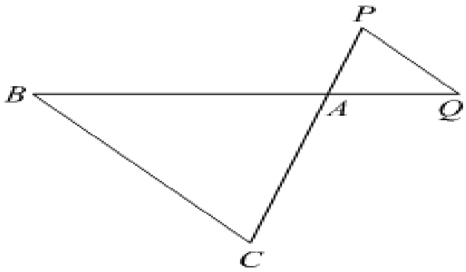
Hence, the two triangles are similar by SSS criteria of similarity.

4. In the given, if ∠ACB = ∠CDA, AC = 6 cm and AD = 3 cm, find BD.
   Answer: BD = 9 cm
5. In the given Figure,
   Prove that △DMU ~ △ BMV.
   Answer: Proof



SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. In the given figure, △ACB ~ △APQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.
   (BASIC)
   Answer: Since, △ACB ~ △APQ



$\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$

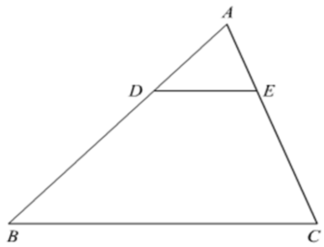
$\frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$

$\frac{AC}{2.8} = \frac{8}{4} \Rightarrow \text{AC} = 5.6 \text{ cm}$

And

$\frac{6.5}{AQ} = \frac{8}{4} \Rightarrow \text{AQ} = 3.25 \text{ cm}$

2. In the given figure, DE || BC such that $AE = \frac{1}{4}AC$. If AB = 6 cm, find AD.
   (BASIC)
   Answer: It is given that, DE || BC & $AE = \frac{1}{4}AC$ and AB = 6 cm.
   We have to find *AD.*
   Since ΔADE ~ ΔABC

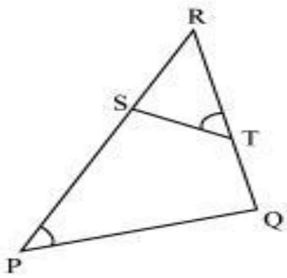


$\frac{\text{AD}}{\text{AB}} = \frac{\text{AE}}{\text{AC}}$

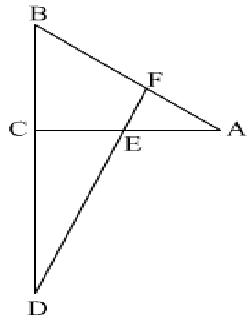
So $\frac{\text{AD}}{6} = \frac{1}{4} \Rightarrow\ 4\ X\ AD = 6\ \Rightarrow\ AD = \frac{6}{4} = \frac{3}{2} \Rightarrow \text{AD} = 1.5 \text{ cm}$

3. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side.
   (BASIC)
   Answer: Proof

4. S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS.
   Show that ΔRPQ ~ ΔRTS.
   Answer: Proof



5. In the given figure, line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point of CA and ∠AEF = ∠AFE. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$.



Answer: Proof

LONG ANSWER TYPE QUESTIONS (5 MARKS)

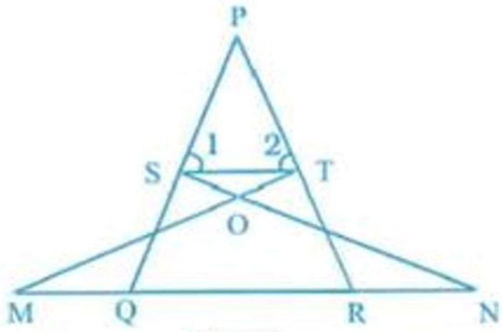
1. If ∠1 =∠2 and Δ NSQ ≅ Δ MTR, then prove that Δ PTS ~ Δ PRQ. **(BASIC)**

Answer: Since, Δ NSQ ≅ Δ MTR

SQ = TR (CPCT) -------(i)

We have PS = PT-------(ii)

and SQ = TR---------(iii)



Now divide (ii) by (iii), we have

$\Rightarrow \frac{PS}{SQ} = \frac{PT}{TR}$

Add both sides by 1, we have

$$\Rightarrow \frac{PS}{SQ} + 1 = \frac{PT}{TR} + 1$$

$$\Rightarrow \frac{PS + SQ}{SQ} = \frac{PT + TR}{TR}$$

$$\Rightarrow \frac{PQ}{SQ} = \frac{PR}{TR}$$

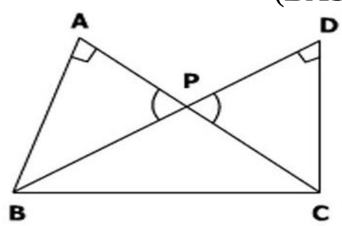
PQ = PR (As SQ = TR from (i))

Now, we can write $\frac{PS}{PQ} = \frac{PT}{PR}$ *and* ∠P = ∠P

Δ PTS ~ Δ PRQ (SAS similar criterion)

2. Two right triangles ABC and DBC are drawn on the same hypotenuse BCand on the same sides of BC. If AC and DB intersect at P, prove that AP x PC = BP x PD. (**BASIC**)

Answer: To prove: AP x PC = BP x PD.



Proof: In AABP and ADCP

∠A = ∠D = 90° (given)

∠APB = ∠DPC (vertically opposite angles)

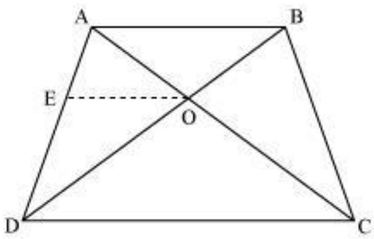
.. Δ ABP ~ Δ DCP (AA similarity axiom)

.. $\frac{AB}{DC} = \frac{BP}{CP} = \frac{AP}{DP}$ (corresponding sides of similar ▲s are proportional) ..(1)

From (1) $\frac{BP}{CP} = \frac{AP}{DP}$

By cross multiplication, BP × DP = AP x PC (proved).

3. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show



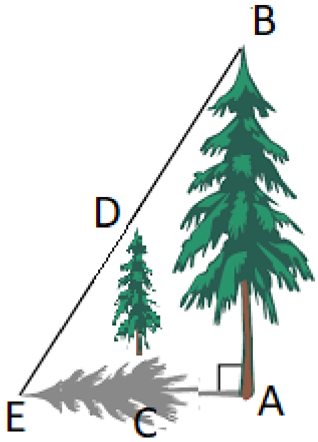
that ABCD is a trapezium.
(**BASIC**)
Answer: proof.

4. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.
Answer: proof

5. If D, E, F are the mid-points of sides BC, CA and AB respectively of a △ABC, then prove ΔABC ~ ΔDEF.
Answer: Proof

CASE STUDY QUESTIONS (4 Marks)

1. In Ravi's school garden. Two trees are standing to each other. The bigger tree is 8 m high, casts a shadow of 6m.
(BASIC)



Answer the following questions:

i) If AB and CD are the two trees and AE is the shadow of the longer tress. Then write the symbolic form of similar triangles.
ii) Write the corresponding ratios of heights of the tress and length of their shadows.
iii) If the ratio of the height of two trees is 3:1, then the shadow of the smaller trees is.
iv) Find the distance of point B from E.

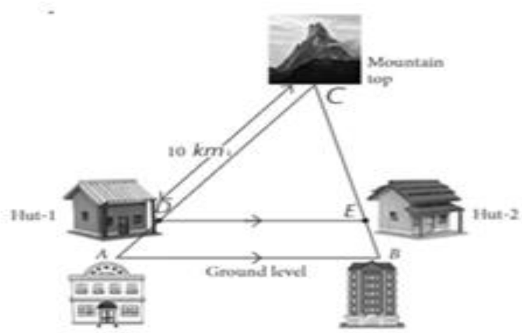
Answer: i) ΔEDC ~ ΔEBA

ii) $\frac{AB}{DC} = \frac{AE}{EC} = \frac{8}{6} = \frac{4}{3}$

iii) $\frac{AB}{CD} = \frac{6}{EC} \Rightarrow \frac{3}{1} = \frac{6}{CE} \Rightarrow CE = 2\text{ m}$

iv) the distance of point B from E = 10 m

2. Mountain Trekking: Two hotels are at the ground level on either side of a mountain. On moving a certain distance towards the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to hut-2 and that of hut-2 to mountain top is 3:6. Based on the above information, answer the following questions.



(i) What is the ratio of the distance between mountain top and Hut-1 to the distance between Hut-1 and hotel A?
(ii) The distance between the hotel A and hut-1 is_________
(iii) If the horizontal distance between the hut-1 and hut-2 is 8 km, then the distance between the two hotels

(iv) If the distance between the mountain top and Hut-2, the distance between the hotel B and hut-2 is__________

Answer:

i) DE//AB, ΔABC ~ ΔDEC (AA – similarity criterion)

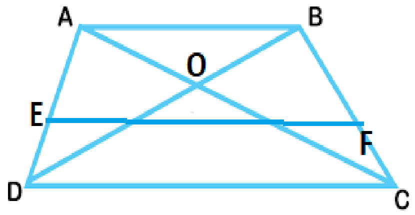
$\frac{DC}{AD} = \frac{CE}{EB} = \frac{6}{3} = \frac{2}{1}$

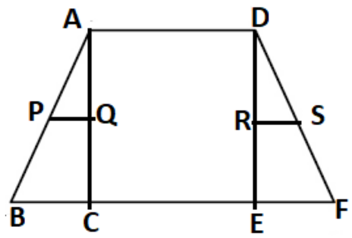
ii) $\frac{10}{AD} = \frac{2}{1} \Rightarrow AD = 5$ km. (The distance between the hotel A and hut-1 is 5 km)

iii) $\frac{DE}{AB} = \frac{CE}{CB} = \frac{6}{9} \Rightarrow \frac{8}{AB} = \frac{2}{3} \Rightarrow AB = 12$ km. (the distance between the two hotels 12 km)

iv) $\frac{DC}{AD} = \frac{EC}{EB} \Rightarrow \frac{10}{5} = \frac{EC}{2} \Rightarrow EC = 4$ km (the distance between the hotel B and hut-2 is 4 km)

3. Class teacher draw the shape of quadrilateral on board. Akansha observed the shape and explored on his notebook in different ways as shown below.





(i) If ABCD is a trapezium with AB// CD, E and F are points on non- parallel sides AD and BC respectively such that EF//AB, then AE/ED = ?

(ii) If OD = 3x- 1, OB = 5x – 3, OC = 2x +1 and AO = 6x – 5, Find the value of x.

(iii) In ΔABC, if PQ// BC and AP= 2.4 cm, AQ = 2cm, QC = 3 cm and BC = 6 cm, Find AB + PQ.

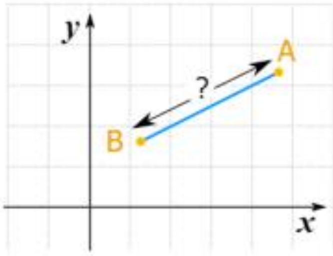
(iv) In Δ DEF, RS// EF, DR=4x – 3, DS = 8x-7, ER = 3x – 1 and FS = 5x -3, Find the value of x.

Answer: i) BF/FC ii) 2 iii)8.4 cm iv)1

COORDINATE GEOMETRY

GIST OF THE TOPIC

DISTANCE FORMULA



Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be two points in the Cartesian plane.

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \;\; AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

DISTANCE OF A POINT P (X, Y) FROM ORIGIN.

Since coordinate of origin is (0,0), Then by applying distance formula,

Distance from P (x, y) is OP $=\sqrt{x^2 + y^2}$

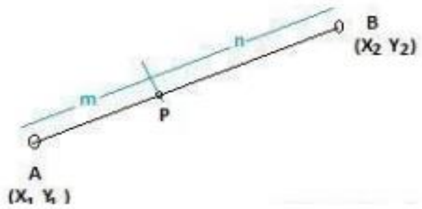
COLLINEAR POINTS:

A given number of points are said to be collinear if they lie on the same line. To prove that three points A, B and C are collinear (using distance formula), we need to prove that sum of any two of the distances AB, BC and AC is equal to the third distance.

SECTION FORMULA

The coordinates of the point P (x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio m:n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



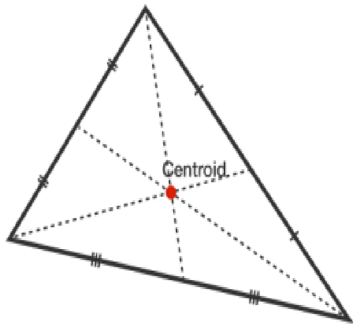
MID POINT FORMULA

If point P(x, y) divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio 1:1 (i.e. P is the mid-point of AB) Then coordinates of point P are given by,

P $(x, y) = \frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}$

CENTROID OF TRIANGLE

The centroid of a triangle is the centre of the triangle. It is referred to as the point of concurrency of medians of a triangle.



The coordinates of the vertices of a triangle are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3), then centroid C (x, y) of given triangle ABC can be find out using,

$$C\ (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

PLEASE KEEP IN MIND

- To check whether the three points form an isosceles triangle or an equilateral triangle, find out the distance between all the three points and if the two sides or three sides are same, we can conclude the answer respectively.
- To calculate the area of any quadrilateral, divide it into two triangles then find the area of individual triangle and add them.
- To check whether the three points A, B and C are collinear either show AB + BC = AC or you can prove it by calculating the area of a triangle formed by these three points is zero.
- Please try to remember all the properties of quadrilaterals and triangles, for the questions which ask to check whether the points form any geometrical shape or not.

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. If $(\frac{a}{3}, 4)$ is the mid-point of the segment joining the points P(-6, 5) and R(-2, 3), then the value of 'a' is
   (Basic)
   (a) 12 (b) -6 (c) -12 (d) -4
   Answer: c
   Explanation:

   Reason: mid-point = $\left(\frac{-6-2}{2}, \frac{5+3}{2}\right) \Rightarrow \left(\frac{a}{3}, 4\right) = (-4, 4)$ $\therefore$ $\frac{a}{3} = -4 \Rightarrow a = -12$

2. The points (1,1), (-2, 7) and (3, -3) are
   (a) vertices of an equilateral triangle (b) collinear

(c) vertices of an isosceles triangle (d) none of these
(Basic)

Answer: b

Explanation: Let A (1, 1), B(-2, 7) and C(3, 3) are the given points, Then, we have

$$AB = \sqrt{(-2-1)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$
$$BC = \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$$
$$\text{and } AC = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Clearly BC = AB + AC. $\therefore$ A, B, C are collinear.

3. Find distance between A (10 cos θ, 0) and B (0, 10 sin θ).
(a) 10 (b) 10 cos θ (c) 10 sin θ (d) none of these
(Basic)
Answer: a
Explanation:

$$AB = \sqrt{(0-10\cos\theta)^2 + (10\sin\theta - 0)^2}$$
$$= \sqrt{100\cos^2\theta + 100\sin^2\theta}$$
$$= \sqrt{100(\cos^2\theta + \sin^2\theta)}$$
$$= \sqrt{100} = 10 \text{ units}$$

4. If the point P (k, 0) divides the line segment joining the points A (2, -2) and B (-7, 4) in the ratio 1:2, then the value of k is:
(a) 1 (b) 2 (c) -1 (d) -2
Answer: (c) -1

5. The point P on x- axis is equidistant from the points A (-1, 0) and B (5, 0) is:
(a) (2, 2) (b) (0, 2) (c) (2, 0) (d) (3, 2)
Answer: (c) (2, 0)

ASSERTION – REASON BASED QUESTION (1 Mark)

1. Assertion: The distance point P (2, 3) from the x-axis is 3.
Reason: The distance from x-axis is equal to its ordinate.
a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c.) Assertion is true but the reason is false.
d.) Both assertion and reason are false.
Answer: a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

2. Assertion: The points (-3,5) and (5,-3) are at different positions in the coordinate plane.
Reason: The position of (x,y) in the Cartesian plane is different from the position of (y, x).
a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c.) Assertion is true but the reason is false.
d.) Both assertion and reason are false.
(Basic)
Answer: The correct option is A

Both assertion and reason are true and the reason is the correct explanation of assertion.
(-3,5) and (5,-3) are different point as (-3,5) lies in II quadrant and (5,-3) lies in IV quadrant.
x≠y,(x,y)≠(y,x)
∴ Assertion and reason are both correct and reason is correct explanation of assertion

SHORT ANSWER TYPE QUESTION (2 MARKS)

1. Find the point on the x-axis which is equidistant from the points (2, -5) and (-2, 3)
(Basic)

Solution: The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Here we are to find out a point on the x-axis which is equidistant from both the points A (−2,5) and B(2,−3)
Let this point be denoted as C(x, y).
Since the point lies on the x-axis the value of its ordinate will be 0. Or in other words we have $y = 0$.
Now let us find out the distances from 'A' and 'B' to 'C'

$$AC = \sqrt{(-2-x)^2 + (5-y)^2}$$
$$= \sqrt{(-2-x)^2 + (5-0)^2}$$
$$AC = \sqrt{(-2-x)^2 + (5)^2}$$
$$BC = \sqrt{(2-x)^2 + (-3-y)^2}$$
$$= \sqrt{(2-x)^2 + (-3-0)^2}$$
$$BC = \sqrt{(2-x)^2 + (-3)^2}$$

We know that both these distances are the same. So equating both these we get,

$$AC = BC$$
$$\sqrt{(-2-x)^2 + (5)^2} = \sqrt{(2-x)^2 + (-3)^2}$$

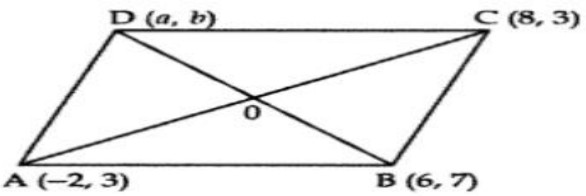
Squaring on both sides we have,

$$(-2-x)^2 + (5)^2 = (2-x)^2 + (-3)^2$$
$$4 + x^2 + 4x + 25 = 4 + x^2 - 4x + 9$$
$$8x = -16$$
$$x = -2$$

Hence the point on the x-axis which lies at equal distances from the mentioned points is $\boxed{(-2,0)}$

2. Find the fourth vertex of parallelogram ABCD whose three vertices are A (- 2, 3), B(6, 7) and C(8,3).
Q
(Basic)

Answer: Diagonals of parallelogram bisect each other, mid-point of AC = mid-point of BD



$$\left(\frac{6+a}{2}, \frac{7+b}{2}\right) = \left(\frac{-2+8}{2}, \frac{3+3}{2}\right)$$

$$\left(\frac{6+a}{2}, \frac{7+b}{2}\right) = (3, 3)$$

$$\frac{6+a}{2} = 3, \qquad \frac{7+b}{2} = 3$$

$$a = 0, \qquad b = -1$$

$$D(0, -1)$$

3. Find the distance of the point P (2, 3) from the x-axis.
   (Basic)
   Answer: In a rectangular coordinate system the distances of a point P(x, y) from the axes are y from x axis and x from y axis. Here the point is P (2, 3) i.e. x=2 & y=3. The distance of the point P (2,3) from the X−axis is 3 units.
4. Let P and Q be the points of trisection of the line segment joining the points A (2, -2) and B (-7,4) such that P is nearer to A. Find the coordinates of P and Q.
   Answer: (-1, 0) and (-4, 2)
5. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3). Hence find m
   Answer: m=0

SHORT ANSWER TYPE QUESTION (3 MARKS)

1. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear
   (Basic)
   Answer: The given points are A (1, 5), B (2, 3) and C (−2, −11).
   Let us calculate the distance: AB, BC and CA by using distance formula.

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5} \text{ units}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2}$$

$$= \sqrt{16+196} = \sqrt{212} = 2\sqrt{53} \text{ units}$$

$$CA = \sqrt{(-2-1)^2 + (-11-5)^2}$$

$$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

$$= \sqrt{5} \times \sqrt{53} \text{ units}$$

From the above we see that: AB+BC is not equal to CA

AB+AC≠BC and BC+AC≠AB and AB+BC≠AC

Hence, the above stated points A (1, 5), B (2, 3) and C (−2, −11) are not collinear.

2. Find a relation between x and y such that the point (x,y) is equidistant from the points (7,1) and (3,5).
   (Basic)

Answer: Let P(x, y) be equidistant from the points A(7,1) and B(3,5).

So, AP = BP

Squaring on both sides, we get

$\Rightarrow (AP)^2 = (BP)^2$

Using, distance formula,

Distance between (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.....(1)

Now,

$\Rightarrow (AP)^2 = (BP)^2$

$\Rightarrow (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$ **[Using eq(1)]**

$\Rightarrow x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

$\Rightarrow -14x + 50 - 2y = -6x + 34 - 10y$

$\Rightarrow -7x + 25 - y = -3x + 17 - 5y$

$\Rightarrow -4x + 8 + 4y = 0$

$\Rightarrow 4x - 4y = 8$

$\Rightarrow 4(x - y) = 8$

$\Rightarrow x - y = 2$

Hence, this is the required relation between x and y.

3. Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right-angled isosceles triangle.
   (Basic)

Answer: Proof

4. If (1,p/3) is the mid-point of the line segment joining the points (2, 0) and (0,2/9), then show that the line 5x + 3y + 2 = 0 passes through the point (- 1, 3p)

Answer: Proof

5. Determine the ratio in which the line 2x + y - 4 = 0 divides the line segments joining A (2, -2) and B (3, 7).

Answer: 2: 9

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. The vertices of quadrilateral ABCD are A (5, -1), B (8,3), C (4, 0) and D (1, -4). Prove that ABCD is a rhombus.
   (Basic)

Answer: The vertices of the quadrilateral ABCD are

A(5, −1), B(8, 3), C(4, 0) and D(1, − 4)

$$\therefore \quad AB = \sqrt{(8-5)^2+(3+1)^2}$$
$$= \sqrt{3^2+4^2} = 5 \text{ units}$$
$$BC = \sqrt{(8-4)^2+(3-0)^2}$$
$$= \sqrt{4^2+3^2} = 5 \text{ units}$$
$$CD = \sqrt{(4-1)^2+(0+4)^2}$$
$$= \sqrt{(3)^2+(4)^2} = 5 \text{ units}$$
$$AD = \sqrt{(5-1)^2+(-1+4)^2}$$
$$= \sqrt{(4)^2+(3)^2} = 5 \text{ units}$$
$$\text{Diagonal } AC = \sqrt{(5-4)^2+(-1-0)^2}$$
$$= \sqrt{1^2+1^2} = \sqrt{2} \text{ units}$$
$$\text{Diagonal } BD = \sqrt{(8-1)^2+(3+4)^2}$$
$$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2} \text{ units}$$

As the length of all the sides are equal but the length of the diagonals are not equal. Hence, ABCD is a rhombus.

2. Find the coordinates of the points of trisection (i.e., Points dividing in three equal parts) of the line segment joining the points A (2, -2) and B (-7, 4).
(Basic)
Answer:

Given:- A line segment joining the points A (2, −2) and B (−7, 4).

Let P and Q be the points on AB such that,

AP = PQ = QB

Therefore,

P and Q divides AB internally in the ratio 1 : 2 and 2 : 1 respectively.

As we know that if a point (h, k) divides a line joining the point (x_1, y_1) and

(x_2, y_2) in the ration m : n, then coordinates of the point is given as-

$$(h, k) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Therefore,

$$\text{Coordinates of P} = \left(\frac{1\times(-7)+2\times 2}{1+2}, \frac{1\times 4+2\times(-2)}{1+2}\right) = (-1, 0)$$

$$\text{Coordinates of Q} = \left(\frac{2\times(-7)+1\times 2}{1+2}, \frac{2\times 4+1\times(-2)}{1+2}\right) = (-4, 2)$$

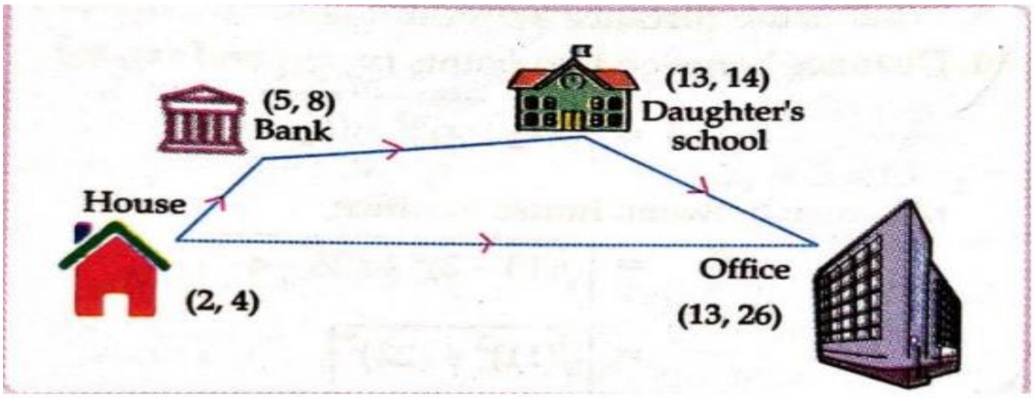
Therefore, the coordinates of the points of trisection of the line segment joining A and B are (−1, 0) and (−4, 2).

3. if A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.
(Basic)
Answer: a=1,b=1; length of each side √10 units

4. Find the centre and radius of the circumcircle (i.e., circumcentre and circum-radius) of the triangle whose vertices are (-2, 3), (2, -1) and (4, 0).
   Answer: Circumcentre of the ΔABC is (3/2,5/2) and Circumradius of ΔABC is $5\sqrt{2}/2$
5. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are (-4, 0) and the origin is the mid-point of the base. Find the co-ordinates of the point P and R
   Answer: Coordinates of P are $(0, 4\sqrt{3})$ or $(0, -4\sqrt{3})$

CASE STUDY-BASED QUESTIONS (4 Marks)

1. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. (Assume that all distances covered are in straight lines and co-ordinates are in km). Answer the following questions:
   a. what is the distance between house and bank?
   b. What is the total distance travelled by Ayush to reach the office?
   c. what is the distance between house and office?
   (Basic)



a.

$$\text{Distance b/w house and bank} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-2)^2 + (8-4)^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= \sqrt{5^2}$$

$$= 5 \text{ km}$$

b.

The distance between Ayush's house and office directly :

D =

$= \sqrt{(13-2)^2 + (26-4)^2}$

$= \sqrt{121 + 484}$

$= \sqrt{605}$

$= 24.6\ Km$

c.

The distance covered by him while going to bank, school and office.

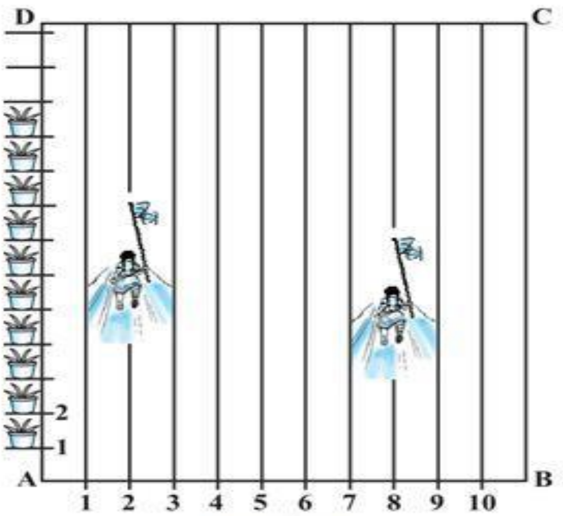
$= \sqrt{(5-2)^2 + (8-4)^2} + \sqrt{(13-5)^2 + (14-8)^2} + \sqrt{(13-13)^2 + (26-14)^2}$

$= \sqrt{9+16} + \sqrt{64+36} + \sqrt{144}$

$= 5 + 10 + 12$

$= 27km$

2. In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th distance AD on the eighth line and posts a red flag.



a. Find the position of green flag

Answer: (2, 25)

It can be observed that Niharika posted the green flag at 14th of the distance AD i.e., (1×100) m = 25 m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

b. Find the position of red flag

Answer: (8,20)

Preet posted red flag at 15th of the distance AD i.e., (1×100) m= 20 m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

c. What is the distance between both the flags?

Answer: $\sqrt{61}$

Distance between these flags by using distance formula = GR

$=\sqrt{(8-2)^2+(25-20)^2}=\sqrt{36+25}=\sqrt{61}$m

d. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

(Basic)

Answer: (5, 22.5)

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points.

Let this point be A (x, y)

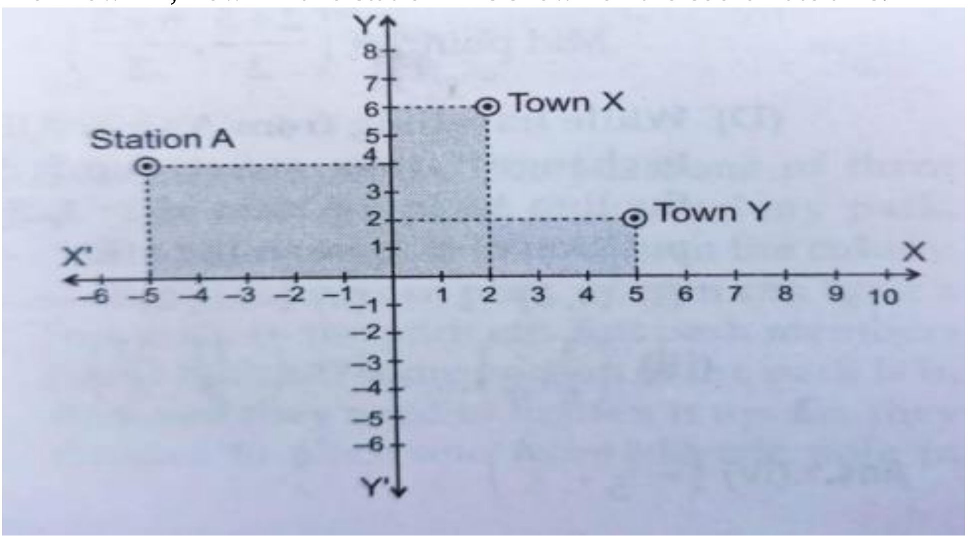
$x=\frac{2+8}{2}$, $y=\frac{25+20}{2}$

x=10/2, y=45/2=22.5

Hence, A (x, y) = (5,22.5)

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

3. Two friends Dalvin and Alice works in the same office in Toronto. In the Christmas vacation, they both decided to go to their home towns represented by Town X and Town Y. Town X and Town Y are connected by trains from the same station C in Toronto. The situation of Town X, Town Y and station A is shown on the coordinate axis.



Based on the given situation, answer the following questions:

i. What is the distance that Dalvin have to travel to reach his hometown X?

Answer: $\sqrt{53}$ units

ii. What is the distance that Alice has to travel to reach her hometown Y?

Answer: $2\sqrt{26}$ units

iii. Now, both of them plan to meet at a place between Town X and Town Y, such that it is a mid-point between both. Calculate the coordinates of the mid-point of X and Y.

Answer: (3.5, 4)

Or

While travelling from A to Y, Alice had to change the train, at a station, it divides the line AY in the ratio of 2: 3, find the position of station on the grid.

Answer: (− 11/5, 24/5)

TRIGONOMETRY

INTRODUCTION TO TRIGONOMETRY

GIST OF THE TOPIC

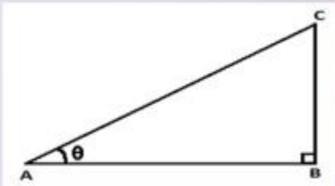
It is the branch of mathematics which deals with the study of relationships between the sides and angles of a triangle.

IMPORTANT CONCEPTS/RESULTS

1. Pythagoras Theorem: In right angled triangle ΔABC, right angled at B:
   $AC^2 = AB^2 + BC^2$

2. Trigonometric ratios:

- Trigonometric ratio of an acute angle of a right angled triangle :-
  Let ABC be right angled at B and <CAB = θ be an acute angle, then



- $\sin\theta = \frac{\text{opposite side of }\theta}{\text{hypotenuse}} = \frac{BC}{AC}$ and $\text{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side of }\theta} = \frac{AC}{BC}$
- $\cos\theta = \frac{\text{adjacent side of }\theta}{\text{hypotenuse}} = \frac{AB}{AC}$ and $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side of }\theta} = \frac{AC}{AB}$
- $\tan\theta = \frac{\text{opposite side of }\theta}{\text{adjacent side of }\theta} = \frac{BC}{AB}$ and $\cot\theta = \frac{\text{adjacent side of }\theta}{\text{opposite side of }\theta} = \frac{AB}{BC}$

Relation between Trigonometric Ratios

Reciprocal Relation :-

- $\text{Sin}\,\theta = \frac{1}{\text{cosec}\theta} \Rightarrow \text{cosec}\theta = \frac{1}{\text{Sin}\,\theta} \Rightarrow \text{Sin}\,\theta\,.\,\text{cosec}\theta = 1$
- $\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta} \Rightarrow \cos\theta\,.\,\sec\theta = 1$
- $\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta} \Rightarrow \tan\theta\,.\,\cot\theta = 1$

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES:

θ	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\mathrm{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Trigonometric Identities:

$\sin^2\theta + \cos^2\theta = 1 \rightarrow 1 - \sin^2\theta = \cos^2\theta \rightarrow 1 - \cos^2\theta = \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \rightarrow \sec^2\theta - 1 = \tan^2\theta \rightarrow \sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \mathrm{cosec}^2\theta \rightarrow \mathrm{cosec}^2\theta - 1 = \cot^2\theta \rightarrow \mathrm{cosec}^2\theta - \cot^2\theta = 1$

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. If cos θ = 1/2 then cosec θ is:

   a) 2 b) 1/√3 c) 2/√3 d) √3/2

   (Basic)

   Sol: cos θ = 1/2 = cos 60°
   (as per table) θ = 60°
   cosec θ = cosec 60° = 2/√3
   (option c is the correct answer)

2. Sec^2 45° – Tan^2 45° =?

   a) 0 b) 1 c) - 1 d) √2

   (Basic)

   Sol: Sec^2 45° – Tan^2 45° =1
   (according to identity 1+ $Tan^2\theta$ = Sec^2 θ)
   (option b is the correct answer)

3. cos1°· cos2°. cos3° ………cos99°. cos100° = ?

   (Basic)

   a) 1 b) -1 c) 0 d) cannot be determined

   Sol: cos1°. cos2°. cos3° ………cos99°. cos100°
   = cos1°. cos2°. cos3° …… cos90° …cos99°. cos100°
   = 0 (cos90° =0 as per table)
   (option c is the correct answer)

4. If 4tanA=3, then $(4\sin A - \cos A)/(4\sin A + \cos A)$

   (a) 2/3 (b) 1/3 (c) 1/2 (d) ¾.

   Answer: (option c is the correct answer)

5. The value of (1-cos θ) (1+cos θ) cosec θ is

   a) $cot\theta$ b) $sin\theta$ c) $cosec\theta$ d) $cos\theta$.

   Answer: (option b is the correct answer)

ASSERTION – REASON BASED QUESTIONS (1 MARK)

1. Statement-1 (Assertion): For any acute angle θ, the value of sin θ cannot be greater than1 Statement-2 (Reason): Hypotenuse is the longest side in any right-angled triangle.
   a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
   b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
   c.) Assertion is true but the reason is false.
   d.) Both assertion and reason are false.
   (Basic)
   Answer: Both statements are true and statement-2 is the correct explanation for statement-1, because sinθ = (Perpendicular / Hypotenuse) < 1

2. Statement-1 (Assertion): For $0 \leq \theta \leq 90o$, $sec\ x + cos\ x \geq 2$
   Statement-2 (Reason): For any x > 0, $x + 1\ x \geq 2$
   a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
   b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
   c.) Assertion is true but the reason is false.
   d.) Both assertion and reason are false.
   Answer: option (a) is correct

SHORT ANSWER TYPE QUESTIONS (2marks)

1. If cot θ = 7/8 then evaluate $\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$$\frac{(1-\sin\theta)(1+\sin\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} \qquad [(a+b)(a-b) = a^2 - b^2]$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta \qquad (\sin^2\theta + \cos^2\theta = 1\ \&\ \cot\theta = \cos\theta / \sin\theta)$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

(Basic)

2. If sec θ = 13 /12 find all other trigonometric ratios.
   (Basic)
   Sol: sec θ = $\frac{\text{Hypotenuse}}{\text{adjacent side}} = \frac{13}{12}$
   Hypotenuse = 13x
   adjacent side = 12x
   hypotenuse^2 = opposite side^2+ adjacent side^2
   $(13x)^2$ = opposite side^2+ $(12x)^2$
   $169x^2$ = opposite side^2+ $144x^2$

$169x^2 - 144x^2$ = opposite side^2
$25x^2$ = (opposite side)^2
5x = opposite side

$$\sin\theta = \frac{opposite side}{hypotenuse} = \frac{5x}{13x} = \frac{5}{13} \qquad \cos\theta = \frac{adjacent side}{hypotenuse} = \frac{12x}{13x} = \frac{12}{13}$$

$$\tan\theta = \frac{opposite side}{adjecent side} = \frac{5x}{12x} = \frac{5}{12} \qquad \cos ec\theta = \frac{hypotenuse}{Opposite side} = \frac{13x}{5x} = \frac{13}{5}$$

$$\cot\theta = \frac{adjecent side}{opposite side} = \frac{12x}{5x} = \frac{12}{5}$$

3. Evaluate $\dfrac{\sin 30^0 + \tan 45^0 - \cos ec60^0}{\sec 30^0 \cos 60^0 + \cot 45^0}$

Sol : $\dfrac{\sin 30^0 + \tan 45^0 - \cos ec60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0} = \dfrac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} - 1} = \dfrac{\sqrt{3} + 2\sqrt{3} - 4}{4 + \sqrt{3} - 2\sqrt{3}} = \dfrac{3\sqrt{3} - 4}{4 - \sqrt{3}}$

(Basic)

4. If in a triangle ABC right angled at B , AB =6 units and BC=8 units then the value of the $sinA\ cosC + cosAsinC$
   Answer: 1
5. If sin (A – B) = 1/2 and cos (A + B) = 1/2 ,then find the value of A and B.
   Answer: A = 45°& B=30°

SHORT ANSWER TYPE QUESTIONS (3marks)

1. If 7 sin^2 A + 3 cos^2 A = 4 , then show that tan A = 1/√3
   (Basic)
   Sol: Given 7 sin^2 A + 3 cos^2 A =4
   ⟹4sin^2A+3(sin^2A+cos^2A)=4
   ⟹4sin^2A+3=4
   ⟹4sin^2A=1
   ⟹sin^2A=1/4
   ⟹sinA=1/2
   ⟹A=30∘
   tanA=tan30∘=1/√3

2. If sin2x= sin30° cos60°+sin60° cos30°, then find the value of x
   (Basic)

Solution

Correct option is A)

sin2x = sin60cos30 − cos60sin30

$\sin 2x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$\sin 2x = \frac{3}{4} - \frac{1}{4}$

$\sin 2x = \frac{1}{2}$

sin2x = sin30

2x = 30

x = 15°

3. If $sec\theta + tan\theta = p$ then find the value of cosecθ.
(Basic)

Sol: $(p^2 + 1)/p^2 - 1)$

4. Prove that $(\sec\theta + tan\theta)(1 - sin\theta) = cos\theta$
5. Prove that $(1+secA)/secA = sin^2A/(1-cos\theta)$

LONG ANSWER TYPE QUESTIONS (5 marks)

1. Prove that $3(\sin x - \cos x)^4 + 4(\sin^6 x + \cos^6 x) + 6(\sin x + \cos x)^2 = 13$
(Basic)

Sol:

Consider $4(\sin^6 x + \cos^6 x)$

$= 4\left[(\sin^2 x)^3 + (\cos^2 x)^3\right]$

$= 4\left[(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x)\right]$

$= 4\left[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x - 2\sin^2 x\cos^2 x\right]$

$= 4\left[1 - 3\sin^2 x\cos^2 x\right]$

$= 4 - 12\sin^2 x\cos^2 x$ ....(1)

$6[\sin x + \cos x]^2$

$= 6\left[\sin^2 x + \cos^2 x + 2\sin x\cos x\right]$

$= 6[1 + 2\sin x\cos x]$

$= 6 + 12\sin x\cos x$ ......(2)

$$3(\sin x - \cos x)^4$$

$$= 3\left[(\sin x - \cos x)^2\right]^2$$

$$= 3\left[\sin^2 x + \cos^2 x - 2\sin x\cos x\right]^2$$

$$= 3[1 - 2\sin x\cos x]^2$$

$$= 3\left[1 - 4\sin x\cos x + 4\sin^2 x\cos^2 x\right]$$

$$= 3 - 12\sin x\cos x + 12\sin^2 x\cos^2 x \quad .....(3)$$

Adding (1), (2) and (3) we get

$$3(\sin x - \cos x)^4 + 4\left(\sin^6 x + \cos^6 x\right) + 6[\sin x + \cos x]^2$$

$$= 3 - 12\sin x\cos x + 12\sin^2 x\cos^2 x + 4 - 12\sin^2 x\cos^2 x + 6 + 12\sin x\cos x$$

$$= 13$$

Hence proved

2. Evaluate $4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$ (Basic)

Sol.: We have,

$$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ \quad ......(1)$$

Now,

$$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2},\ \cos 45^\circ = \frac{1}{\sqrt{2}},\ \tan 60^\circ = \sqrt{3},\ \tan 45^\circ = 1$$

So by substituting above values in equation (1)

We get,

$$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$$

$$= 4\left(\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right) - 3\left(\left(\sqrt{3}\right)^2 - 1^2\right) + 5\times\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 4\left(\frac{\left(\sqrt{3}\right)^4}{2^4} + \frac{\left(\sqrt{3}\right)^4}{2^4}\right) - 3(3-1) + 5\times\frac{1^2}{\left(\sqrt{2}\right)^2}$$

$$= 4\left(\frac{9}{16} + \frac{9}{16}\right) - 3(2) + 5\times\frac{1}{2}$$

$$= 4\left(\frac{9+9}{16}\right) - 6 + \frac{5}{2}$$

$$= 4\left(\frac{18}{16}\right) - 6 + \frac{5}{2}$$

$$= 4\left(\frac{9}{8}\right) - 6 + \frac{5}{2}$$

$$= \frac{36}{8} - 6 + \frac{5}{2}$$

$= \frac{9-12+5}{2}$

$= \frac{14-12}{2}$

$= \frac{2}{2}$

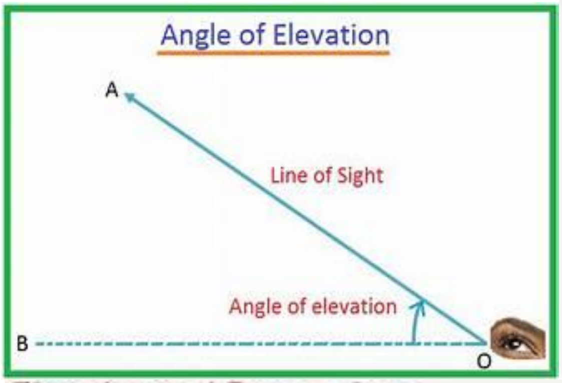
$= 1$

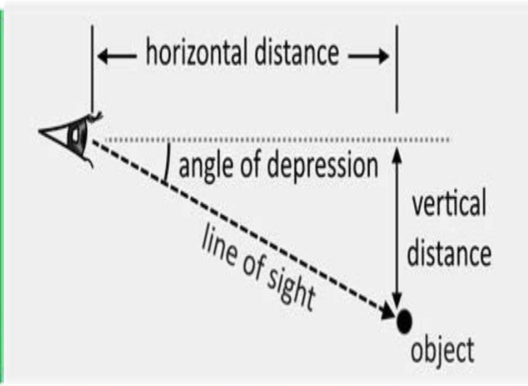
3. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$
   (Basic)
   Answer: $\tan(\theta) + \sin(\theta) = m$
   $\tan(\theta) - \sin(\theta) = n$
   $m^2 - n^2 = [\tan(\theta) - \sin(\theta)]^2 - [\tan(\theta) + \sin(\theta)]^2$
   $\Rightarrow (m+n)(m-n)$
   $= (\tan(\theta) - \sin(\theta) + \tan(\theta) + \sin(\theta))(\tan(\theta) + \sin(\theta) - \tan(\theta) + \sin(\theta))$
   $[\because a^2 - b^2 = (a-b)(a+b)]$
   $\Rightarrow m^2 - n^2 = (2\tan(\theta))(2\sin(\theta)) \Rightarrow m^2 - n^2 = 4(\tan(\theta))(\sin(\theta))$
   $\sqrt{mn} = \sqrt{(\tan(\theta) + \sin(\theta))(\tan(\theta) - \sin(\theta))}$
   $\because [(a-b)(a+b) = a^2 - b^2]$
   $\Rightarrow \sqrt{mn} = \sqrt{\tan^2(\theta) - \sin^2(\theta)}$
   $\Rightarrow \sqrt{mn} = \sqrt{\frac{\sin^2(\theta)}{\cos^2(\theta)} - \sin^2(\theta)} \because \left[\tan(x) = \frac{\sin(x)}{\cos(x)}\right]$
   $\Rightarrow \sqrt{mn} = \sqrt{\frac{\sin^2(\theta)[1 - \cos^2(\theta)]}{\cos^2(\theta)}}$
   $\Rightarrow \sqrt{mn} = \sqrt{\tan^2(\theta)\sin^2(\theta)} \because \left[\tan(x) = \frac{\sin(x)}{\cos(x)}, \sin^2(x) + \cos^1(x) = 1\right]$
   $\Rightarrow mn = \tan(\theta)\sin(\theta)$
   $\Rightarrow m^2 - n^2 = 4\sqrt{mn}$
4. RPQ is a right-angled triangle at Q. If PQ = 5 cm and RQ = 10 cm, find
   (i) $\sin^2 P$
   (ii) $\cos^2 R$ and tan R
   (iii) sin P x cos P
   (iv) $\sin^2 P - \cos^2 P$
   Answer- (i) 4/5 ii) 4/5 , 1/2 (iii) 2/5 (iv) 3/5
5. If sin (A+B) =1 and tan (A+B) = 1/√3. Find the value of
   (i) tan A + cot B
   (ii)sec A + cosec B
   Answer-(i) 2√3, (ii) 0

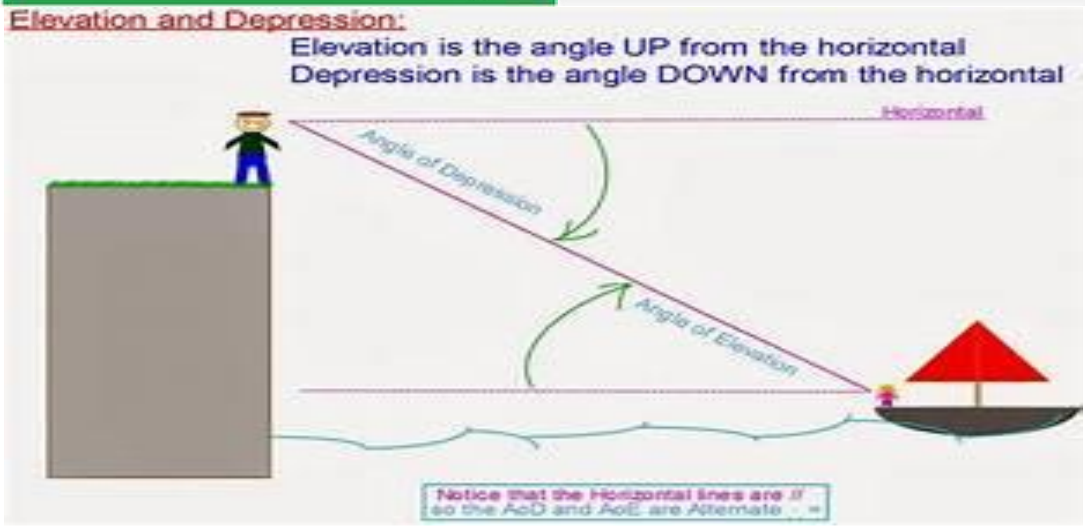
SOME APPLICATIONS OF TRIGONOMETRY

GIST OF THE TOPIC

1. The line drawn from the eye of an observer to a point in the object where the person is viewing is called the line of sight.
2. The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.
3. The angle formed by the line of sight with the horizontal when the object is below the horizontal level is called the angle of depression.
4. The height of an object or distance between distant objects can be determined with the help of trigonometry ratios.







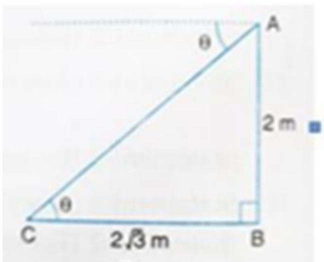
MULTIPLE CHOICE QUESTION (1 Mark)

1. In the given figure point C is observed from point A. The angle of depression is

   A. 60° B. 30°

   C.45° D.75°

   (Basic)

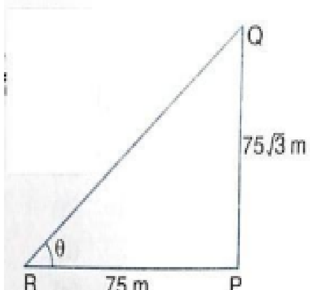
   Sol. (B) Let θ be the angle of depression of point C from A.

   In rt.ΔABC

   $\tan\theta = AB/BC$

   $\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ =tan30°

2. The measure of the angle of elevation of the top of the tower $75\sqrt{3}$ m high from a point at a distance

of 75 m from the foot of the tower in a horizontal plane is

A. 60° B. 30°

C.45° D.90°

(Basic)

Soln. (A) Let PQ be a tower of height $75\sqrt{3}$ m and let θ be the angle of elevation of its top Q from a point R at a distance of 75 from the tower.

In right ΔRPQ ,

we obtain $\tan\theta = PQ/PR$ =>$\tan\theta = 75\sqrt{3}/75 = \sqrt{3}$

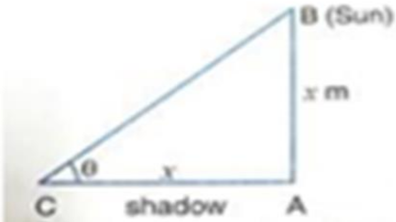
$\tan\theta = \tan 60°$

3. If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of sun's altitude is

A.45° B.60°

C. 30° D. 75°

(Basic)



Soln. (A) Let AB be the vertical pole of height x m and let AC be the length of its shadow when the angle of elevation of sun is θ. It is given that AC= x m.

In Δ ACB,

Tan θ = AC/AB = x/x =1=tan 45°

=>θ=45°

Sun's altitude is 45°

4. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower is

a) $50\sqrt{3}$ b) $150\sqrt{3}$ c) $150\sqrt{2}$ d) 75

Ans - (b)

5. The angle of elevation of the top of a tower from a point on the ground 30 m away from the foot of the tower is 30°. The height of the tower is

a) 30 m b) $10\sqrt{3}$ m c) 20 m d) $10\sqrt{2}$ m

Ans - (b

ASSERTION-REASON BASED QUESTIONS (1 MARK)

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason(R).

Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion(A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c)Assertion (A) is true but reasons (R) is false.

(d) Assertion (A) is false but reasons(R) is true.

1. Assertion: If shadow of pole is $1/\sqrt{3}$ of its height, then the altitude of the sun is 60°

Reason: If the sun's altitude is 45° , then the shadow of a vertical pole is same as height.

(Basic)

Soln. (B) Let PQ be height of the pole =h and RP=length of shadow = $h\sqrt{3}$.

sun's altitude =θ

Then tanϑ= h/h√3 = 1/√3

=>tanϑ = tan 60°

=> ϑ=60°

Let PQ be height of the pole =h and RP be length of shadow

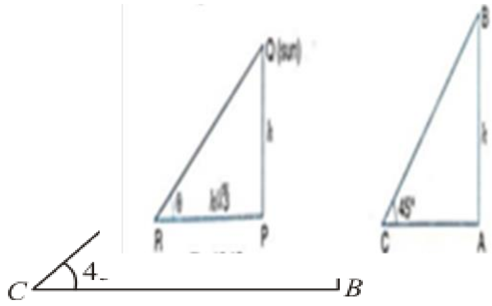
when. Sun's altitude is 45°

In rt.Δ BAC

tan 45° = AB/AC

=> 1 = h/AC

=> AC =h



Both statements are true but statement 2 is not the correct reason of statement 1.

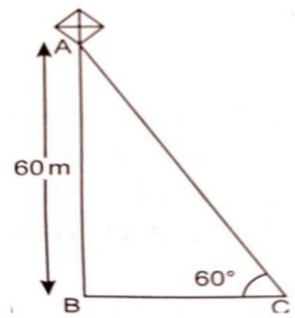
2. Assertion: The line of sight is the line drawn from the eye of an observer to the point of the object viewed by the observer.

   Reason: Trigonometric ratios are used to find height or length of an object or length of an object or distance between two distant objects

   Answer: (b)both the statement and the reasoning are true, but it is not a correct explanation of the statement.

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string. (Basic)

   Solution - The height of the kite is 60 m.

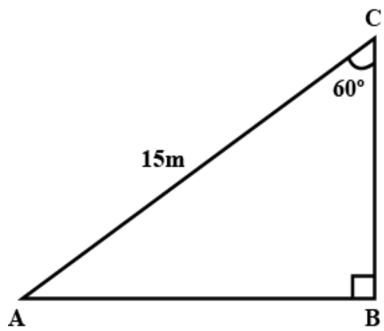
   In $\Delta ABC,$

   AB/AC = sin60°

   $\frac{60}{AC} = \frac{\sqrt{3}}{2}$

   AC = 40√3 m.

2. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall. (Basic)

   Solution -Let the length of the ladder be =15m (hypotenuse)

   The angle between the ladder and the wall ∠BCA=60∘

   Angle between ladder and the ground

   ∠CAB=90∘−60∘=30∘

   Height of the wall =BC

   sin30∘=BC/15 ⇒1/2=BC/15 ⇒BC=15/2=7.5m

3. The height of the tower is 10 m. What is the length of its shadow when sun's altitude is 45°? (Basic)

   Solution -Let BC be the length of shadow is x m

   Given that: Height of tower is 10 meters and altitude of sun is 45°

   Here we have to find length of shadow.

   So we use trigonometric ratios.

   In a triangle ABC ,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow 1 = \frac{10}{x}$

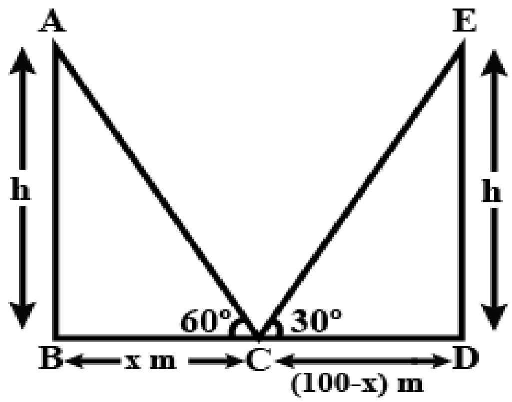
$\Rightarrow x = 10$

Hence the length of shadow is 10 m.

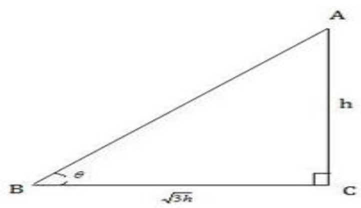
4. An observer, 1.7 m tall, is 20√3 m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.
   Answer: 21.7 m
5. The top of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of line joining their feet, then find x : y.
   Answer: 1: 3

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Two pillars are of equal height on either side of a road which is 100 m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars and height of each pillar
   (Basic)
   Solution- Let AB and ED be two pillars each of height h metres Let C be a point on the road BD such that
   BC = x metres
   Then CD = (100 - x) metres
   Given ∠ACB=60° and ∠ECD=30°



   In ΔABC,
   tan60°=AB/BC
   ⇒√3=h/x
   ⇒h=√3x..........(i)
   In ΔECD,
   tan30°=ED/CD
   ⇒1/√3=h/(100−x)
   ⇒h√3=100−x...........(ii)
   Subst. the value of h from (i) in (ii) we get
   √3.x=(100−x)/√3 ⇒3x=100−x ⇒4x=100 ⇒x=25m
   h=(√3×25)=25×1.732m=43.3m
   The required point is at a distance of 25 m from the pillar B and the height of each pillar is 43.3 m.
2. Find the angle of elevation of the Sun when the shadow of a pole h m high is √3h m long. (Basic)
   Solution - Let the angle of elevation of the Sun is θ.
   Given, height of pole = h
   Now, in ΔABC,
   tan θ=AC/BC=h/√3h
   tan θ=1/√3=tan 30°
   θ=30°
   Hence, the angle of elevation of the Sun is 30°.



3. From the top of a 300 metre high light-house, the angles of depression of two ships, which are due south of the observer and in a straight line with its base, are 60° and 30°.

Find their distance apart?

(Basic)

Solution - Height of tower= AB=300m

Let distance between boats= DC=y m

In △ABC,

$\tan 60^o = \frac{300}{x}$

$\Rightarrow x = \frac{300}{\sqrt{3}}$

$\Rightarrow x = 100\sqrt{3}$m

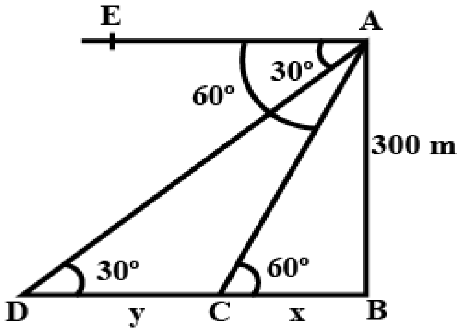
In △ABD,

$\tan 30^o = \frac{300}{x+y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{300}{x+y}$

$\Rightarrow 100\sqrt{3} + y = 300\sqrt{3} \Rightarrow y = \sqrt{3}(300-100)$

$y = 200\sqrt{3}$m

$y = 346.4$m



4. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower
   Answer: the required distance and height are $10\sqrt{3}$m and 10 m, respectively.
5. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower
   Answer: the height of the tower is $25\sqrt{3}$m

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60∘. When he moves 30 metres away from the bank, he finds the angle of elevation to be 30∘. Find the height of the tree and the width of the river. [Take $\sqrt{3}$=1.732]

(Basic)

Solution:

Let AB be the tree and AC be the river. Let C and D be the two positions of the person.

Then, $\angle ACB=60°, \angle ADB=30°, \angle DAB=90°$ and

CD = 30 m

Let AB = h metres and AC = x metres.

From right ΔCAB, we have

$\frac{AC}{AB} = \cot 60° = \frac{1}{\sqrt{3}}$

$\frac{x}{h} = \frac{1}{\sqrt{3}}$

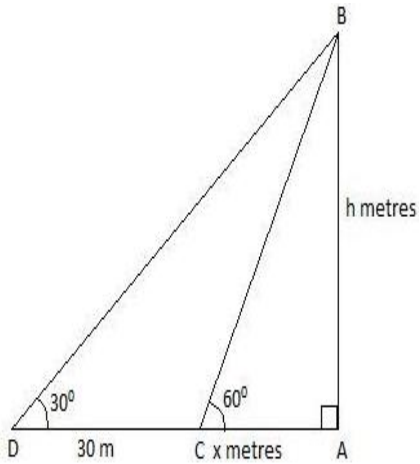
$x = h\sqrt{3}$ .... (i)

From right ΔDAB, we have

$\frac{AD}{AB} = \cot 30° = \sqrt{3}$

$\frac{x+30}{h} = \sqrt{3}$

$x = \sqrt{3}h - 30$ .... (ii)



Equating the values of x from (i) and (ii), we get

$h/\sqrt{3} = \sqrt{3}\,h - 30$

$h = 3h - 30\sqrt{3}$

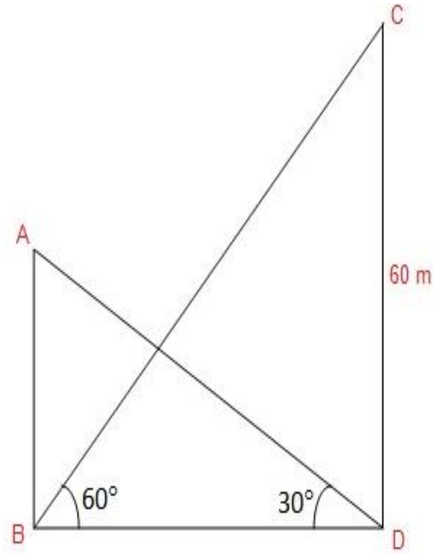
$2h = 30\sqrt{3}$

$h=15\sqrt{3} = 15\times1.732 = 25.98$

Putting $h=15\sqrt{3}$ (i), we get $x=\frac{15\sqrt{3}}{\sqrt{3}}=15$.

Hence, the height of the tree is 25.98 m and the width of the river is 15 metres.

2. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.
(Basic)



Solution: Given, height of tower CD = 60 m

Let the height of the building, AB = h

in right angled triangle BDC,

$$\tan 60^\circ = \frac{CD}{BD}$$

$$\sqrt{3} = \frac{60}{BD}$$

$$BD = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}\,m$$

In right triangle ABD

$$\tan 30^\circ = \frac{AB}{BD}$$

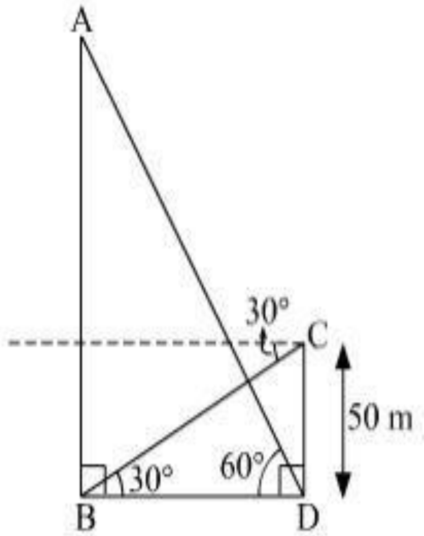
$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

∴ h = 20 m

Thus the height of the building is 20 m.

3. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower to the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill. (Basic)

Solution:



Let AB be the hill and CD be the tower.

Angle of elevation of the hill at the foot of the tower is 60°.i.e.,∠ADB=60° and the angle of depression of the foot of hill from the top of the tower is 30°,

i.e., ∠CBD=30°.

Now in right angled ΔCBD:

$\tan 30^\circ=\frac{CD}{BD}$

$\Rightarrow BD=CD/\tan 30^\circ$

$\Rightarrow BD=\frac{50}{\frac{1}{\sqrt{3}}}$

$\Rightarrow BD=50\sqrt{3}$ m

In right ΔABD:

$\tan 60^\circ=\frac{AB}{BD}$

$\Rightarrow AB=BD\times\tan 60^\circ = 50\sqrt{3}\times\sqrt{3} = 50\times3$ m = 150 m

Hence, the height of the hill is 150 m.

4. A boy whose eye level is 1.3m from the ground, spots a balloon moving with wind in a horizontal line at some height from the ground. The angle of elevation of the balloon

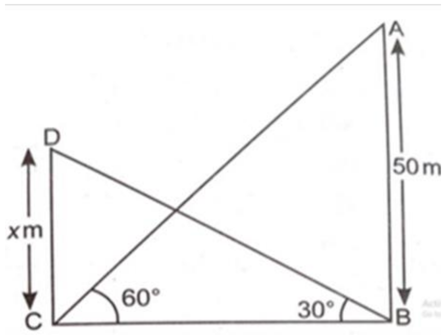
from the eyes of the boy at any instant is 60°. After 2 seconds, the angle off elevation reduces to 30°. If the speed of wind at that moment is 29√3 m/s, then find the height of the balloon from the ground.
Answer: 88.3m

5. A man standing on the deck of the ship, which is 16m above the water level, observes the angle of elevation of the top of the clip as 60° and the angle of depression of the base of the cliff as 30°. Calculate the distance of the cliff from the ship and height of the cliff.
Answer : 16√3m

CASE STUDY BASED QUESTIONS (4 Marks)

CASE STUDY – 1

The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. The height of the tower is 50m. Observe the given figure and give the answer of the following:



(i) What is the distance between building CD and tower AB? (1)
(ii) What is the distance between root of building CD and the top of the tower AB? (1)
(iii) Find the height of building CD? (2)

OR

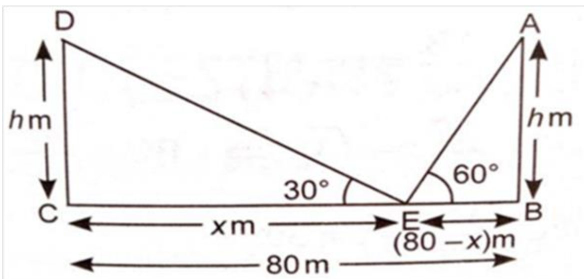
Find the distance between root of tower AB and the top of the building CD?
(Basic)

Solution: (i) In Right triangle ABC, tan 60° = AB/BC, So BC=50/√3
(ii) In Right triangle ABC, Sin60° = 50/AC So, AC =100/√3
(iii) In Right triangle DCB, tan30° =DC/BC , CD= x = 50/3
OR,
In Right triangle DCB, sin30° = CD/BD So, BD=100/3

CASE STUDY – 2

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the angles of elevation of the top of poles are 600 and 300 respectively. Find the height of the poles and the distances of the point from the poles.
(i) Draw a neat labelled diagram to show the above situation diagrammatically. (1)
(ii) Find the height of the pole? (2)
(iii) Find the distance of the point from the pole? (1)
(Basic)

Solution: (i)



(ii) In Δ ,
tan 30° = CD/CE
X = √3h -----(i)
In Δ ABE,
tan 60° = AB/BE
$\frac{h}{(80-x)} = \sqrt{3}$
h= 80√3 - √3x -------(ii)
Solving (i) and (ii), h=20√3 m.

(iii) x= √3h = √3 x 20√3 =60 m
And
80-x = 80- 60 = 20 m.

CASE STUDY – 3

A group of students of class x visited India Gate on an education trip. The teacher and students had interest in History as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 meters) in height.

I. What is the angle of elevation if they are standing at a distance of 42m away from the monument?
II. They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance.
III. When the altitude of the sun is at 60°, find the height of a vertical tower that casts a shadow of 20m length.
IV. What is the angle of elevation of the sun when the ratio of the height of the tower to its shadow is 1:1

Answer: i. 45°, ii. 14√3m, iii. 20√3m, iv.45°

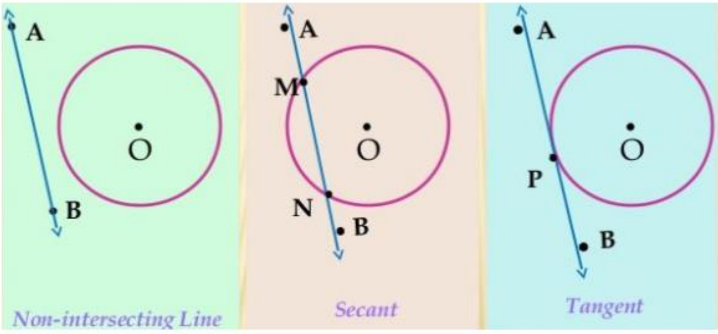
CIRCLE

Gist of the Topic:

The Relation between a Circle and a Line in a Plane

There could be three situations when there are a line and a circle.

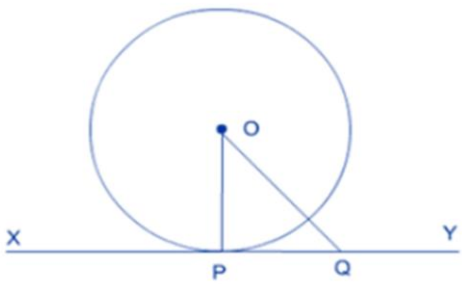
1. Non-intersecting Line: When a line and a circle have no common point then it is called a Non-intersecting Line with respect to the circle.
2. Secant: When a line intersects a circle in such a way that there are two common points then that line is called Secant.
3. Tangent: When a line touches the circle in such a way that they have only one common point then that line is called a Tangent. That common point is called the point of contact.



Tangent to a Circle

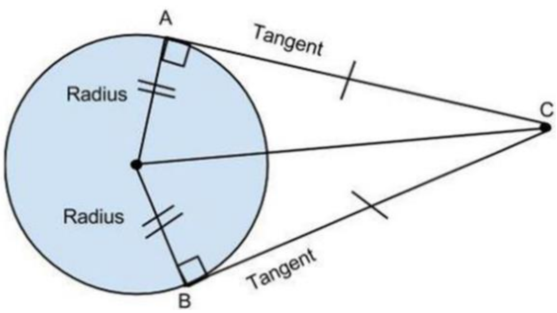
All the tangents of a circle are perpendicular to the radius through the point of contact of that tangent.

OP is the radius of the circle and Q is any point on the line XY which is the tangent to the circle. As OP is the shortest line of all the distances of the point O to the points on XY. So OP is perpendicular to XY. Hence, OP⊥ XY



Number of Tangents from a Point on a Circle

1. There could be only one tangent at one point of contact.
2. Tangent could not be drawn from any point inside the circle.
3. There could be only two tangents to a circle from any point outside the circle.
4. The lengths of tangents drawn from an external point to a circle are equal.

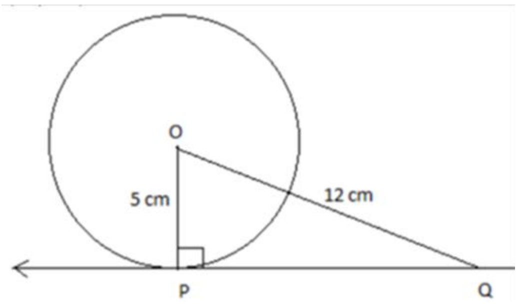


Here, two tangents are drawn from the external point C. As the tangent is perpendicular to the radius, it formed the right-angled triangle.

So ΔAOC and ΔBOC are congruent right-angled triangle. Hence AC = BC.

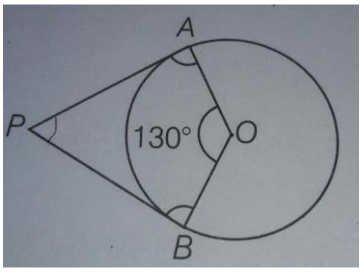
MULTIPLE CHOICE QUESTIONS (1 MARK)

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:
(Basic)
(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm
Answer: (D)
Solution: Consider the figure drawn. PQ represents the tangent drawn to the circle. O is the centre of the circle. Thus OP = 5 cm, which is the radius of the circle, and OQ = 12 cm.



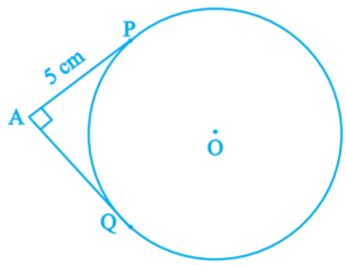
From the figure, we can say that OP is perpendicular to the tangent, i.e. OP⊥PQ.
The tangent at any point of circle is perpendicular to the radius through the point of contact.
$\therefore \angle OPQ = 90^\circ$
Hence, ΔOPQ forms a right angled triangle. We need to find the length of PQ. Consider ΔOPQ, right angled at P.
By using Pythagoras theorem, we can say that,
$(\text{hypotenuse})^2 = (\text{height})^2 + (\text{base})^2$
$\Rightarrow OQ^2 = OP^2 + PQ^2$
$\therefore PQ^2 = OQ^2 - OP^2$
$PQ = \sqrt{OQ^2 - OP^2} = \sqrt{12^2 - 5^2} = \sqrt{144 - 25} = \sqrt{119}$cm

2. If angle between two radii of a circle is 130°, the angle between the tangents at the ends of the radii is:
(Basic)
(A) 90° (B) 50° (C) 70° (D) 40°
Answer: (B)
Solution: We know tangents are ⊥ to radius at point of contact.



$\therefore \angle PAO = \angle PBO = 90^\circ$
Now in quadrilateral AOPB,
$\Rightarrow \angle P + \angle A + \angle B + \angle O = 360^\circ$
[Sum of four angles of a quadrilateral is 360o]
$\Rightarrow \angle P + 90^\circ + 90^\circ + 130^\circ = 360^\circ$
$\Rightarrow \angle P + 310^\circ = 360^\circ$
$\Rightarrow \angle P = 360^\circ - 310^\circ$
$\therefore \angle P = 50^\circ$
∴ Required measure of an angle is 50°.

3. In the given figure, the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. Then the radius of the circle is (Basic)



(A) 10 cm (B) 7.5 cm
(C) 5 cm (D) 2.5 cm
Answer: (C)

Solution: According to Question when we draw figure we get tangents AP = AQ (Also Adjacent sides of quadrilateral APOQ) and OP = OQ (radii of the circle)

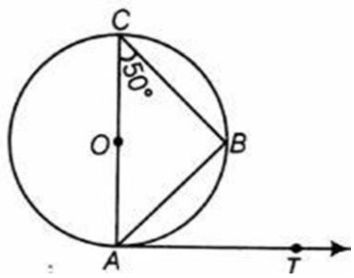
Now as we know that angles at the point of contact are 90^o

$\angle P = \angle Q = 90^o \therefore \angle O = 90^o$

thus, we can say that OP = OQ = AP = AQ (As APOQ is a square)

thus, the radius of the circle is 5 cm.

4. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to



(A) 65° (B) 60°

(C) 50° (D) 40°

Answer: (C) (Competency Based Question)

5. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(A) 60 cm^2 (B) 65 cm^2 (C) 30 cm^2 (D) 32.5 cm^2

Answer: (A)

ASSERTION-REASON BASED QUESTIONS (1 MARK)

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of Reason (R) . Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion(A) is true but reason(R) is false.

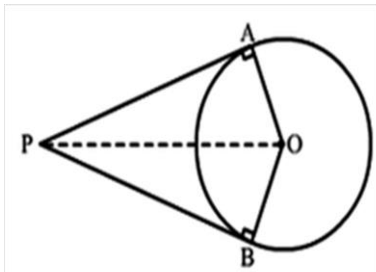
(d) Assertion(A) is false but reason(R) is true.

1. Assertion: If length of a tangent from an external point to a circle is 8 cm, then length of the other tangent from the same point is 8 cm.

Reason: length of the tangents drawn from an external point to a circle are equal. (basic)

Answer: (a)

Solution: Let AP and BP be the two tangents to the circle with centre O.



In ΔAOP and ΔBOP

OA = OB (radii of the same circle)

$\angle OAP = \angle OBP = 90°$ (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

OP = OP (common)

$\therefore \Delta AOP \cong \Delta BOP$ (by R.H.S. congruence criterion)

$\therefore$ AP = BP (corresponding parts of congruent triangles)

Hence the length of the tangents drawn from an external point to a circle are equal.

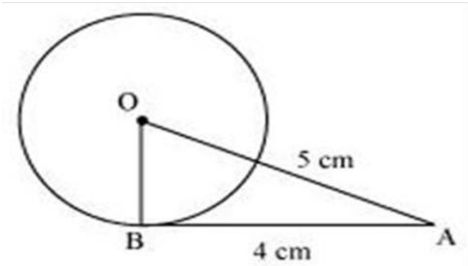
2. Assertion: If in a cyclic quadrilateral, one angle is 40°, then the opposite angle is 140°

Reason: Sum of opposite angles in a cyclic quadrilateral is equal to 360°

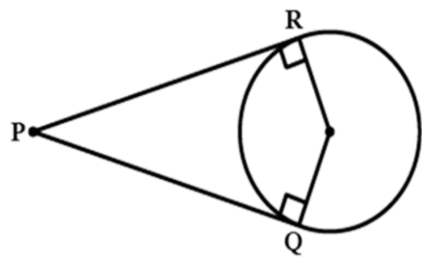
Answer: C

SHORT ANSWER TYPE -I QUESTIONS (2 MARKS)

1. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
   (Basic)
   Solution: Let AT be the tangent drawn from a point A to a circle with centre O and OA=5 cm and AT=4 cm. Since tangent at a point is perpendicular to the radius through the point of contact
   $\therefore OT \perp AT$
   $\therefore$ from right angled $\triangle OAT$,
   $(OA)^2=(OT)^2+(TA)^2$
   $\Rightarrow OT)^2 =(5)^2- (4)2 = 25-16 = 9$
   $\Rightarrow OT=3$ cm
   $\therefore$ radius of the circle =3 cm.



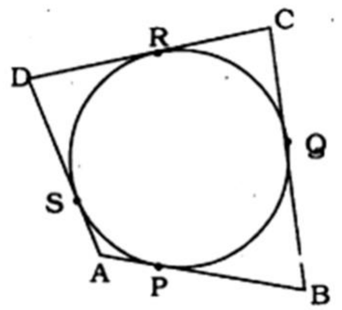
2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
   (Basic)
   Solution:
   Tangents PR and PQ from an external point P to a circle with centre O.
   To prove: Quadrilateral QORP is cyclic.
   Proof: RO and RP are the radius and tangent respectively at contact point R.
   $\therefore \angle PRO=90^\circ$
   Similarly, $\angle PQO=90^\circ$
   In quadrilateral OQPR, we have
   $\angle P+\angle R+\angle O+\angle Q=360^\circ$
   $\Rightarrow \angle P+\angle 90^\circ+\angle O+\angle 90^\circ=360^\circ$
   $\Rightarrow \angle P+\angle O=360^\circ-180^\circ=180^\circ$
   These are opposite angles of quadrilateral QORP and are supplementary.
   $\therefore$ Quadrilateral QORP is cyclic, hence, proved.



3. A circle touches the side BC of $\triangle$ ABC at P and touches AB and AC produced at Q and R respectively. Prove that AQ=$\frac{1}{2}$ (perimeter of $\triangle$ ABC).
   (Basic)
   Answer: Proof

4. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.
   Answer: Proof

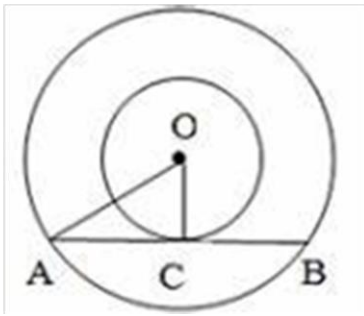
5. A quadrilateral ABCD is drawn to circumscribe a circle (see the fig.). Prove that AB + CD = AD + BC.
   Answer: Proof



SHORT ANSWER TYPE -II QUESTIONS (3 MARKS)

1. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
   (Basic)

   Solution: Given Two circles have the same centre O and AB is a chord of the larger circle touching the smaller circle at C; also, OA = 5 cm and OC = 3 cm

   In $\Delta OAC, OA^2 = OC^2 + AC^2$

   $\therefore AC^2 = OA^2 - OC^2 = 5^2 - 3^2 = 25 - 9 = 16$

   $AC = 4\ cm$

   AB = 2 AC (Since perpendicular drawn from the centre of the circle bisects the chord)

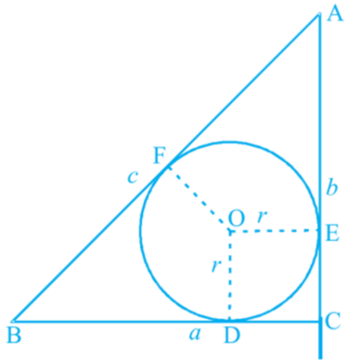
   AB = 2 x 4 = 8 cm

   The length of the chord of the larger circle is 8 cm.

2. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by

   r $= \frac{a+b-c}{2}$

   (Basic)

   Solution: Let the circle touches the sides BC, CA, AB of the right triangle ABC at D, E and F respectively, where BC = a, CA = b and AB = c (see Fig. 9.12). Then AE = AF and BD = BF.

   Also, CE = CD = r.

   i.e., b – r = AF, a – r = BF

   or AB = c = AF + BF = b – r + a – r

   This gives r $= \frac{a+b-c}{2}$

3. PC is a tangent to the circle at C. AOB is the diameter which when extended meets the tangent at P. Find ∠CBA, ∠AOC and ∠BCO, if ∠PCA=110°

   (Basic)

   Answer: ∠CBA=70^o, ∠AOC= 140^o and ∠BCO= 7^o.

4. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

   Answer: Proof

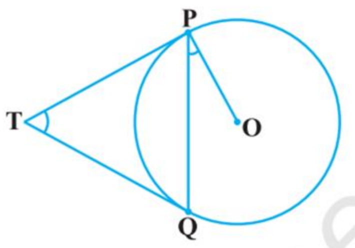
5. Prove that the parallelogram circumscribing a circle is a rhombus.

   Answer: Proof

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠ PTQ = 2 ∠ OPQ. (Basic)

   Solution: We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact (see Fig. 10.9). We need to prove that ∠ PTQ = 2 ∠ OPQ

   Let ∠ PTQ = θ

   Now, by Theorem 10.2, TP = TQ. So, TPQ is an isosceles triangle.

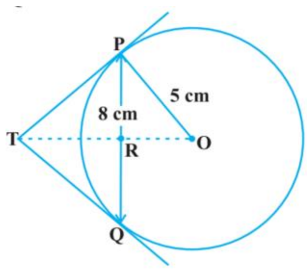
Therefore, ∠ TPQ = ∠ TQP = $\frac{1}{2}$ $(180^o - \theta) = 90^o - \frac{1}{2}\,\theta$

Also, by Theorem 10.1, ∠ OPT = 90°

So, ∠ OPQ = ∠ OPT – ∠ TPQ = $90^o - \left(90^o - \frac{1}{2}\,\theta\right) = \frac{1}{2}\,\theta = \frac{1}{2}$∠PTQ

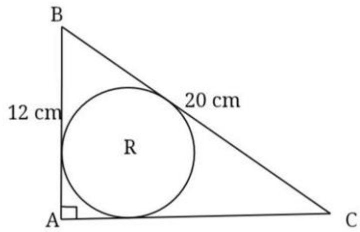
This gives ∠ PTQ = 2 ∠ OPQ

2. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig. 10.10). Find the length TP.
(Basic)

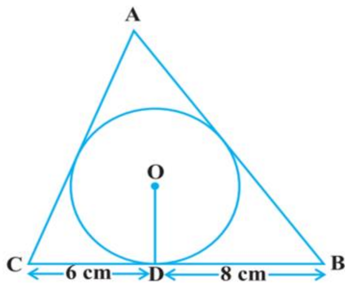


Solution: Join OT. Let it intersect PQ at the point R. Then Δ TPQ is isosceles and TO is the angle bisector of ∠ PTQ. So, OT ⊥ PQ and therefore, OT bisects PQ which gives
PR = RQ = 4 cm.
Also, OR = $\sqrt{OP^2 - PR^2} = .\ \sqrt{5^2 - 4^2}$=3 cm
Now, ∠ TPR + ∠ RPO = 90° = ∠ TPR + ∠ PTR
So, ∠ RPO = ∠ PTR
Therefore, right triangle TRP is similar to the right triangle PRO by AA similarity.
This gives $\frac{TP}{PO} = \frac{RP}{RO}$, i.e., $\frac{TP}{5} = \frac{4}{3}$ or TP =$\frac{20}{3}$ cm

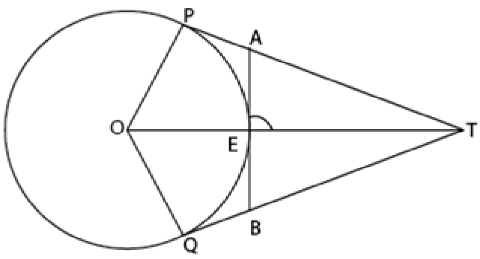
3. A model of a traffic signal on the road has a triangular base ABC with ∠A=90∘ and with a red circular light within it as shown in figure. If AB=12 cm and BC=20 cm and R is the centre of the ΔABC, find the area used for the red light.
(Basic)
Answer: Proof



4. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see the figure). Find the sides AB and AC.
Answer: Proof



5. In the given figure, O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E and it intersects the tangents PT and QT at A and B, find the length of AB.
(Competency Based Question)



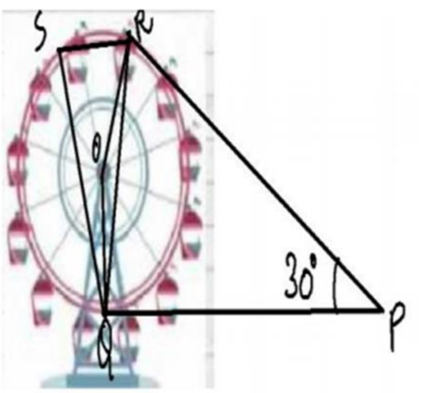
Answer: 6.67 cm

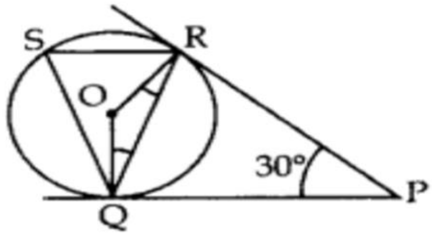
CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.

(Basic)





(i) In the given figure find $\angle$ ROQ

Solution: Here, $\angle ROQ+\angle OQP+\angle P+\angle ORP=360^o$

$\angle ROQ+90^o+30^o+90^o=360^o$

$\angle ROQ= 360^o-210^o=150^o$

or

Find RQP

Solution: Here in ΔORQ, OR=OQ

or $\angle ORQ=\angle OQR=x$

Also, $\angle ROQ+\angle ORQ+\angle ROQ =180^o$

or $x+x+150^o =180^o$

or $x=\frac{180^o-150^o}{2}=15^o$

Now, $\angle$ RQP= 90o- $\angle OQR= 90^o-15^o = 75^o$

(ii) Find $\angle$ RSQ

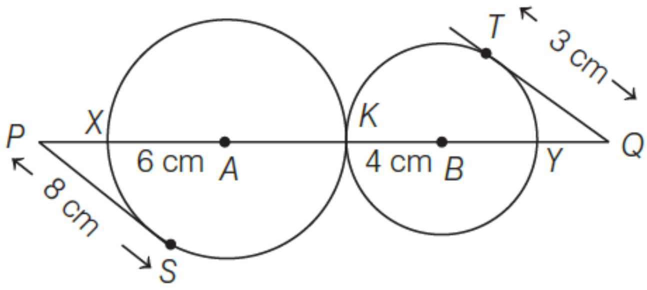
Solution: Here, $\angle RSQ=\frac{1}{2}\angle ROQ=\frac{1}{2}\times 150^o = 75^o$

(iii) Find $\angle$ OQR or $\angle$ORQ

Solution: In OQR, ∠OQR+∠ORQ+∠ROQ=180^o
or ∠OQR+∠OQR+150^o= 180^o (Since ∠OQR=∠ORQ)
or ∠OQR=$\frac{180^o-150^o}{2} = 15^o$

CASE STUDY 2:

A student draws two circles that touch each other externally at point K with centres A and B and radii 6 cm and 4 cm, respectively as shown in the figure.
(Basic)

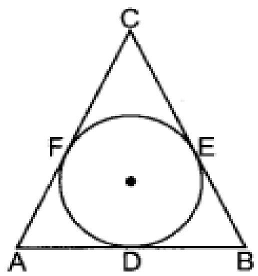


Based on the above information, answer the following questions.
(i) Find the length of PA.
or
Find the length of BQ.
(ii) Find the length of PK.
(iii) Find the length of QY.
Answer: (i) 10 cm or 5cm (ii) 16 cm (iii)1 cm

CASE STUDY 3:

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.

(Competency Based Question)



(i) Find the length of AD
(ii)Find the Length of BE
(iii) If radius of the circle is 4cm, Find the area of ΔOAB
or
Find area of ΔABC
Answers:
(i) 7 (ii) 5 (iii) 24 or 60

AREAS RELATED TO CIRCLE

GIST OF THE LESSON:

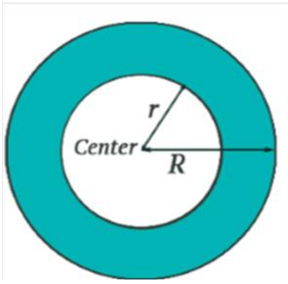
Circumference (perimeter) of the circle = 2πr = πd ($\pi = \frac{22}{7}$)

r = Radius and d = 2r

Area of the circle = πr^2

Perimeter of the Semi-circle = πr+2r = $\frac{\pi d}{2}$ +d($\pi = \frac{22}{7}$)

Area of the Semi-circle: $\frac{\pi r^2}{2}$



[Note: Area of the semi-circle is just half of the area of the circle.]

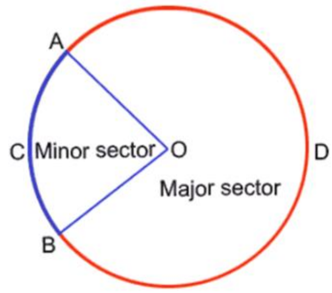
Area of a Ring: Area of the ring i.e. the coloured part in the above figure is calculated by subtracting the area of the inner circle from the area of the bigger circle.

Area of the ring = $\pi R^2 - \pi r^2 = \pi (R^{2} - r^2)$

Where, R = radius of outer circle

r = radius of inner circle

Areas of Sectors of a Circle: The area formed by an arc and the two radii joining the endpoints of the arc is called Sector.



Minor Sector: The area including ∠AOB with point C is called Minor Sector. So OACB is the minor sector. ∠AOB is the angle of the minor sector.

Area of the sector of angle $\theta = \frac{\theta}{360^o} \times \pi r^2$

Major Sector: The area including ∠AOB with point D is called the Major Sector. So OADB is the major sector. The angle of the major sector is 360° – ∠AOB.

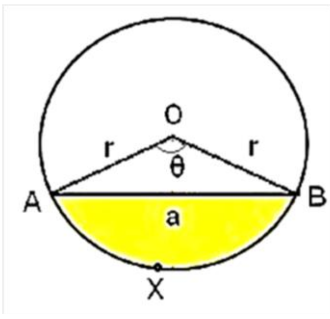
Area of Major Sector = πr^2 - Area of the Minor Sector

[Remark: Area of Minor Sector + Area of Major Sector = Area of the Circle]

Length of an Arc of a Sector of Angle θ: An arc is the piece of the circumference of the circle so an arc can be calculated as the θ part of the circumference.

Length of an arc f a sector of angle $\theta = \frac{\theta}{360^2} \times 2\pi r$

Areas of Segments of the Circle: The area made by an arc and a chord is called the Segment of the Circle.



Minor Segment: The area made by chord AB and arc X is the minor segment. The area of the minor segment can be calculated by

Area of Minor Segment = Area of Minor Sector – Area of ΔABO $= \frac{\theta}{360^o} \times \pi r^2 - \frac{1}{2} r^2 sin\theta$

Major Segment: The other part of the circle except for the area of the minor segment is called a Major Segment.

Area of Major Segment = πr^2 - Area of Minor Segment

[Remark: Area of major segment + Area of minor segment = Area of circle]

Areas of Combinations of Plane Figures: As we know how to calculate the area of different shapes, so we can find the area of the figures which are made with the combination of different figures.

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. The area of a quadrant of a circle with circumference of 44 cm is (Basic)

   (A) 77cm^2 (B) 38.5 cm^2 (C) 35.5cm^2 (D) 38cm^2

   Answer: B

   Solution: C=2πr = 44 ⇒ r = 7cm

   Area of quadrant $=\frac{1}{4}\times\frac{22}{7}\times7^2=38.5\text{cm}^2$

2. The area of the circle that can be inscribed in a square of side 6 cm is (Basic)

   (A) 36 π cm^2 (B) 18 π cm^2 (C) 12 π cm^2 (D) 9 π cm^2

   Answer: D

   Solution: From figure, AB=BC=CD=DA=6 cm

   PR is the diameter of the circle, inscribed in the square ABCD

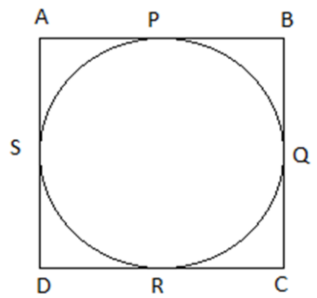
   ∴PR = side of the square = 6 cm

   ∴Radius$=\frac{6}{2}=3$

   Area = $\pi r^2 = \pi\times3^2$

   $= 9\pi$

   ∴Area of circle=9 π cm^2



3. If θ is the angle (in degrees) of a sector of a circle of radius r, then area of the sector is

   (A) $\frac{\pi r^2\theta}{360^o}$ (B) $\frac{\pi r^2\theta}{180^o}$ (C) $\frac{2\pi r\theta}{360^o}$ (D) $\frac{2\pi r\theta}{180^o}$

   (Basic)

   Answer: A

4. A wheel has diameter 84 cm. The number of complete revolutions it will take to cover 792 m is.

   (A) 100 (B) 150 (C) 200 (D) 300

   Answer: D

5. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

   (A) 31 cm (B) 25 cm (C) 50 cm (D) 62 cm

   Answer: C

ASSERTION-REASON BASED QUESTIONS (1 MARK)

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of Reason (R) . Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion(A) is true but reason(R) is false.

(d) Assertion(A) is false but reason(R) is true.

1. **Assertion (A)**: The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is 77/6 cm2.

   Reason (R): The length of an arc of a sector of angle θ and radius r is given by

   $l=\frac{\theta}{360}\times2\pi r$

   (Basic)

Answer: (b)

Solution: For a minute hand, 60 minutes is equivalent to 360^o and so 5 minutes will be 15^o

Area swept in 60 minutes is area of full circle.

So, area swept in 5 minutes will be $\frac{1}{12}$ of area of the circle.

Thus, area swept = $\frac{1}{12} \times \frac{22}{7} \times 7^2 = \frac{77}{6}$ cm^2

Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

2. **Assertion (A)**: If the outer and inner diameter of a circular path is 10 m and 6 m respectively, then area of the path is 16π m^2.

   Reason (R): If R and r be the radius of outer and inner circular path respectively, then area of circular path = $\pi (R^2 - r^2)$.

   Answer: (a) (Competency Based Question)

SHORT ANSWER TYPE -I QUESTIONS (2 MARKS)

1. Find the area of a sector of circle of radius 21 cm and central angle 120°.
   (Basic)

   Solution: Area of the sector = $\frac{\theta}{360^o} \times \pi r^2 = \frac{120^o}{360^o} \times \frac{22}{7} \times 21^2 = 22 \times 21 = 462\ cm^2$

2. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.
   (Basic)

   Solution: Radius r_1= 15 cm and radius r_2=18 cm.

   ∴ The circumference $C_1=2\pi r_1=2\pi\times15cm=30\pi$ cm and

   The circumference $C_2=2\pi r_2=2\pi\times18$ cm$=36\pi$ cm

   So $C_1+C_2=(30+36)\pi$ cm$=66\pi$ cm.

   This is the circumference C of the resulting circle.

   ∴$C=C_1+C_2=66\pi$ cm

   Now, the radius of a circle $=\frac{circumference}{2\pi} = \frac{66\pi}{2\pi}$ cm=33 cm.

3. A sector has an area of 30π sq units and a central angle of 72^o, find the radius of the circle.

   Answer: $5\sqrt{6}$ units
   (Basic)

4. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

   Answer: $\frac{154}{3} cm^2$

5. The outer and inner diameter of a circular ring are34 cm and 32 cm respectively, then find the area of the ring.

   Answer: 33π cm^2

SHORT ANSWER TYPE -II QUESTIONS (3 MARKS)

1. Radius of a circle is 10 cm. Area of the minor sector is 100 cm^2. Find the area of its corresponding major sector. (π= 3.14)
   (Basic)

   Solution: Radius of the circle, r = 10 cm

   Area of the minor sector = 100 cm^2

   ∴ Area of the corresponding major sector = Area of the circle − Area of the minor sector

$= \pi r^2 - 100cm^2 = 3.14 \times 10^2 - 100 = 314 - 100 = 214\ cm^2$

Thus, the area of the corresponding major sector is 214 cm^2.

2. Area of a sector of a circle of radius 36 cm is 54 π cm^2. Find the length of the corresponding arc of the sector.
(Basic)
Solution: Let the central angle (in degrees) be θ.
So, $\frac{\theta}{360^o} \times \pi \times 36^2 = 54\pi$ $\qquad or, \theta = \frac{54\times360}{36\times36} = 15^o$
Now, length of the arc = $\frac{\theta}{360^o} \times 2\pi r = \frac{15^o}{360^o} \times 2\pi \times 36$ cm = 3 π cm

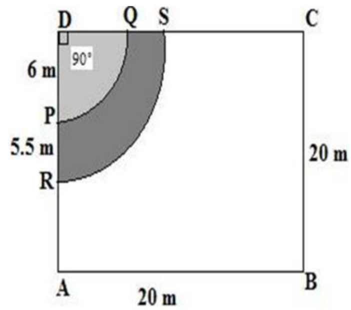
3. A race track is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. find the width of the track.
(Basic)
Answer: 7 m (Competency Based Question)

4. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and 3 = 1.73)
Answer: 20.4375 cm^2; 686.0625 cm^2

5. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.
Answer: 45.03 m^2

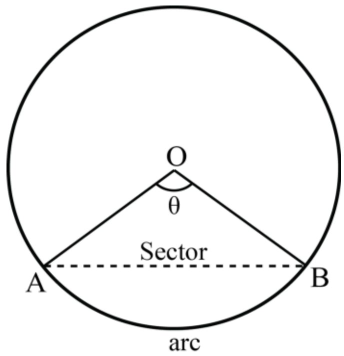
LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5m, find the increase in area of the grassy lawn in which the calf can graze.
(Basic)



Solution: Let the calf be tied at the corner A of the square lawn.
Then, the increase in area = Difference of the two sectors of central angle 90° each and radii 11.5 m (6 m + 5.5 m) and 6 m, which is the shaded region in the figure.
So, required increase in area
$= [\frac{90^o}{360^o} \times \pi \times 11.5^2 - \frac{90^o}{360^o} \times \pi \times 6^2] m^2$
$= \frac{\pi}{4} \times (11.5 - 6) \times (11.5 + 6) = \frac{22}{7 \times 4} \times 5.5 \times 17.5 = 75.625\ m^2$

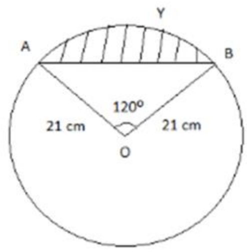
2. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.
(Basic)



Solution: Let the central angle of the sector be θ
Given that, radius of the sector of a circle (r) = 5cm
And arc length (l) =3.5 cm
∴ Central angle of the sector $= \theta = \frac{arc\ length\ l}{2\pi r} \times 360^o$
$= \frac{3.5}{2 \times \pi \times 5} \times 360^o = \frac{0.7}{\pi} \times 180^o$
Now, area of sector with angle θ
$= \frac{\theta}{360} \pi r^2 = \left(\frac{0.7}{\pi \times 360^o} \times 180^o\right) \times \pi \times 5^2$
$= 0.35 \times 25 = 8.75\ cm^2$

Hence, the required area of the sector of a circle is 8.75 cm^2

3. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)
(Basic)
Answer: 189.97 km^2
4. Find the area of the segment AYB shown in the figure, if radius of the circle is 21 cm and $\angle$AOB = 120°. (Use $\pi = \frac{22}{7}$)
Answer: $\frac{21}{4}(88 - 21\sqrt{3})$ cm^2

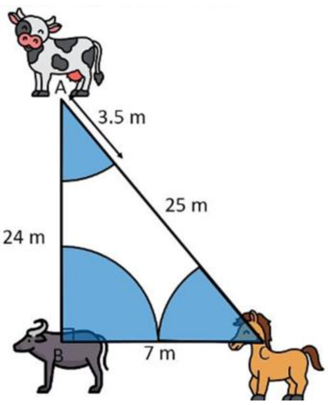


5. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circle is drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. [Take $\sqrt{3}$=1.73]
(Competency Based Question)
Answer: 7.77 cm^2

CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

Gayatri has a triangular shaped grass field. At the three corners of the field, a cow, a buffalo and a horse are tied separately to the pegs by means of ropes of3.5 m each to graze in the field, as shown in the figure. Sides of the triangular field are 25 m, 24 m and 7 m. Based on the above information, answer the following questions.
(Basic)



(i) Find the area of the triangular field.
Solution: Since $\triangle$ABC is a right angled triangle
$\therefore$ Area of triangle ABC=$\frac{1}{2}$×7×24=84 m^2

(ii) Find the area of the region grazed by the cow.
Solution: $\frac{\angle A}{360^o} \times \pi \times (3.5)^2$
or
Find the area of region grazed by the buffalo and the horse.
Solution: $\frac{\angle B}{360^o}\times\pi\times(3.5)^2+\frac{\angle C}{360^o}\times\pi\times(3.5)^2$
$= \frac{\angle B+\angle C}{360^o}\times\pi\times(3.5)^2$

(iii) Find the total area grazed by the cow, the buffalo and the horse.
Solution: Given, the length of the rope used to tie the cow, the buffalo, and the horse is 3.5 cm each.
So, the radius of the sector at A, B, and C is 3.5 cm

Total area grazed by the cow, the buffalo and the horse = Area of sector at point A+ Area of sector at point B + Area of sector at point C

$=\frac{\angle A}{360^o} \times \pi \times 3.5^2 \text{ m}^2 + \frac{\angle B}{360^o} \times \pi \times 3.5^2 \text{ m}^2 + \frac{\angle C}{360^o}° \times \pi \times 3.5^2 \text{ m}^2$

$= \frac{\angle A+\angle B+\angle C}{360^o} \times \pi \times 3.5^2 \text{ m}^2$

$=\frac{180^o}{360^o} \times \frac{22}{7} \times 3.5^2 m^2$ [Sum of angle of a triangle=180°].

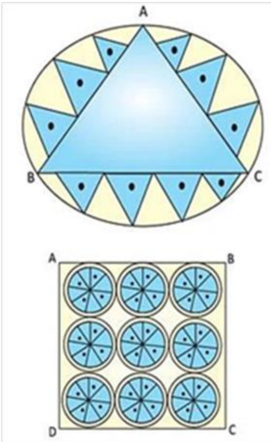
$= \frac{77}{4} = 19.25 \text{ m}^2$

CASE STUDY 2:

Pookalam is the flower bed or flower pattern designed during Onam in Kerala. It is similar as Rangoli in North India and Kolam in Tamil Nadu. During the festival of Onam, your school is planning to conduct a Pookalam competition. Your friend who is a partner in competition, suggests two designs given below.

(Basic)

Observe these carefully.



Design I: This design is made with a circle of radius 32cm leaving equilateral triangle ABC in the middle as shown in the given figure.

Design II: This Pookalam is made with 9 circular design each of radius 7cm.

Refer Design I:

(i) Find the side of equilateral triangle.

(ii) Find the altitude of the equilateral triangle.

Refer Design II:

(iii) Find the area of square ABCD.

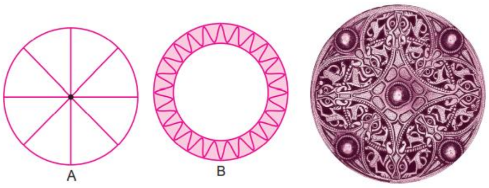
or

Find the area of each circular design.

Answers: (i) $32\sqrt{3}$ cm^2, (ii) 48cm, (iii) 1764 cm^2 or 154 cm^2

CASE STUDY 3:

A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat. Designs of some brooch are shown below. Observe them carefully.



Design A: Brooch A is made with silver wire in the form of a circle with diameter 28mm. The wire used for making 4 diameters which divide the circle into 8 equal parts.

Design B: Brooch b is made two colours_ Gold and silver. Outer part is made with Gold. The circumference of silver part is 44mm and the gold part is 3mm wide everywhere.

Refer to Design A

1. Find the total length of silver wire.
2. Find the area of each sector of the brooch.

Refer to Design B

3. Find the circumference of outer part (golden).

or

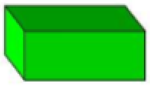
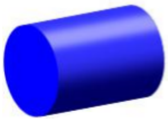
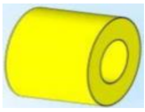
Find the difference of areas of golden and silver parts.

Answers: (i) 200mm (ii) 77mm^2 (iii) 62.86 mm or 51πmm

SURFACE AREAS & VOLUMES

GIST OF THE LESSON:

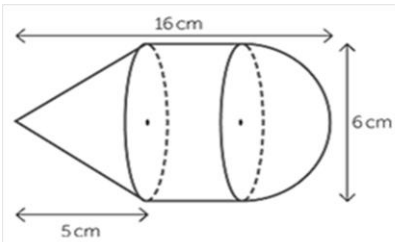
Surface Area is the area of the outer part of any 3D figure and **Volume** is the capacity of the figure i.e. the space inside the solid. To find the surface areas and volumes of the combination of solids, we must know the surface area and volume of the solids separately. Some of the **formulas** of solids are -

Name	**Figure**	**Lateral or Curved Surface Area**	**Total Surface Area**	**Volume**	**Nomenclature**
Cube		$4l^2$	$6l^2$	l^3	l = edge of the cube
Cuboid		2h(l +b)	2(lb + bh + hl)	lbh	l = length b = breadth h = height
Cylinder		$2\pi rh$	$2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$	πr^2h	r = radius h = height
Hollow cylinder		$2\pi h\ (R + r)$	$2\pi h\ (R + r)$ $+ 2\pi\ (R^2 - r^2)$	-	R = outer radius r = inner radius
Cone		πrl	$\pi r^2 + \pi rl$ $= \pi r(r + l)$	$\frac{1}{3}\pi r^2h$	r = radius h = height l = slant height
Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r = radius

Surface Area of a Combination of Solids

If a solid is moulded by two or more than two solids then we need to divide it in separate solids to calculate its surface area.

Example: This solid is the combination of three solids i.e. cone, cylinder and hemisphere.



Total surface area of the solid
= Curved surface area of cone
+ Curved surface area of cylinder
+ Curved surface area of hemisphere

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(Basic)
(A) $4\pi r^2$ (B) $6\pi r^2$ (C) $3\pi r^2$ (D) $8\pi r^2$
Answer: A
Solution: We know that,
Curved surface area of a hemisphere $=2\pi r^2$
So, when two solids' hemispheres of same base radius are joined together along their bases, then, curved surface area of newly formed solid sphere$=2\times2\pi r^2=4\pi r^2$.

2. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is (Basic)
(A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9
Answer: D
Solution: Given that,
Volume of two spheres are in ratio=64:27
We know that, volume of sphere $=\frac{4}{3}\pi r^3$
Then, $\frac{Volume of sphere(1)}{Volume of sphere(2)} = \frac{64}{27}$
or, $\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$
or, $\frac{r_1^3}{r_2^3} = \frac{64}{27}$ or, $\frac{r_1}{r_2} = \frac{4}{3}$
Then, Ratio of areas both spheres $=\frac{area\ of\ sphere(1)}{area\ of\ sphere(2)} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

3. A cylindrical pencil sharpened at one edge is the combination of (Basic)
(A) a cone and a cylinder (B) two cones and a cylinder
(C) a hemisphere and a cylinder (D) two cylinders.
Answer: A

4. A cylinder with radius 4 cm and height 12 cm is combined with a hemisphere of the same radius. The total volume of the combination is
(A) $\frac{704}{3}\pi\ cm^3$ (B) $\frac{112}{9}\pi\ cm^3$ (C) $304\pi\ cm^3$ (D) $\frac{200}{4}\pi\ cm^3$
Answer: A

5. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed, the volume of the remaining solid is
(A) 240 cm^3 (B) 330 π cm^3 (C) 240 π cm^3 (D) 440 π cm^3
Answer: C

ASSERTION-REASON BASED QUESTIONS (1 MARK)

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion(A) is true but reason(R) is false.
(d) Assertion(A) is false but reason(R) is true.

1. Assertion: If a ball is in the shape of a sphere has a surface area of 221.76 cm^2, then its diameter is 8.4 cm.

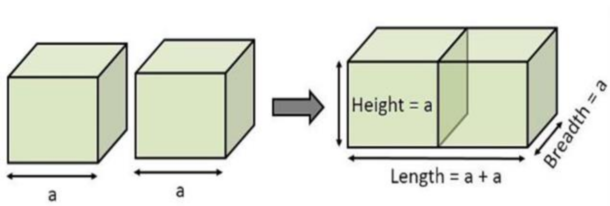
   Reason: If the radius of the sphere be r, then surface area, $S = 4\pi r^2$, i.e., $r = \sqrt{\left(\frac{S}{4\pi}\right)}$

   Answer: (a)
   (Basic)
   Solution: Given, surface area of sphere =221.76cm^2
   We know, surface area of sphere =$4\pi r^2$
   $\Rightarrow 4(\pi)r^2 = 221.76$
   $\Rightarrow r^2 = \frac{7}{22} \times 221.76 \times \frac{1}{4} = 7 \times (10.08) \times \mathbf{\frac{1}{4}}$
   $\Rightarrow$r=4.2cm
   Then, diameter equals 2r=2×4.2=8.4 cm, which is true and reason is also correct.
   Moreover, reason is the correct explanation of assertion. Hence, option (a) is correct.

2. Assertion: Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.
   Reason: If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$.
   Answer: (a) (Competency Based Question)

Short Answer Type -I Questions (2 Marks)

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
   (Basic)
   Solution: The volume of cube 64cm^3
   Side of cube = $\sqrt[3]{64}$ = 4cm
   Length of resulting cuboid 4+4=8 cm
   Surface area =2(lb + hl + bh)
   =2[4(4) +4(8) +8(4)]
   =2(16+32+32)
   =2(80) = 160 cm^2



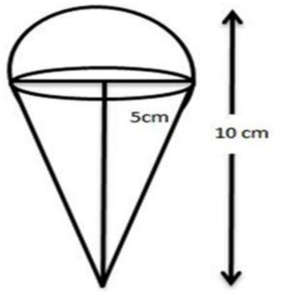
2. Find the volume of the largest circular cone that can be cut out from a cube of edge 4.2 cm.
   (Basic)
   Solution: As can be observed from the figure of largest cone in a cube,
   the radius, r=2a and Height, h=a
   Volume of cone = $3\pi r^2 h = 3\pi(2a)^2 a = 12\pi a^3 = 12\pi\,(4.2)^3\,cm^3 = 19.38\,cm^3$.

3. What is the volume of a cube with edge length 4 cm combined with a hemisphere of radius 2 cm?
   (Basic)
   Answer: 80.75 cm^3
4. If each edge of a cube is increased by 50%, then find the percentage increase in the surface area of the cube.
   Answer: 125%

5. The perimeter of the base of a cone is 44 cm and the slant height is 25 cm. Find the volume and the curved surface area of the cone.
Answer: volume of the cone = 1232 cm^3, curved surface area= 550 cm^2

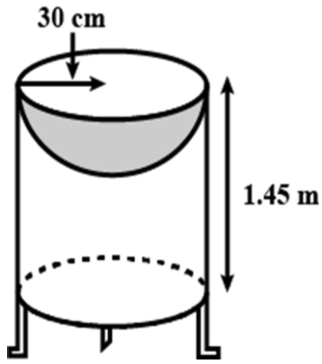
SHORT ANSWER TYPE -II QUESTIONS (3 MARKS)

1. An ice cream cone full of ice cream having radius 5 cm and height 10 cm as shown in the given figure. Calculate the volume of ice cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.
(Basic)



Solution:
Volume of ice cream
= Volume of hemisphere + Volume of cone
$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$
Radius of hemisphere = Radius of cone
Since height of hemisphere is 5 cm, then height of cone will be 10−5=5cm
∴ Volume of ice cream $= \frac{2}{3}\pi (5)^3 + \frac{1}{3}\pi (5)^2 \times 5 = 125\pi = 392.85$ cm^3
$\frac{1}{6}$ th of ice cream $= \frac{392.85}{6} = 65.475$ cm^3
Volume of required portion of ice cream =392.85−65.47=327.375cm^3

2. Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (see the figure). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)
(Basic)



Solution: Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.
Then, the total surface area of the bird-bath
= CSA of cylinder + CSA of hemisphere
$= 2\pi rh + 2\pi r^2 = 2\pi r (h + r)$
$= 2 \times \frac{22}{7} \times 30(145 + 30) = 2 \times \frac{22}{7} \times 30 \times 175 = 33000$ cm^2 = 3.3 m^2

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
(Basic)
Answer: 214.5 cm^2

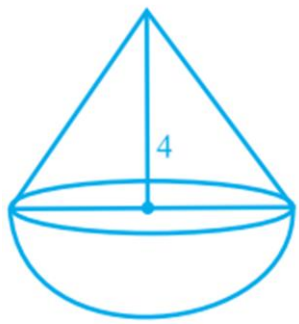
4. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.
Answer: 100 (Competency Based Question)

5. A petrol tank is a cylinder of base diameter 28 m and length 15 m, fitted with conical ends each of height 6 m. Determine the capacity of the tank.
Answer: 11704 m^3

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy. (Basic)

Solution: Let r be the radius of the hemisphere and the cone and h be the height of the cone.



Volume of the toy

= Volume of the hemisphere + Volume of the cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times 4^3 + \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 4$

$= \frac{1408}{7}\ cm^3$.

A cube circumscribes the given solid. Therefore, edge of the cube should be 8 cm.

Volume of the cube = 8^3 cm^3 = 512 cm^3

Difference in the volumes of the cube and the toy

$= \frac{1408}{7} - 512\ cm^3$ = 310.86 cm^3

Total surface area of the toy

= Curved surface area of cone + curved surface area of hemisphere

$= \pi r l + 2\pi r^2$, where $l = \sqrt{r^2 + h^2}$

$= \pi r\ (l + 2r) = \frac{22}{7} \times 4\left(\sqrt{4^2 + 4^2} + 2 \times 4\right) = \frac{22}{7} \times 4 \times (4\sqrt{2} + 8)$ cm^2

$= \frac{88}{7} \times 4(\sqrt{2} + 2)$ = 171.68 cm^2

2. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? (Basic)

Solution: Given that, a heap of rice is in shape of a cone of height h=3.5 m

Radius r = $\frac{9}{2}$ m = 4.5m.

Volume of heap = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4.5 \times 4.5 \times 3.5\ m^3$

Volume =74.25m^3

Canvas cloth require to cover the help = CSA of cone = πrl

Where l = $\sqrt{r^2 + h^2}$

l = $\sqrt{(4.5)^2 + (3.5)^2} = 5.70$ m

Now CSA = $\pi rl = \frac{22}{7} \times 4.5 \times 5.70$ m^2 =80.61m^2

3. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. (Basic)

Answer: 18 cm^2

4. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use π= 3.14)

Answer: 892.26 kg

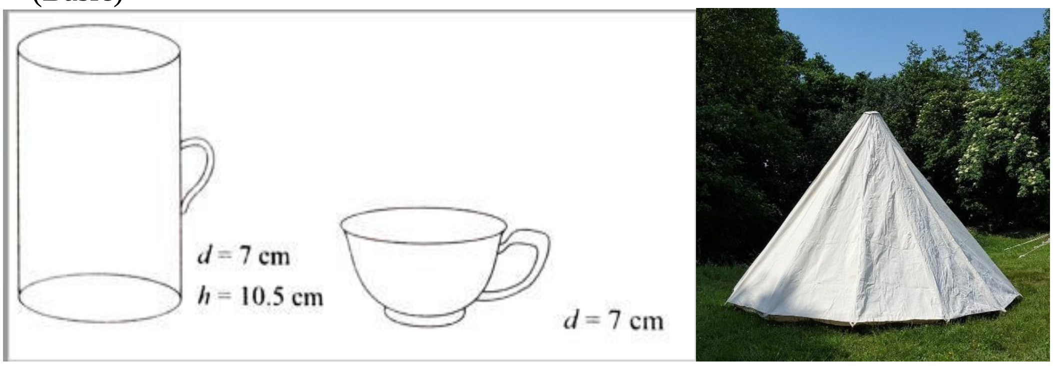
5. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [Use $\pi = 3.14$].
Answer: surface area of the rocket = 301.44 cm^2, volume of the rocket = 377.1 cm^3

CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551m^2. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1m^2, the students put the tents. The radius of the tent is 7m.

(Basic)



(i) Find the volume of cylindrical cup.

Solution: Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10.5 = 404.25 cm^3$

(ii) Find the volume of hemispherical cup.

Solution: Volume of hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 89.83\ cm^3$

(iii) Find the height of the conical tent prepared to accommodate four students.

Solution: Here $\pi r l = 551 - 1 = 550$

$l = \frac{550}{\pi r} = \frac{550 \times 7}{22 \times 7} = 25$

Now, $h^2 = l^2 - r^2 = 25^2 - 7^2 = 625 - 49 = 576$

$h = \sqrt{576} = 24\, m$

or

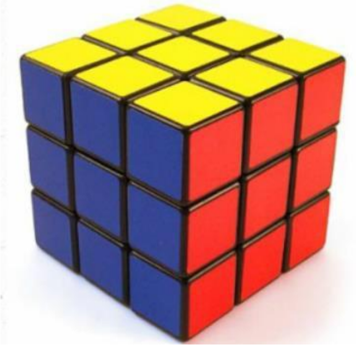
How much space on the ground is occupied by each student in the conical tent?

Solution: Area of the base of the tent= $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ m^2$

$\therefore$ space on the ground occupied by each student= $\frac{154}{4} = 38.5\ m^2$

CASE STUDY 2:

On a Sunday, your Parents took you to a fair. You could see lot of toys displayed, and you wanted them to buy a RUBIK's cube and strawberry ice-cream for you. Observe the figures and answer the questions-: (Basic)



(i) Find the length of the diagonal if each edge measures 6cm.

or

Find the volume of the solid figure if the length of the edge is 7cm.

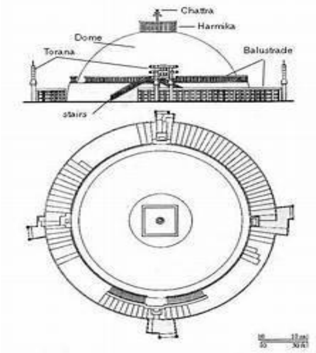
(ii) Find the curved surface area of hemisphere (ice cream) if the base radius is 7cm.

(iii) Find the total surface area of cone with hemispherical ice cream.

Answers: (i) $6\sqrt{3}$ cm or 343 cm^3 (ii) 308 cm^2 (iii) 858 cm^2

CASE STUDY 3:

The Great **Stupa** at **Sanchi** is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. .It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. (Take =)



(i) Calculate the volume of the hemispherical dome if the height of the dome is 21 m.

(ii) Find the volume of the cuboidal shaped top is with dimensions 8m ×6m ×4m.

(iii) Find the cloth require to cover the hemispherical dome if the radius of its base is 14m.

or

Find the total surface area of the combined figure i.e. hemispherical dome with radius 14m and cuboidal shaped top with dimensions 8m ×6m ×4m.

Answers: (i)19404 cu. m (ii) 192m^3 (iii) 1232 sq. m or 1392 sq. m

STATISTICS

GIST OF THE LESSON:

Statistics is one of the parts of mathematics in which we study about the collecting, organizing, analyzing, interpreting and presenting data. Statistics is very helpful in real life situations as it is easy to understand if we represent a data in a particular number which represents all numbers. This number is called the **measure of central tendency**. Some of the central tendencies commonly in use are -

Mean: It is the average of "n" numbers, which is calculated by dividing the sum of all the numbers by n.

The mean $\bar{X}$ of n values $x_1, x_2, x_3, \ldots\ldots x_n$ is given by $\bar{x} = \frac{x_1+x_2+x_2+\cdots+x_n}{n}$

Median: If we arrange the numbers in an ascending or descending order then the middle number of the series will be median. If the number of series is even then the median will be the average of two middle numbers.

If n is odd then the median is $\left(\frac{n+1}{2}\right)^{th}$ observation.

If the n is even then the median is the average of $\left(\frac{n}{2}\right)^{th}$ *and* $\left(\frac{n+1}{2}\right)^{th}$ observation.

Mode: The number which appears most frequently in the series then it is said to be the mode of n numbers.

Mean of Grouped Data (Without Class Interval):

If the data is organized in such a way that there is no class interval then we can calculate the mean by $\bar{x} = \frac{f_1x_1+f_2x_2+\cdots+f_nx_n}{f_1+f_2+\cdots+f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$

where, $x_1, x_2, x_3, \ldots\ldots x_n$ are the observations

$f_1, f_2, f_3, \ldots\ldots f_n$ are the respective frequencies of the given observations.

Mean of Grouped Data (With Class-Interval):

When the data is grouped in the form of class interval then the mean can be calculated by three methods.

1. Direct Method

In this method, we use a midpoint which represents the whole class. It is called the **class mark**. It is the average of the upper limit and the lower limit.

Class Mark= $\frac{upper\ class\ limit - lower\ class\ limit}{2}$

Or, $\bar{x} = \frac{\sum f_ix_i}{\sum f_i}$

2. Deviation or Assumed Mean Method:

If we have to calculate the large numbers then we can use this method to make our calculations easy. In this method, we choose one of the x's as **assumed mean** and let it as "a". Then we find the deviation which is the difference of assumed mean and each of the x. The rest of the method is the same as the direct method.

$$\bar{x} = a + \frac{\sum f_id_i}{\sum f_i}$$

a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i

3. Step Deviation Method:

In this method, we divide the values of d with a number "h" to make our calculations easier.

$\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$, where a is the assumed mean, h is the class size and $u_i = \frac{x_i - a}{h}$

Mode of Grouped Data:

In the ungrouped data the most frequently occurring no. is the mode of the sequence, but in the grouped data we can find the class interval only which has the maximum frequency number i.e**. the modal class.**

The value of mode in that modal class is calculated by

Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

l = lower class limit of the modal class

h = class interval /size

f_1 =frequency of the modal class

f_0 =frequency of the preceding class

f_2 = frequency of the succeeding class

Median of Grouped Data:

To find the median of a grouped data, we need to find the cumulative frequency and n/2, then we have to find the median class, which is the class of the cumulative frequency near or greater than the value of n/2.

Cumulative Frequency is calculated by adding the frequencies of all the classes preceding the given class.

Then substitute the values in the formula

Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

where l = lower limit of median class

n = no. of observations

cf = cumulative frequency of the class preceding to the median class

f = frequency of the median class

h = size of class

Remark: The empirical relation between the three measures of central tendency is

3 Median = Mode + 2 Mean

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. In the formula $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$, for finding the mean of grouped frequency distribution, u_i =

   (A) $\frac{x_i + a}{h}$ (B) $h\,(x_i - a)$ (C) $\frac{x_i - a}{h}$ (D) $\frac{a - x_i}{h}$

   Answer: C

2. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

   the sum of lower limits of the median class and modal class is

   (Basic)

   (A) 15 (B) 25 (C) 30 (D) 35

   Answer: B

Solution: The cumulative frequency Table is:-

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Here, $\frac{n}{2} = \frac{66}{2} = 33$
Thus 37 is just greater than 33,
∴ The median class is "10−15."
The lower limit of the median class is 10.
Modal class = class with maximum frequency.
∴ Modal class is 15−20.
The lower limit of the modal class is 15.
Sum of lower limits of median class and modal class = 10 + 15 = 25

3. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below:

Class	13.8-14	14-14.2	14.2-14.4	14.4-14.6	14.6-14.8	14.8-15
Frequency	2	4	5	71	48	20

The number of athletes who completed the race in less than 14.6 seconds is: (Basic)
(A) 11 (B) 71 (C) 82 (D) 130
Answer: C
Solution: The number of athletes who completed the race in less than 14.6 is equal to sum of all frequencies up to14.4−14.6 class interval =2+4+5+71=82

4. While computing mean of grouped data, we assume that the frequencies are
(A) evenly distributed over all the classes
(B) centred at the class marks of the classes
(C) centred at the upper limits of the classes
(D) centred at the lower limits of the classes
Answer: B

5. In the following distribution:

Monthly income range (in Rs)	Number of families
Income more than Rs 10000	100
Income more than Rs 13000	85
Income more than Rs 16000	69
Income more than Rs 19000	50
Income more than Rs 22000	33
Income more than Rs 25000	15

the number of families having income range (in Rs) 16000 – 19000 is
(A) 15 (B) 16 (C) 17 (D) 19
Answer: D

ASSERTION-REASON BASED QUESTIONS (1 MARK)

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of Reason (R) . Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion(A) is true but reason(R) is false.

(d) Assertion(A) is false but reason(R) is true.

1. Assertion: If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.
   (Basic)

   Reason: Median = $\frac{mode + 2\ mean}{2}$

   Answer: (c)

   Solution: Median =$\frac{1}{3}$ (mode + 2 mean) =$\frac{\mathbf{1}}{\mathbf{3}}$ (60+2×66) = 64

   Hence the reason is incorrect and the assertion is correct.

2. Assertion: The arithmetic mean of the following given frequency distribution table is 13.81.

x	4	7	10	13	16	19
f	7	10	15	20	25	30

   Reason: $\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

   Answer: (a)

SHORT ANSWER TYPE -I QUESTIONS (2 MARKS)

1. Construct the cumulative frequency distribution of the following distribution:
   (Basic)

Class	12.5-17.5	17.5-22.5	22.5-27.5	27.5-32.5	32.5-37.5
Frequency	2	22	19	14	13

   Solution: The required cumulative frequency distribution of the given distribution is given below:

Class	Frequency	Cumulative frequency
12.5-17.5	2	2
17.5-22.5	22	24
22.5-27.5	19	43
27.5-32.5	14	57
32.5-37.5	13	70

2. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	18	23	18	8	3

   Determine the median percentage of marks.
   (Basic)

Solution:

Marks (Class)	No. of students (Frequency)	Cumulative frequency
30-35	14	14
35-40	16	30
40-45	18	48
45-50	23	71
50-55	18	89
55-60	8	97
60-65	3	100

Here, n = 100.

Therefore, $\frac{n}{2}$= 50, This observation lies in the class 45-50.

l (the lower limit of the median class) = 45

cf (the cumulative frequency of the class preceding the median class) = 48

f (the frequency of the median class) = 23

h (the class size) = 5

Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 45 + \left(\frac{50-48}{23}\right) \times 5 = 45 + \frac{10}{23} = 45.4$

So, the median percentage of marks is 45.4

3. The following table gives the number of pages written by Sarika for completing her own book for 30 days:

Number of pages written per day	16-18	19-21	22-24	25-27	28-30
Number of days	1	3	4	9	13

Find the mean number of pages written per day.
(Basic)

Answer: 26

4. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows:

Speed (km/h)	85-10	100-115	115-130	130-145
Number of players	11	9	8	5

Calculate the median bowling speed.

Answer: 109.17 km/h

5. The median class of a frequency distribution in 125-145. The frequency of the median class and cumulative frequency of the class preceding the median class are 20 and 22 respectively. Find the sum of frequencies if the median is 137.

Answer: 68

SHORT ANSWER TYPE -II QUESTIONS (3 MARKS)

1. The frequency distribution table of agricultural holdings in a village is given below:

Area of land (in hectares)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holdings of the village.
(Basic)

Solution: Here the maximum class frequency is 80, and the class corresponding to this frequency is 5-7.

So, the modal class is 5-7.

l (lower limit of modal class) = 5

f_1(frequency of the modal class) = 80

f_0(frequency of the class preceding the modal class) = 45

f_2(frequency of the class succeeding the modal class) = 55

h (class size) = 2

Mode = = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h == 5 + \frac{80-45}{2\times 80-45-55} \times 2 = 5 + \frac{35}{60} \times 2 =$ = 5 + 1.2 = 6.2

Hence, the modal agricultural holdings of the village is 6.2 hectares

2. In a hospital, the ages of diabetic patients were recorded as follows. Find the median age.

(Basic)

Age in years	0-15	15-30	30-45	45-60	60-75
Number of patients	5	20	40	50	25

Solution:

Class interval	Frequency	cf
0-15	5	5
15-30	20	25
30-45	40	65
45-60	50	115
60-75	25	140

Here,

n=140 $\therefore \frac{n}{2}$=70

cf>70 is 115

Median class = 45−60

So, l=45, h=15, f=50 and cf=cf of preceding class i.e. 65

Now, Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 45 + \left(\frac{70-65}{50}\right) \times 15$

$= 45 + \frac{5}{50} \times 15$= 45+1.5=46.5

Therefore, median age of diabetic patients is 46.5 years.

3. Find the unknown entries a, b, c, d, e, f in the following distribution of heights of students in a class:

(Basic)

Height (in cm)	Frequency	Cumulative frequency
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
	Total	50

Answer: a = 12, b = 13, c = 35, d = 8, e = 5, f = 50

4. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.
Answer: 65.625 hours

5. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Answer: x = 8, y = 7

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.
(Basic)

Class Interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	12	a	12	15	b	6	6	4

Solution: We prepare the cumulative frequency table, as shown below:

Class	Frequency(f_i)	Cumulative frequency(cf)
0-5	12	12
5-10	a	12+a
10-15	12	24+a
15-20	15	39+a
20-25	b	39+a+b
25-30	6	45+a+b
30-35	6	51+a+b
35-40	4	55+a+b
	Total	70

Total $\quad n = \sum f_i = 70 \Rightarrow \frac{n}{2} = 35$

Let a and b be the missing frequencies of class intervals 5−10 and 20−25 respectively. Then, 55 + a + b = 70 ⇒ a + b = 15 ...(1)

Median is 16, which lies in 15−20. So, the median class is 15−20.

∴ l = 15, h = 5, $\frac{n}{2} = 35$, f = 15 and cf = 24 + a

Now, Median= $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 15 + \left(\frac{35-(24+a)}{15}\right) \times 5 = 15 + \frac{11-a}{3}$

$\Rightarrow 16 = 15 + \frac{11-a}{3}$

$\Rightarrow 1 = \frac{(11-a)}{3}$

⇒3=11-a $\qquad$ ⇒a=11-3 = 8

∴ b = 15 − a [From (1)]

$\Rightarrow b = 15 - 8$ $\Rightarrow b = 7$

Hence, a = 8 and b = 7.

2. Find the mean, median and mode of the following data:

(Basic)

Class interval	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency	2	3	5	6	5	3	1

Solution:

Class interval	Mid value xi	Frequency fi	fixi	cf
0−50	35	2	50	2
50−100	75	3	225	5
100−150	125	5	625	10
150−200	175	6	1050	16
200−250	225	5	1127	21
250−300	275	3	825	24
300−350	325	1	325	25
		∑fi=25	∑fixi=4225	

$\Rightarrow$ Mean = $\frac{\sum fixi}{\sum fi}$ = $\frac{4225}{25}$ = 169

$\Rightarrow$ We have n=25, then, $\frac{n}{2}$ =12.5

$\Rightarrow$ So, median class is 150−200.

$\therefore$ l=150, h=200−150=50, f=6, cf=10

$\Rightarrow$ Median= $l + \left(\frac{\frac{n}{2}-cf}{f}\right) \times h = 150 + \left(\frac{12.5-10}{6}\right) \times 50 = 150 + \frac{125}{6}$

$\therefore$ Median=150+20.83=170.83.

$\Rightarrow$ Here maximum frequency is 6, then the corresponding class 150−200 is the modal class.

$\Rightarrow$ l=150, h=50, f_1=6, f_15 =5, f_2 =5

$\Rightarrow$ Mode= $150 + \frac{6-5}{2\times 6-5-5} \times 50 = 150 + \frac{1}{2} \times 50 = 150 + 25 = 175$

3. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

(Basic)

No. of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

Answer: Mode = 44.7 cars (Competency Based Question)

4. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Answer: Median weight = 56.67 kg

5. The mean of the following frequency distribution is 57.6 and the sum of the observation is 50. Find the missing frequencies f_1 and f_2.

Class interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	f_1	12	f_2	8	5

Answer: f_1 = 8, f_2 = 10

CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

The COVID-19 pandemic, also known as coronavirus pandemic, is an ongoing pandemic of coronavirus disease caused by the transmission of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) among humans.

(Basic)

The following tables shows the age distribution of case admitted during a day in two different hospitals

Table 1

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	6	11	21	23	14	5

Table 2

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	8	16	10	42	24	12

Refer to table 1

(i) What is the average age for which maximum cases occurred?

Solution: Here, Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 35 + \left(\frac{23-21}{2\times23-21-14}\right) = 35 + \frac{20}{11} = 36.82$

(ii) Find the mean of the given data.

Solution: Required mean = $\frac{6\times10+11\times20+21\times30+23\times40+14\times50+5\times60}{6+11+21+23+14+5} = \frac{2830}{80} = 35.375 =$ 35.4

Refer to table 2

(iii) Find the mode of the given data.

Solution: Here, mode == $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) = 35 + \left(\frac{42-10}{84-10-24}\right) \times 10 = 35 + \frac{32}{5} = 41.4$

or

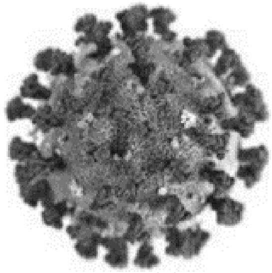
Find the median of the given data.

Solution: Here, median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 35 + \left(\frac{56-34}{42}\right) \times 10 = 35 + \frac{220}{42} = 40.24$

CASE STUDY 2:

Direct income in India was drastically impacted due to the COVID-19 lockdown. Most of the companies decided to bring down the salaries of the employees upto 50%. The following table shows the salaries (in percent) received by 50 employees during lockdown.

Salaries received (in %)	50-60	60-70	70-80	80-90
Number of employees	18	12	16	4



Based on the above information, answer the following questions.
(Basic)
(i) Find the total number of persons whose salary is reduced by more than 20%.
or
Find the total number of persons whose salary is reduced by atmost 40%.
(ii) Find the median class of the given data.
(iii) What is the empirical relationship among mean, median and mode?
Answers: (i) 46 or 32 (ii) 60-70 (iii) Mode = 3 Median- 2 Mean

CASE STUDY 3:

Electricity energy consumption is the form of energy consumption that uses electric energy. Global electricity consumption continues to increase faster than world population, leading to an increase in the average amount of electricity consumed per person (per capita electricity consumption).

Tariff	: LT - Residential	Bill Number	: 384756
Type of Supply	: Single Phase	Connected Load	: 3 kW
Meter Reading Date	: 31-11-13	Meter Reading	: 65789
Previous Reading Date	: 31-10-13	Previous Meter Reading	: 65500
		Units Consumed	: 289

A survey is conducted for 56 families of a Colony A. The following tables gives the weekly consumption of electricity of these families.

Weekly consumption (in units)	0-10	10-20	20-30	30-40	40-50	50-60
No. of families	16	12	18	6	4	0

The similar survey is conducted for 80 families of Colony B and the data is recorded as below:

Weekly consumption (in units)	0-10	10-20	20-30	30-40	40-50	50-60
No. of families	0	5	10	20	40	5

Refer to data received from Colony A

(i) Find the median weekly consumption.

or

Find the mean weekly consumption.

Refer to data received from Colony B

(ii) What is the modal weekly consumption?

(iii) What is the mean weekly consumption?

Answers: (i) 20 units or 19.64 units, (ii) 43.6 units, (iii) 38.75 units

PROBABILITY

GIST OF THE LESSON:

Probability is the study of mathematics which calculates the degree of uncertainty. There are two types of approaches to study probability-

1. **Experimental or Empirical Probability**

The result of probability based on the actual experiment is called experimental probability. In this case, the results could be different if we do the same experiment again.

$$P(E) = \frac{Number\ of\ trials\ in\ which\ the\ event\ happened}{total\ no.of\ trials}$$

2. **Probability — A Theoretical Approach**

In the theoretical approach, we predict the results without performing the experiment actually. The other name of theoretical probability is classical probability.

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

Where the outcomes are equally likely.

Equally Likely Outcomes: If we have the same possibility of getting each outcome then it is called equally likely outcomes.

Not Equally Likely: If we don't have the same possibility of getting each outcome then it is said to be the not equally likely outcome.

Elementary Event: If an event has only one possible outcome then it is called an elementary event.

Remark: The sum of the probabilities of all the elementary events of an experiment is 1.

Impossible Events: If there is no possibility of an event to occur then its probability is zero. This is known as an impossible event.

Sure or Certain Event: If the possibility of an event to occur is sure then it is said to be the sure probability. Here the probability is one.

This shows that the probability of an event could be $0 \leq P(E) \leq 1$

•For an event E, P(E) + P (E) = 1, where E is the event 'not E'. E is called the complement of the event E.

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. The probability that a non-leap year selected at random will contain 53 Sundays is (Basic)

   (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

   Answer: A

   Solution: A non-leap year has 365 days i.e. 52 weeks and 1 odd days.

   So, there are 52 Sundays always.

   But the odd day can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday.

So, there are 7 possibilities for odd day and out of which one is favourable.

So, the probability of getting Sunday on last week is $\frac{1}{7}$

Thus, the probability of getting 53 Sundays in a non-leap year is $\frac{1}{7}$

2. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is
(Basic)

(A) 7 (B) 14 (C) 21 (D) 28

Answer: B

Solution: Given, Total number of eggs = 400

The probability of getting a bad egg P (E) = 0.035

Let the number of bad eggs = x

We know that, Probability of getting a bad egg, P(E) = $\frac{Number\ of\ \ bad\ eggs}{Total\ number\ of\ eggs}$

$\Rightarrow$ 0.035 = $\frac{x}{400}$

$\Rightarrow \frac{35}{1000} = \frac{x}{400}$

$\Rightarrow \frac{35}{1000} \times 400 = x \quad \Rightarrow x = 14$

Therefore, the number of bad eggs = 14

3. Which of the following cannot be the probability of an event?
(Basic)

$(A)\,\frac{1}{3}$ (B) 0.1 (C) 3% (D) $\frac{17}{16}$

Answer: D

4. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(A) 40 (B) 240 (C) 480 (D) 750

Answer: C

5. Someone is asked to take a number from 1 to 100. The probability that it is a prime is

(A) $\frac{1}{5}$ (B) $\frac{6}{25}$ (C) $\frac{1}{4}$ (D) $\frac{13}{50}$

Answer: C

ASSERTION-REASON BASED QUESTIONS (1 MARK)

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of Reason (R) . Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion(A) is true but reason(R) is false.

(d) Assertion(A) is false but reason(R) is true.

1. Assertion: Card numbered as 1, 2, 3..………..15 are put in a box and mixed thoroughly, one card is then drawn at random. The probability of drawing an even number is 7/15.
Reason: For any event E, we have $0 \leq P(E) \leq 1$. (Basic)

Answer: (b)

Solution: We have,

Total number of cards =15

Even cards=2,4,6,8,10,12,14

Even card number=7

Probability of an even card= $\frac{7}{15}$

In an experiment, the probability of an event is the likelihood of the occurrence of that event.
If P(E) represents the probability of an event E, then
P(E)= $\frac{Number\ of\ all\ possible\ outcomes}{Number\ of\ outcomes\ favorable\ to\ event\ E}$, 0≤P(E)≤1
P(E)=0 if and only if E is an impossible event.
P(E)=1 if and only if E is a sure event.

2. If a box contains 5 white, 2 red and 4 black marbles, then the probability of not drawing a white marble from the box is $\frac{5}{11}$.
Reason: P($\bar{E}$) = 1 – P(E), where E is any event.
Answer: (d)

SHORT ANSWER TYPE -I QUESTIONS (2 MARKS)

1. If P(E) = 0.05, what is the probability of 'not E'?
(Basic)
Solution: We know that the sum of the probabilities of all the elementary events of an experiment is 1.
If an experiment has two elementary events say E & not E then,
P(E)+P(notE)=1
⟹ P(notE)=1−P(E)
∴P(notE)=1−0.05=0.95 (Given P(E)=0.05)
2. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a prime number?
(Basic)
Solution: According to the question, two dice are thrown simultaneously.
So, that number of possible outcomes =36
Sum of the numbers appearing on the dice is a prime number i.e. 2,3,5,7 and 11
So, the possible ways are
(1,1),(1,2),(2,1),(1,4),(2,3),(3,2),(4,1),(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(5,6) and (6,5)
Number of possible ways =15
∴ Required probability = $\frac{15}{36} = \frac{5}{12}$
3. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?
(Basic)
Answer: $\frac{5}{13}$
4. A coin is tossed two times. Find the probability of getting at most one head.
Answer: $\frac{3}{4}$
5. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
(i) an orange flavoured candy? (ii) a lemon flavoured candy?
Answer: (i) 0, (ii) 1

SHORT ANSWER TYPE -II QUESTIONS (3 MARKS)

1. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.
(Basic)
Solution: Total no. of possible outcomes = 6^2 = 36
i.e. (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Out of these, product of outcomes that would be less than 9 would be
(1,1), (1,2), (2,1), (1,3), (3,1), (1,4), (4,1), (1,5), (5,1), (1,6), (6,1), (2,2), (2,3), (3,2), (2,4), (4,2).

Probability = $\frac{16}{36} = \frac{4}{9}$

2. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
   (Basic)

   Solution: Number of good pens = 132
   Number of defective pen = 12
   ∴ Total pens = 132 + 12 = 144
   Probability of getting good pens = $\frac{132}{144} = \frac{11}{12}$

3. Two dice are thrown at the same time. Find the probability of getting
   (i) same number on both dice.
   (ii) different numbers on both dice
   (Basic)

   Answer: (i) $\frac{1}{6}$ (ii) $\frac{5}{6}$

4. A bag contains 24 balls of which x are red, 2x are white and 3x are blue. A ball is selected at random. What is the probability that it is
   (i) not red? (ii) white?

   Answer: (i) $\frac{5}{6}$ (ii) $\frac{1}{3}$

5. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

   Answer: $\frac{3}{4}$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is
   (Basic)
   (i) a heart (ii) a king (iii) Clubs

   Solution: Here, King, queen and jack of club are removed from the Deck of 52 playing cards.
   So remaining cards in deck=52−3=49
   Total number of outcomes=49
   (i) Cards which are heart
   Total number of heart cards = 13
   We know that PROBABILITY = $\frac{Number\ of\ favourable\ outcomes}{number\ of\ total\ outcomes}$
   Hence probability of getting a heart card = $\frac{13}{49}$
   (ii)We know that there are 4 Kings in a deck. After remove removing a king of club we left with 3 kings.

Hence probability of getting a king = $\frac{3}{49}$

(iii) Cards which are clubs

Total number of club cards =13

The king, queen and jack of clubs are removed

Hence total number of club cards left =10

Hence probability of getting a club card = $\frac{10}{49}$

2. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a (Basic)

   (i) triangle (ii) square (iii) square of blue colour (iv) triangle of red colour

   Solution: Total number of articles n(s)=10+8=18

   The total number of blue triangles is 3.

   The total number of red triangles is 5.

   The total number of blue squares is 6.

   The total number of red squares is 4.

   (i) The probability that lost piece is a triangle is calculated as follows:

   P(triangle) = $\frac{Number\ of\ triangle}{number\ of\ articles}$

   ⇒P(triangle) = $\frac{8}{18} = \frac{4}{9}$

   (ii) The probability that lost piece is a square is calculated as follows:

   P(square) = $\frac{Number\ of\ square}{number\ of\ articles}$

   ⇒P(square) = $\frac{10}{18} = \frac{5}{9}$

   (iii) The probability that lost piece is a square of blue colour is calculated as follows:

   P(blue square) = $\frac{Number\ of\ blue\ square}{number\ of\ articles}$

   ⇒P(blue square) = $\frac{6}{18} = \frac{1}{3}$

   (iv) The probability that lost piece is a triangle of red color is calculated as follows:

   P(red triangle) = $\frac{Number\ of\ red\ triangle}{number\ of\ articles}$

   ⇒P(red triangle) = $\frac{5}{18}$

3. A bag contains 18 balls out of which x ball is red. (Basic)

   (i) If one ball is drawn at random from the bag, what is the probability that it is red ball?

   (ii)If 2 more red ball put in the bag, the probability of drawing a red ball will be 9/8 times that of probability of red ball coming in part (i). find x.

   Answer: (i) x/18 (ii) x=8

4. In a game, the entry fee is Rupees 5. The game consists of tossing a coin three times. If one or two heads shows, Sweta gets her entry fee back. If she tosses 3 heads, See receives double the entry fee. Otherwise, She will loss. For tossing a coin three times, find the probability that she

   (i) Loses the entry fee.

   (ii) gets double entry

   (iii) Third just gets her entry fee.

   Answer: (i) $\frac{1}{8}$, (ii) $\frac{1}{8}$, (iii) $\frac{3}{4}$ (Competency Based Question)

5. A bag contains white, black and red balls only. A ball is drawn at random from the bag. The probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$. Find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

   Answer: P (Red Ball) = $\frac{3}{10}$, total number of balls in the bag = 50

CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

On a weekend Rani was playing cards with her family. The deck has 52 cards. If her brother drew one card.

(Basic)

(i) Find the probability of getting a king of red colour.

Solution: No. of cards of a king of red colour = 2

Total no. of cards = 52

Probability of getting a king of red colour

$= \frac{No.of\ king\ of\ red\ colour}{Total\ number\ of\ cards} = \frac{2}{52} = \frac{1}{26}$

(ii) Find the probability of getting a jack of hearts.

Solution: Since there is only one jack of hearts.

∴ Favourable number of elementary events =1

∴ Probability of getting the jack of heart = $\frac{1}{52}$

or

Find the probability of getting a face card.

Solution: No. of face card = 12

Total no of cards = 52

Probability of getting a face card = $\frac{No.of\ face\ cards}{Total\ no.of\ cards} = \frac{12}{52} = \frac{3}{13}$

(iii) Find the probability of getting a spade.

Solution: No. of spade card = 13

Total no of cards = 52

Probability of getting a face card = $\frac{No.of\ spade\ cards}{Total\ no.of\ cards} = \frac{13}{52} = \frac{1}{4}$

CASE STUDY 2:

Rahul and Ravi planned to play Business (board game) in which they were supposed to use two dice.

(Basic)

(i) Ravi got first chance to roll the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 8?

(ii) Rahul got next chance. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 13?

(iv) Rahul got next chance. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is equal to 7?

or

Now it was Ravi's turn. He rolled the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is greater than 8?

Answers: (i) $\frac{5}{36}$ (ii) 0 (iii) $\frac{1}{6}$ or $\frac{5}{18}$

CASE STUDY 3:

A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes.

(i) What is the probability that arrow will point at 8 ?

(ii) What is the probability that arrow will point at an odd number?

(iii) What is the probability that arrow will point at a number greater than 2?

or

What is the probability that arrow will point at a number less than 9?

Answers: (i) $\frac{1}{8}$, (ii) $\frac{1}{2}$, (iii) $\frac{3}{4}$, (iv) 1

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CLASS - X (2023-24)

SAMPLE QUESTION PAPER (SOLVED)

SUBJECT – MATHEMATICS (STANDARD)

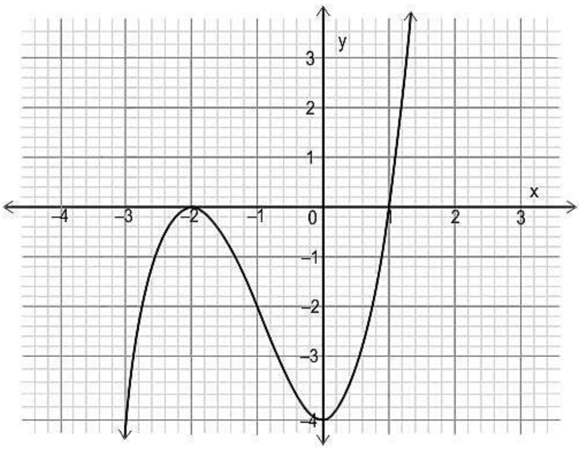
TIME: 3 hours **MAX.MARKS: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

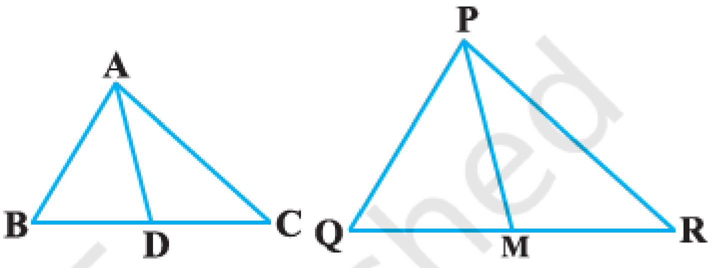
SECTION - A

Section - A consists of 20 questions of 1 mark each.

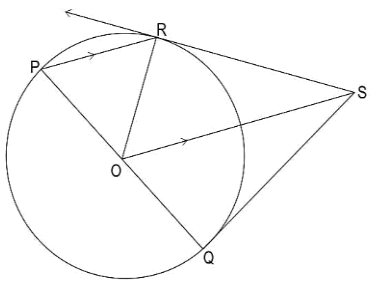
1.	The sum of exponents of prime factors in the prime factorization of 1771 is. (a) 3. (b) 4 (c) 5 (d) 6	1
2.	The number of zeroes for a polynomial p(x) where graph of y = p(x) in given figure, is  (a) 3 (b) 4 (c) 2 (d) 5	1
3.	A pair of linear equationsx = 2 and x = 5 has (a)no common solution (b)infinitely many solutions (c)unique solution (d)none of these	1

4.	The quadratic equation $2x^2 - 6x + 3 = 0$ has (a) two distinct real roots (b) two equal real roots (c) no real roots (d) more than two real roots	1
5.	For what value of p, are (2p+1), 13,(5p-3) three consecutive terms of an AP? (a) 1 (b) 4 (c) 5 (d) 6	1
6.	In triangle ABC, D and E are points on the sides AB and AC respectively, such that DE ǁ BC. If AD=x,DB=x-2,AE =x+2 and EC=x-1.Wnat is the value of x? (a) 4 (b) 2 (c) 1 (d) none of these	1
7.	A triangle with vertices (3,2),(-2,-3) and (2,3) is a/an (a)equilateral triangle (b)right-angled triangle (c)isosceles triangle (d)none of these	1
8.	The co-ordinates of the point which divides the line–segment joining the points (4,-3) and(8,5)in the ratio 3:1,internal; is :- (a) (3,7) (b) (-3,-7) (c) (7,3) (d) none of these	1
9.	A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ=12cm.Length PQ is :- (a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$cm	1
10.	If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, the ∠POA is equal to :- (a) 50° (b) 60° (c) 70° (d) none of these	1
11.	What is the value of $\frac{4\, sin60°}{1+tan^2 60°}$? (a) $sin60°$ (b) $cos60°$ (c) $tan60°$ (d) $sin30°$	1
12.	If tanA + cotA =5, then the value of $\tan^3 A + \cot^3 A$=? (a) 100 (b) 115 (c) 110 (d) none of these	1
13.	The angle of elevation of the top of a tower from a point on the ground is 30m away from the foot of the tower is 60°. The height of the tower is :- (a) $10\sqrt{3}$ m (b) $18\sqrt{3}$ m (c) 18m (d) none of these	1
14.	The area of quadrant of a circle whose circumference is 44cm be	1

	(a) 77cm^2 (b)38.5cm^2 (c)115.5cm^2 (d) none of these	
15.	The areas of two circles are in the ratio of16:49. what is the ratio of their perimeters:- (a)64:243 (b)16:49 (c)4:7 (d) none of these	1
16.	One card is drawn from a well -shuffled deck of 52 cards. The probability of "not be an ace" is:- (a) $\frac{10}{13}$ (b) $\frac{11}{13}$ (c) $\frac{12}{13}$ (d) none of these	1
17.	A coin is tossed twice. The probability of getting both heads is :- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) none of these	1
18.	The difference of mode and median of a data is 24,then the difference of median and mean is:- (a) 8 (b) 12 (c) 24 (d) 36	1
19.	DIRECTION: In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option Statement A (Assertion): If the volume of a cuboid, whose dimensions are in the ratio of 5:4:3 is 480m^3.Then, its one side is 8m. Statement R (Reason): The total surface area of a cuboid = 2(lb + bh + hl). a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) c) Assertion (A) is true but reason (R) is false. d) Assertion (A) is false but reason (R) is true.	1
20.	Statement A (Assertion): Sum of first 51 terms of an AP:-10, 14, 18,…….. is 5610. Statement R (Reason): Sum of n term of an AP=$\frac{n}{2}[2a+(n-1)d]$, where a= first term and d=common difference. a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) c) Assertion (A) is true but reason (R) is false. d) Assertion (A) is false but reason (R) is true.	1
	SECTION - B	
	Section - B consists of 5 questions of 2 marks each.	
21.	Check whether 4^n can end with the digit 0 for any natural number n.	2
22.	Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.	2

		
23.	Prove that the lengths of tangents drawn from an external point to a circle are equal.	2
24.	If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; where $0 < A + B \leq 90^0$; $A > B$. Find the value of A and B.	2
	[OR]	
	Find the value of $\sin 60^0$, geometrically.	
25.	A chord of a circle of radius 10cm subtends a right-angle at the centre.Find the area of the corresponding minor sector.	2
	OR	
	The length of the minute hand of clock is 21 cm. Find the area swept by the minute hand in 10 minutes.	
	SECTION - C	
	Section - C consists of 6 questions of 3 marks each	
26.	Prove that $\sqrt{5}$ is an irrational number.	3
27.	Find the zeroes of the quadratic polynomial $(6x^2 - 3 - 7x)$, and verify the relationship between the zeroes and the coefficients.	3
28.	Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden graphically.	3
	[OR]	
	Solve: $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.	

29. In the given figure, PQ is the diameter of the circle with centre O. R is a point on the boundary of the circle, at which a tangent is drawn. A line segment is drawn parallel to PR through O ,such that it intersects the tangent at S .In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. — 3

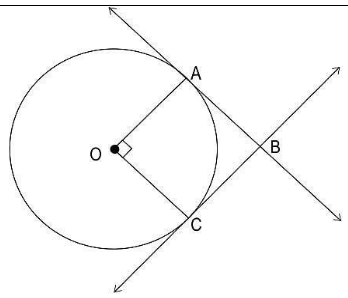


Prove that $\angle AOB = 90°$.

Show that SQ is a tangent of the circle.

[OR]

Shown below is a circle with centre O.Tangents are drawn at points A and C, such that they intersect at point B.



If $OA \perp OC$, then show that quadrilateral OABC is a square.

30. Prove that $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-tanA}$ = 1 + secA .cosecA — 3

31. Calculate the median and mode of the scores of 20 students in a mathematics test :- — 3

Marks obtained	10-20	20-30	30-40	40-50	50-60
No. of students	2	4	7	6	1

SECTION - D

Section - D consists of 4 questions of 5 marks eachu

32. A train travels a distance of 360 km at a uniform speed. If the speed had been 5 km/h more, then it would have taken 1 hour less to cover the same distance. Find the uniform speed and new speed of the train. — 5

[OR]

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 5 more than thrice the number of articles produced on that day. If the total cost of production on that day was ₹100, find the number of articles produced and the cost of each article.

33. (a)State and prove Basic Proportionality Theorem. — 5

	(b)Using B.P.T, prove that a line drawn through the mid- point of one side of a triangle parallel to another side bisects the third side.	
34.	A vessel is in the form of an inverted cone. Its height is 8cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.	5
	[OR]	
	A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take π = 3.14)	
35.	The median of the following data is 28.5. Find the values of x and y, if the total frequency is 60.	5

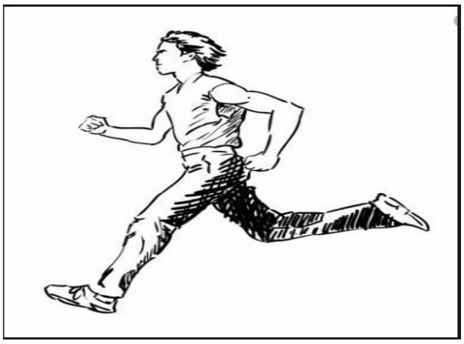
Class interval	0 - 10	10 -20	20 – 30	30 - 40	40 - 50	50 - 60
Frequency	*5*	*x*	*20*	*15*	*y*	*5*

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has twosubquestionsof2markseach.)

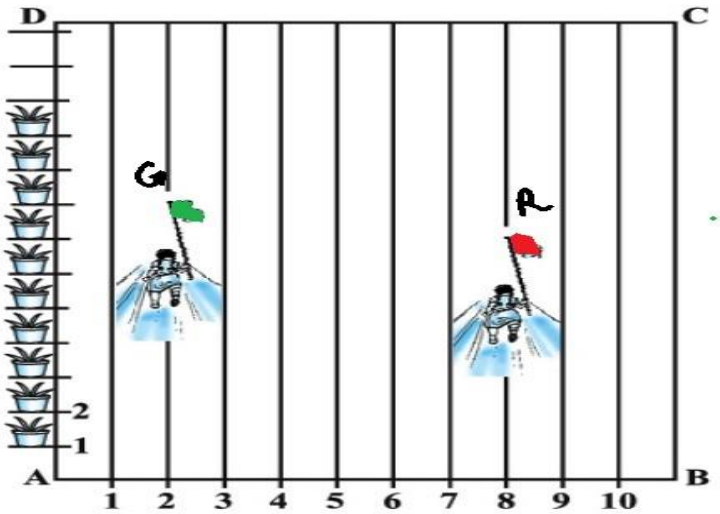
36. A high-quality PE curriculum enables all students to enjoy and succeed in many kinds of physical activities. They develop a wide range of skills and the ability to use tactics, strategies and compositional ideas to perform successfully. When they are performing, they think about what they are doing, they analyse the situation and make decisions. They also reflect on their own and others' performances and find ways to improve upon them. As a result, they develop the confidence to take part in different physical activities and learn about the value of healthy, active lifestyles.

Your friend Gaurav wants to participate in a 200m race competition of Regional Sports Meet organized by Kendriya Vidyalaya Sangathan, Patna region. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less respectively. He wants to do in 31 seconds for achieved the goal.



Based on the above information, answer the following questions:-

	(i)What are the time taken in first five days for the given situation? Is this in AP?	1
	(ii)What is the minimum number of days he needs to practice till his goal is achieved? [OR] On which day Gaurav takes 37 seconds to complete the race?	2

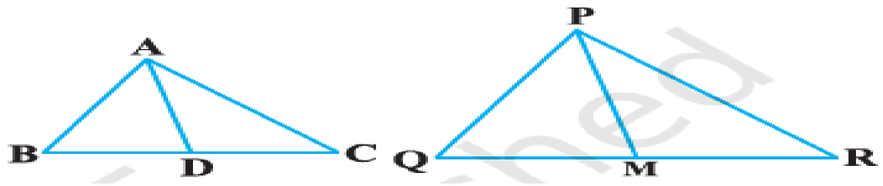
	(iii)Find the total time taken by the Gaurav starts from 51s and end at 31seconds?	1
37.	In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag. 	
	(i)What are the positions of green flag and red flag?	1
	(ii)If Rashmi has to post a blue flag exactly halfway between the line- segments joining the two flags, where should she post her flag? [OR] If Joy has to post a flag at one-fourth distance from green flag, in the line segment joining the green and red flags, then where should he post his flag?	2
	(iii)What is the distance between green flag and red flag?	1
38.	A group of students of class -X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that **India Gate**, official name **Delhi Memorial**, originally called **All-India War Memorial**, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that Indi Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.	

		
	(a)What is the angle of elevation if they are standing at a distance of 42m away from the monument?	1
	(b)They want to see the monument at an angle of 60^0; so, they want to know the distance where they should stand and hence find the distance. [OR] If the altitude of the Sun is at 60^0, then find the height of the vertical tower that will cast a shadow of length 20 m.	2
	(c) The ratio of the length of a rod and its shadow is 1:1. What is the angle of elevation of the Sun?	1

CLASS - X (2023-24)
MARKING SCHEME
SUBJECT – MATHEMATICS (STANDARD)

TIME: 3 hours **MAX.MARKS: 80**

	SECTION - A	
	Section - A consists of 20 questions of 1 mark each.	
1.	(a) 3	1
2.	(c) 2	1
3.	(a)no common solution	1
4.	(a) two distinct real roots	1
5.	(b) 4	1
6.	(a) 4	1
7.	(b)right-angled triangle	1
8.	(c) (7,3)	1
9.	(d) $\sqrt{119}$cm	1
10.	(a) 50°	1
11.	(a) $sin60°$	1
12.	(c) 110	1
13.	(d) none of these	1
14.	(b)38.5cm^2	1
15.	(c)4:7	1
16.	(c) $\frac{12}{13}$	1
17.	(a) $\frac{1}{4}$	1
18.	(b) 12	1
19.	(b)Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)	1
20.	(a)Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).	1
	SECTION - B	
	Section - B consists of 5 questions of 2 marks each.	
21.	Here, $4^n = (2x2)^n = 2^{2n}$ We know that 4^n ends with the digit O , if it has factors 2 and 5. But 4^n has only one factor 2. From the Fundamental Theorem of Arithmetic:- The prime factorization of every composite number is unique. Hence ,4^n can not end with the digit 0 for any natural number n.	½ ½ ½ ½
22.	Given, sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR.	



We have to prove that $\Delta ABC \sim \Delta PQR$. ½

Since AD and PM are medians of ΔABC and ΔPQR,

$\therefore$ BD = $\frac{1}{2}BC$ and QM = $\frac{1}{2}QR$ .... (1) ½

Given that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ .... (2)

From (1) and (2) ½

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ .... (3)

$\therefore$ By SSS criterion of proportionality $\Delta ABD \sim \Delta PQM$ ½

$\therefore \angle B = \angle Q$ (corresponding sides of similar triangles) .... (4)

Now, In ΔABC and ΔPQR

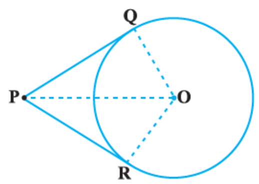
$\frac{AB}{PQ} = \frac{BC}{QR}$ (from (2))

$\angle B = \angle Q$ (from (4))

By SAS criterion of proportionality $\Delta ABC \sim \Delta PQR$.

23. We are given a circle with centre O,

a mpoint P lying outside the circle and two tangents ½



PQ, PR on the circle from P.

We are required to prove that PQ = PR.

For this, we join OP, OQ and OR. ½

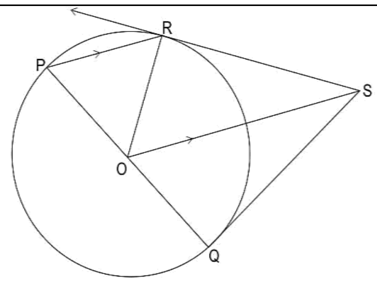
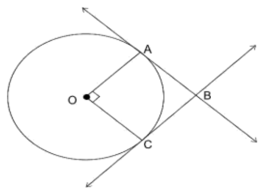
Then $\angle OQP$ and $\angle ORP$ are right angles, because these are angles between the radii and tangents, and according to Theorem 10.1 they are right angles. ½

Now in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

	OP = OP (Common) Therefore, $\Delta OQP \cong \Delta ORP$ (RHS) This gives PQ = PR (CPCT)	½
24.	Given, $\tan(A+B) = \sqrt{3}$ $\Rightarrow \tan(A+B) = \tan 60°$ $\Rightarrow A + B = 60°$ --- Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$ $\Rightarrow \tan(A-B) = \tan 30°$ $\Rightarrow A - B = 30°$ --- To find the value of A=45° -- B=15° . --	1/2 1/2 1/2 1/2
	[OR]	
	For correct construction and figure. -- To find the value of $\sin 60^0 = \frac{\sqrt{3}}{2}$ -------------------------------------	1 1
25.	Here, r= 10cm , $\theta = 90^0$ Area of the corresponding minor ssector $= \frac{\pi R^2 \theta}{360°}$ $= \frac{22 \times 10 \times 10 \times 90}{7 \times 360} = 550/7$ cm^2	1/2 1/2 1
	OR	
	We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360° In 10 minutes, minute hand will rotate $= (\frac{360}{60} \times 10) degree = 60°$ Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 60° in a circle of 21cm radius. Area of sector of angle $\theta = \frac{\pi R^2 \theta}{360°}$ $= \frac{22 \times 21 \times 21 \times 60}{7 \times 360} = 231$ cm^2	½ ½ 1
	SECTION - C	
	Section - C consists of 6 questions of 3 marks each	
26.	Prove that $\sqrt{5}$ is an irrational number. Let its simplest form be $\frac{a}{b}$; where a and b are co-prime &b $\neq$ 0.	1

	Now, $\sqrt{5} = \frac{a}{b} \Rightarrow 5 = \frac{a^2}{b^2}$ $\Rightarrow 5b^2 = a^2 \Rightarrow$ 5 divides a^2 $\Rightarrow$ 5 divides a . Hence, 5 is a factor of 'a'. Let $a = 5c$ for some integer c . $\therefore 5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$. $\Rightarrow$ 5 divides b^2. $\Rightarrow$ 5 divides b. Hence, 5 is a factor of 'b'. Thus, 5 is a common factor of a and b. But, this contradicts the fact that a and b are co-prime, that is a and b have no common factor other than 1. So, our assumption is wrong. Hence, $\sqrt{5}$ is an irrational number.	1 1
27.	Let $p(x) = 6x^2 - 3 - 7x = 6x^2 - 7x - 3) = (2x-3)(3x+1)$. For zeroes, $p(x) = 0$ $(2x-3)(3x+1) = 0$ $\Rightarrow x = \frac{3}{2}$ and $\frac{-1}{3}$. So, both the zeroes are $\frac{3}{2}$ and $\frac{-1}{3}$. a=6, b=-7, c= -3 . Sum of zeroes $= \frac{3}{2} + \left(\frac{-1}{3}\right) = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$ Product of zeroes $= \frac{3}{2} \times \left(\frac{-1}{3}\right) = \frac{-3}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$	1 ½ ½ ½ 1/2
28.	Let length of the garden = x m and breadth(width) = y m. To find two equations, x=y + 4 and x + y =36 .... To complete the tables for two equation... To draw the graph .... To find length = x m =20m and breadth = y m =16 m.	1 1 1
	[OR]	
	Here, 2x + 3y = 11 ....(i) 2x – 4y = - 24 ....(ii) To find x = -2 -- To find y = 5 -- From y = mx + 3 To find m = -1 --	1 1 1

29.	Find that $\angle OPR=\angle ORP$, and $\angle ORP=\angle ROS$.  Finds $\angle QOS=\angle ROS=\angle ORP$. Gives a valid reason. For example: -since, exterior angle property, $\angle OPR+\angle ORP=\angle QOS+\angle ROS$. $\Rightarrow 2\angle ROS=\angle QOS+\angle ROS$ $\Rightarrow \angle QOS=\angle ROS$ Writes that $\Delta ORS\cong\Delta OQS$ by SAS congruence. The working may look as follows: OS=OS (common side) OR=OQ(radius) $\angle ROS=\angle QOS$ **N**otes that as RS is a tangent to the circle, $\angle ORS=90°$. Concludes that SQ is a tangent to the circle as $\angle ORS=\angle OQS=90°$,by CPCT.	½ 1 1 1/2
	[OR]	
	Shown below is a circle with centre O. Tangents are drawn at points A and C, such that they intersect at point B. Writes that AB=BC, as they are tangents from an external point to a circle. Notes that OA=OC as they are radii.  Writes that$\angle BAO= \angle BCO=90°$ as AB and BC are tangents. Notes that OA||BC as $\angle AOC+\angle OCB=180°$(adjacent interior angles) Notes that OC||AB as $\angle AOC+\angle OAB=180°$(adjacent interior angles) Concludes that OABC is a parallelogram. Writes that as opposite sides in a parallelogram are equal, OA=BC and OC= AB. Also, as opposite angles in a parallelogram are equal, $\angle AOC=\angle ABC=90°$ (Award full marks if students first proves that OABC is a rectangle using angle sum property and then shows that the adjacent sides are equal.) Concludes that OABC is a square as all of its angles are 90°,and OA= AB= BC= OC.	1/2 1/2 1/2 1/2 1/2 **1/2**

30.	$$L.H.S = \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-tanA}$$ $= \frac{sin\,A.sinA}{cosA(sinA-cosA)} + \frac{cosA.cosA}{sinA(cosA-sinA)}$-------------------------- $=\left(\frac{sin^3A-cos^3A}{\text{sinA.cosA (sinA}-\text{cosA)}}\right)$.-- $=\frac{(\text{sin}A-\text{cos}A)(sin^2A+\text{sin}A.cosA+cos^2A)}{sinA.cosA\ (sinA-cosA)}$------------------------------ $=\frac{(sin^2A+\text{sin}A.cosA+cos^2A)}{sinA.cosA}$--- $=\frac{1+\text{sin}A.cosA}{sinA.cosA}$-- $=\frac{1}{sinA.cosA}+\frac{\text{sin}A.cosA}{sinA.cosA}$=cosecA.secA+1=1+secA.cosecA=R.H.S-----------------------	½ ½ ½ ½ ½ ½
31.	Formula for Median.-- To find Median = 35.7 (approx)--- Formula for Mode -- To find Mode = 37.5 --	1/2 1 1/2 1
	SECTION - D	
	Section - D consists of 4 questions of 5 marks each	
32.	Let uniform speed of the train = x km/h Then, new speed of the train = (x+5) km/h--- To find the equation $x^2+5x-1800=0$ -- (For each correct step ½ mark should be given). To find x=40 -- Hence, uniform speed of the train = x km/h = 40 km/h Then, new speed of the train = (x+5) km/h = 45 km/h	1 2 1 1
	[OR]	
	Let no. of articles produced in 1 day =x. Then the production cost of each article =₹(3x + 5) Total production cost =₹ x(3x+5)=₹ 100--- $3x^2 + 5x - 100=0$-- (3x-15)(x+ 20)=0-- x=5 and x= -20(negligible)------------------------------------ Hence,no. of articles produced in 1 day =₹x. The production cost of each article =₹(3x + 5)=₹ 20--	1 1 1 1 1
33.	(a)State and prove Basic Proportionality Theorem.--- For correct statement, given, we have to prove, construction and figure ------------------------.	$1\frac{1}{2}$

For correct proof . --- $1\frac{1}{2}$

(b)Using B.P.T, prove that a line drawn through the mid- point of one side of a triangle parallel to another side bisects the third side.

Given, we have to prove, construction and figure--- 1

For correct proof . -- 1

34. We have, height of the conical vessel, h = 8 cm, and radius of the conical vessel, r =5 cm--------- 1/2

Volume of water filled in the vessel cone = $\frac{1}{3}\pi r^2 h = \frac{200}{3}\pi$cm³---------------------------- 1

Also, we have radius of a spherical lead shot = 0.5 cm------------------------------------ 1/2

Volume of each lead shot= $\frac{4}{3}\pi$ r^3= $\frac{4}{3}\pi$ (0.5)3 cm³-- $1\frac{1}{2}$

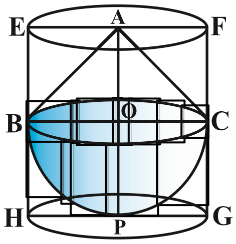
Volume of lead shots dropped =Volume of water flows out =$\frac{1}{4}$ X$\frac{200}{3}\pi$cm³------------------------ $\frac{1}{2}$

Number of lead shots dropped = 100-- 1

Hence required number of lead shots is 100

[OR]

34. For figure.-- 1



Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere .

The radius BO of the hemisphere (as well as of the cone) =$\frac{1}{2}$ × 4 cm = 2 cm. 1

So, volume of the toy = $\frac{2}{3}\pi$ r^3+$\frac{1}{3}\pi r^2 h$

3 1

= 25.12 cm3

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = HP = BO = 2 cm, and its height is

EH = AO + OP = (2 + 2) cm = 4 cm 1

So, the volume required = volume of the right circular cylinder – volume of the toy

= (3.14 × 22 × 4 – 25.12) cm3

= 25.12 cm3

Hence, the required difference of the two volumes = 25.12 cm3. 1

35.

Class-intervals	Frequency	Cumulative Frequency
0-10	5	5
10-20	x	5+x=cf
20-30	20=f	25+x
30-40	15	40+x
40-50	y	40+$x + y$
50-60	5	45+$x + y$
	N=60	

2

for correct table.-- 1

	Here ,n = 60 So, $45 + x + y = 60, i.e., x + y = 15 \ -(1)$--- Median =28.5, which lies in class –interval 20-30. So, $l = 20, f = 20, cf = 5 + x, h = 10$ $Median = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$ --- Putting the values, we have $x = 8$--- $From(1), y = 7$ --	 1 1/2 1/2
	SECTION - E **This section comprises of 3 case-study/passage-based questions of 4 markseach with two sub-questions. First two case study questions have three subquestions of marks 1, 1, 2 respectively. The third case study question has twosubquestionsof2markseach.)**	
36.	(i)51,49,47,45,43 d=--2 is a fixed number.hence, this is in A.P.	1/2 1/2
	(ii)For formula -------------------------------------- To find no. of days (n) = 11	1/2 $1\frac{1}{2}$
	OR,For formula -------------------------------------- To find no. of days (n) = 8	1/2 $1\frac{1}{2}$
	(iii) ,For formula -------------------------------------- To find total time taken = 451 seconds =7 minutes 31 seconds	1/2 1/2
37.	(i)Position of green flag = (2,25) Position of red flag = (8,20)	1/2 1/2
	(ii)To find position of blue flag = (5,22.5)	2
	OR, To find position of Joy flag = (3.5, 23.75)	2
	(ii) Distance between green flag and red flag=$\sqrt{61}m$	1
38.	(i) To find angle =45^0	1
	(ii) To find required distance =14 $\sqrt{3}$m	2
	(ii) Required distance =20 $\sqrt{3}$m	2
	(iii)To find angle =45^0	1

CLASS - X (2023-24)
SAMPLE QUESTION PAPER (UNSOLVED)
SUBJECT – MATHEMATICS (STANDARD)

TIME: 3 hours **MAX.MARKS: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION A	
	SECTION A CONTAINS 1 MARKS QUESTION EACH	
1	The decimal representation of $\frac{11}{2^3 \times 5}$ $will$ a) terminate after 1 decimal place b) terminate after 2 decimal place c) terminate after 3 decimal place d) not terminate76	1
2	If α and β are the zeros of a polynomial f(x) = px2 – 2x + 3p and α + β = αβ, then p is (a)-2/3 (b) 2/3 (c) 1/3 (d) -1/3	1
3	If the system of equations 3x+y =1 and (2k-1)x +(k-1)y =2k+1 is inconsistent, then k = (a) -1 (b) 0 (c) 1 (d) 2	1
4	ΔABC~ΔPQR. If AM and PN are altitudes of ΔABC and ΔPQR respectively and AB2 : PQ2 = 4 : 9, then AM: PN = (a) 3:2 (b) 16:81 (c) 4:9 (d) 2:3	1
5	If sinθ + cosθ = √2, then tanθ + cot θ = (a) 1 (b) 2 (c) 3 (d) 4	1
6	ABCD is a trapezium with AD ‖ BC and AD = 4cm. If the diagonals AC and BD	1

<table>
<tr><td></td><td>intersect each other at O such that AO/OC = DO/OB =1/2, then BC =

(a) 6cm (b) 7cm (c) 8cm (d) 9cm</td><td></td></tr>
<tr><td>7</td><td>If two tangents inclined at an angle of 60^{o} are drawn to a circle of radius 3cm, then the length of each tangent is equal to

(a) $3\sqrt{3}/2$ cm (b) 3cm (c) 6cm (d) $3\sqrt{3}$cm</td><td>1</td></tr>
<tr><td>8</td><td>The area of the circle that can be inscribed in a square of 6cm is

(a) 36π cm2 (b) 18π cm2

(c) 12π cm2 (d) 9π cm2</td><td>1</td></tr>
<tr><td>9</td><td>The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$cm and the length of its diagonal is $2\sqrt{3}$cm. The total surface area of the cuboid is

(a) 48 cm2 (b) 72 cm2 (c) 96 cm2 (d) 108 cm</td><td>1</td></tr>
<tr><td>10</td><td>If the difference of Mode and Median of a data is 24, then the difference of median and mean is

(a) 8 (b) 12 (c) 24 (d) 36</td><td>1</td></tr>
<tr><td>11</td><td>For the following distribution, the sum of the lower limits of the median and modal class is

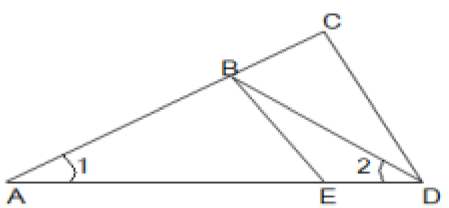
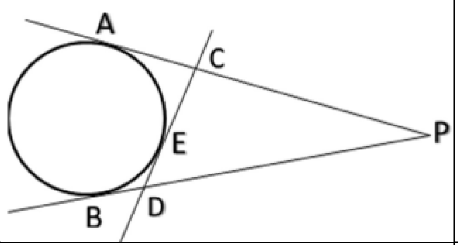
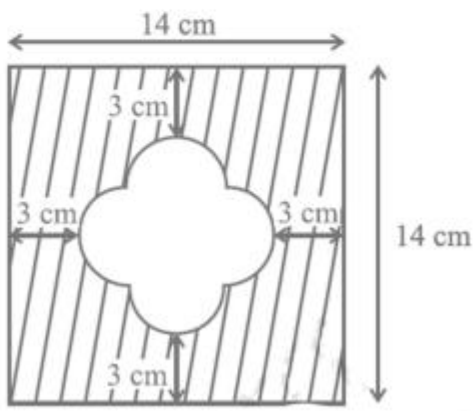
<table>
<tr><th>Classs</th><th>Frequency</th></tr>
<tr><td>0 – 5</td><td>10</td></tr>
<tr><td>5 – 10</td><td>15</td></tr>
<tr><td>10 – 15</td><td>12</td></tr>
<tr><td>15 – 20</td><td>20</td></tr>
<tr><td>20 – 25</td><td>9</td></tr>
</table>

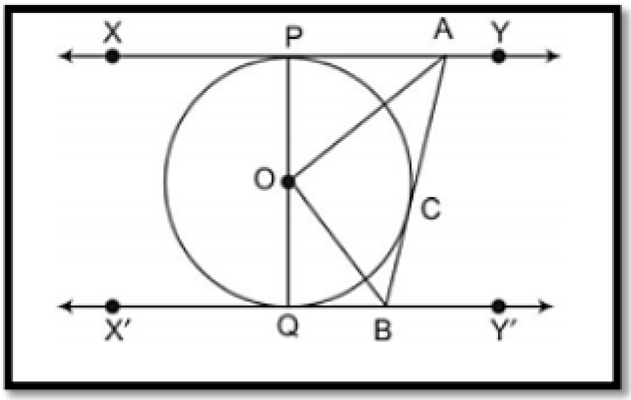
(a) 15 (b) 25 (c) 30 (d) 35</td><td>1</td></tr>
<tr><td>12</td><td>The nature of roots of the quadratic equation 9x2 – 6x – 2 = 0 is:

(a) No real roots (b) 2 equal real roots

(c) 2 distinct real roots (d) More than 2 real roots</td><td>1</td></tr>
<tr><td>13</td><td>Two APs have the same common difference. The first term of one of these is –1</td><td>1</td></tr>
</table>

	and that of the other is – 8. The difference between their 4th terms is (a) 1 (b) -7 (c) 7 (d) 9	
14	What is the ratio in which the line segment joining (2,-3) and (5, 6) is divided by x-axis? (a) 1:2 (b) 2:1 (c) 2:5 (d) 5:2	1
15	(secA + tan A) (1 – sin A) equals: (a) sec A (b) sin A (c) cosec A (d) cos A	1
16	If a pole 6 m high casts a shadow 2 $\sqrt{3}$m long on the ground, then the Sun's elevation is (a) 60° (b) 45° (c) 30° (d) 90°	1
17	If the perimeter and the area of a circle are numerically equal, then the radius of the circle is: (a) 2 units (b) π units (c) 4 units (d) 7 units	1
18	2 cards of hearts and 4 cards of spades are missing from a pack of 52 cards. A card is drawn at random from the remaining pack. What is the probability of getting a black card?	1
	DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option	
19	Statement A (Assertion): Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone. Statement R(Reason) : Top is obtained by joining the plane surfaces of the hemisphere and cone together. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.	1
20	Statement A (Assertion): If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3,5) and E(-3,-3) respectively, then BC = 20 units Statement R (Reason): The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct	1

	explanation of assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason(R) is false. (d) Assertion (A) is false but reason(R) is true.	
	SECTION B	
	Section B contains each question of 2 marks	
21	If 49x+51y= 499, 51 x+49 y= 501, then find the value of x and y	2
22	In the given figure below, AD AE = AC BD and $\angle 1 = \angle 2$. Show that Δ BAE~ ΔCAD.  	2
23	If sin(A+B) =1 and cos(A-B)= $\sqrt{3}/2$, $0° < A+B \leq 90°$ and A> B, then find the measures of angles A and B. OR Find an acute angle θ when $\cos\theta - \sin\theta / \cos\theta + \sin\theta = 1-\sqrt{3} / 1+\sqrt{3}$.	2
24	From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD.  	2
25	With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle. OR  	2

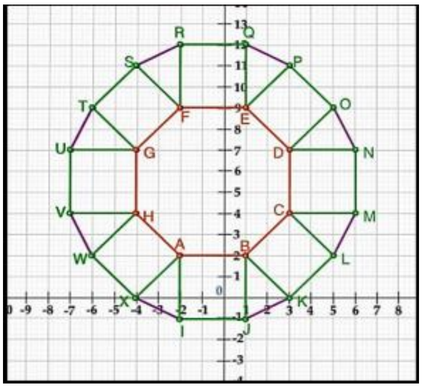
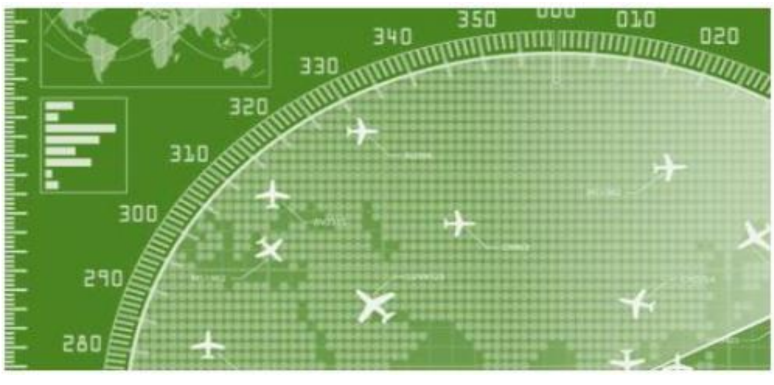
	Find the area of the unshaded region shown in the given figure.	
	SECTION C	
	Section C consists of 6 questions of 3 marks each.	
26	Given that √3 is irrational, prove that 5 + 2√3 is irrational.	3
27	If the zeroes of the polynomial x^2 +px +q are double in value to the zeroes of the polynomial $2x^2$ -5x -3, then find the values of p and q.	3
28	Prove the following that- $$\frac{tan^3\theta}{1+tan^2\theta}+\frac{cot^3\theta}{1+cot^2\theta}=sec\theta cosec\theta-2sin\theta cos\theta$$	3
29	If α , β are zeroes of quadratic polynomial $5x^2$ + 5x + 1, find the value of 1. $\alpha^{2}+\beta^{2}$. 2. $\alpha^{-1}+\beta^{-1}$	3
30	Two coins are tossed simultaneously. What is the probability of getting (i) At least one head? (ii) At most one tail? (iii) A head and a tail?	3
31	Prove that a parallelogram circumscribing a circle is a rhombus OR In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of ∠AOB 	3
	SECTION D	
	Section D consists of 4 questions of 5 marks each.	
32	To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be	5

	filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool? OR In a flight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.	
33	Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non-parallel sides in the same ratio.	5
34	The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data.	5

Marks obtained	Number of students
20 – 30	p
30 – 40	15
40 – 50	25
50 – 60	20
60 – 70	q
70 – 80	8
80 - 90	10

35	Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per m2, find the amount shared by each school to set up the tents. OR	5

	There are two identical solid cubical boxes of side 7cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find (i) the ratio of the total surface area of the two new solids formed (ii) volume of each new solid formed.	
	SECTION E	
	Competency Based Questions	
36	The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.  (i) If the first circular row has 30 seats, how many seats will be there in the 10th row? (ii) For 1500 seats in the auditorium, how many rows need to be there? OR If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row? (iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?	4
37	A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons. A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane.	4

	He used regular octagons, squares and triangles for his floor tessellation pattern  Use the above figure to answer the questions that follow: (i) What is the length of the line segment joining points B and F? (ii) The centre 'Z'of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z? (iii) What are the coordinates of the point on y axis equidistant from A and G? OR What is the area of Trapezium AFGH?	
38	We all have seen the airplanes flying in the sky but might have not thought of how they actually reach the correct destination. Air Traffic Control (ATC) is a service provided by ground-based air traffic controllers who direct aircraft on the ground and through a given section of controlled airspace, and can provide advisory services to aircraft in non-controlled airspace. Actually, all this air traffic is managed and regulated by using various concepts based on coordinate geometry and trigonometry.  At a given instance, ATC finds that the angle of elevation of an airplane from a point on the ground is 60°. After a flight of 30 seconds, it is observed that the angle of elevation changes to 30°. The height of the plane remains constantly as $3000\sqrt{3}$	4

	m. Use the above information to answer the questions that follow- (i) Draw a neat labelled figure to show the above situation diagrammatically. (ii) What is the distance travelled by the plane in 30 seconds? OR Keeping the height constant, during the above flight, it was observed that after 15(√3 -1) seconds, the angle of elevation changed to 45°. How much is the distance travelled in that duration. (iii) What is the speed of the plane in km/hr.	

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION
SAMPLE QUESTION PAPER -2023-24
Class - X
Subject - Mathematics Basic (SOLVED)

Time Allowed:3 Hours **Maximum Marks:80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D,and E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.

6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 5 marks, 2 Questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π=22/7 wherever required if not stated.

SECTION– A

Questions 1 to 20 carry1 mark each.

1. The ratio of LCM and HCF of 5,10 and 15 is

(a)1:6 (b)6:1 (c)1:2 (d)2:1

2. The largest number which divides 70 and 125 leaving remainders 5 and 8 respectively,is

(a)13 (b)65 (c)675 (d)1750

3.If **α** and **β** are the zeroes of the quadratic polynomial $p(x)=x^2-3x+2$,then the value of

(α+β–αx β) is

(a) 3 (b)2 (c)1 (d)None of these

4. The pair of linear equations:- 3x+2y= 5, 2x–3y=7 have

(a)unique solution (b)infinitely many solution (c)no solution (d)none of these

5. The value of k for which the equation $4x^2+kx+9=0$ has real and equal roots, is

(a)-12 (b) 12 (c) +12,-12 (d) 10

6. The distance between two points (a,b) and (-a,-b) is

(a)abunits (b)(a+b) units (c)$2\sqrt{a^2-b^2}$units (d)$2\sqrt{a^2+b^2}$units

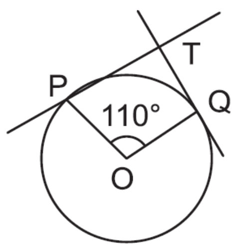
7. If in ΔABC and ΔDEF,$AB/DE = BC/FD$ then they will be similar, when

(a)∠B=∠E (b) ∠A=∠D (c)∠B=∠D (d)∠A=∠F

8. A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ=12cm.Length PQ is

(a)12 cm (b)13 cm (c)8.5 cm (d) √119 cm

9. In the given figure, if TP and TQ are tangents to a circle with centre O, so that ∠POQ=110°,then ∠PTQ is



(a)110° (b)90° (c)80° (d)70°

10. If tanθ =1, then the value of sin θ+ cosθ is:-

(a)3√2 (b)4√2 (c)2√2 (d)√2

11. In ΔABC, right angled at B, AB = 5 cm and sin C = 1/2. Determine the length of side AC.

(a) 10 cm (b) 15 cm (c) 20 cm (d) none of these

12. If sec A=15/7 and A+B =90°, find the value of cosecB.

(a) 8/7 (b)12/7 (c)7/15 (d)15/7

13. The area of a quadrant of a circle where the circumference of circle is 176 m is :
(a)2464 m^2 (b)1232 m^2 (c)616m^2 (d)308m^2

14. If the radii of two circles are in the ratio of 4 : 3,then their areas are in the ratio of:

(a) 4:3 (b)8:3 (c)16:9 (d)9:16

15. If two cubes of edge 3 cm each are joined end to end, then the surface area of resulting cuboid is.
(a)90 cm^2 (b)95 cm^2 (c)92cm^2 (d)94 cm^2

16. For the following distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	13	10	15	8	11

The upper limit of the modal class is:

(a) 12 (b)18 (c)24 (d)12

17. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is

(a)22 (b)23.5 (c)24 (d)24.5

18. Two different dice are thrown together. The probability of getting the sum of the two numbers

less than 7 is:

(a)5/12 (b)7/12 (c)12/5 (d)3/11

19**. Statement A (Assertion):** The number 6^n never end with digit 0 for any natural number n..

Statement R (Reason): The number 9^n never end with digit 0 for any natural number n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion(A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reasons (R) is false.

(d) Assertions (A) is false but reason (R) is true.

20. ***Statement A (Assertion):*** If the co-ordinates of the mid-points of the sides AB and AC of

ΔABC are D(3,5) and E(-3,-3) respectively, then BC =20units

Statement (Reason) :The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) Both assertion(A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion(A) and reason (R) are true and reason(R)is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason(R) is false.

(d)Assertion (A) is false but reason (R) is true.

SECTION– B

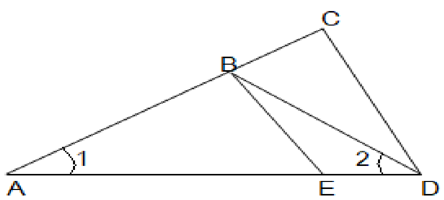
Questions 21 to 25carry 2 marks each.

21. For what values of k will the following pair of linear equations have unique solutions?

$$Kx+2y=3$$

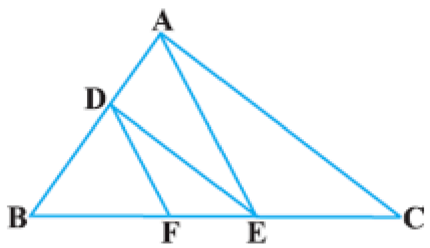
$$3x+6y=10$$

22. In the given figure below, AD/AE = AC/BD and $\angle 1=\angle 2$. Show that $\Delta BAE \sim \Delta CAD$.

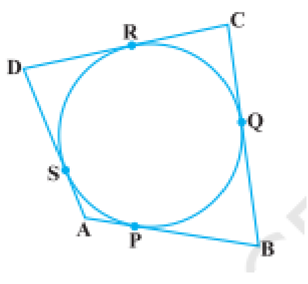


OR

In the figure, DE||AC and DF||AE. Prove that BF/FE= BE/EC



23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC



24. Find A and B, if sin (A + 2B) = √3/2 and cos (A + B) = ½.

25. A sector is cut from a circle of radius 35cm.The angle of the sector is 90^0.Find the length of the arc.

OR

A rope by which a cow is tethered is increased from 16 m to 23 m. How much additional ground does it have now to graze?

SECTION – C

Questions 26 to 31 carry 3 marks each

26. Given that √3 is irrational, prove that 5 + 2√3 is irrational.

27. Find the zeroes of the quadratic polynomial $2x^2 - x - 6$ and verify the relationship between the zeroes and the coefficients of the polynomial.

28. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Mona paid 27 for the book she kept for 7days, while Tanvy paid Rs 21 for the book she kept for 5 days .Find the fixed charge and the charge paid for each extra day.

OR

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

29. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

30. Prove that : $\frac{\cos A}{1+sinA} + \frac{1+sinA}{cosA} =$ 2sec A

OR

Prove that: $\frac{Sin\ A-2Sin^{3}A}{2Cos3_{A}-Cos\ A} = \tan A$

31. One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is (i) a king of red card (ii) a card of red colour (iii) neither a red card nor a queen.

SECTION – D

Questions 32 to 35 carry 5 marks each.

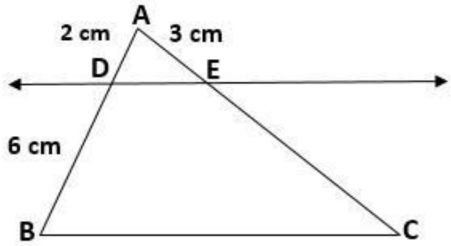
32. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speedfor the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.

OR

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

33. Prove that "If a line is drawn parallel to one side of a triangle to intersect the other two sides indistinct points, the other two sides are divided in the same ratio."

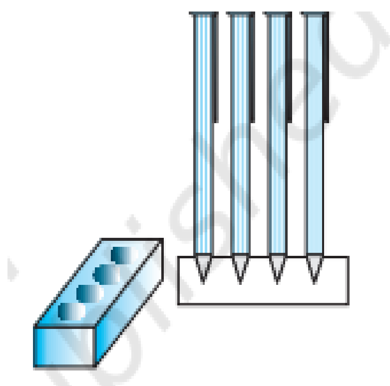
In the figure, find EC if AD/DB = AE/EC using the above theorem



34. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

OR

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



35. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components. Determine the modal lifetimes of the components.

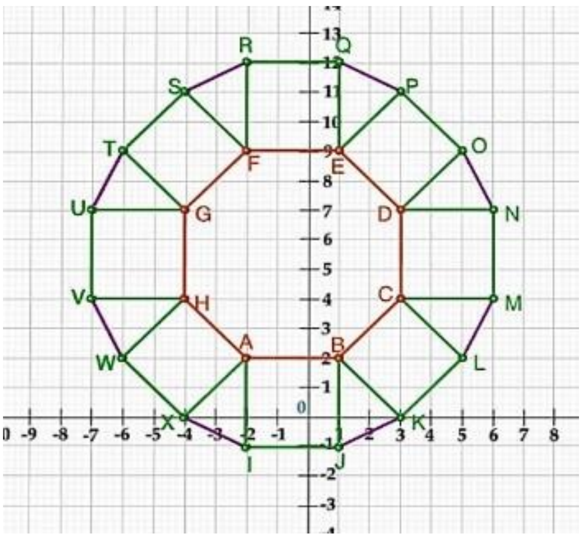
Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

SECTION – E (Case Study Based Questions)

Questions 36 to 38 carry 4 marks each

36. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern



Use the above figure to answer the questions that follow:

(i) What is the length of the line segment joining points B and F? 1 mark

(ii) Find the coordinates of midpoints of points H and C. 1 mark

(iii) What are the coordinates of the point on y axis equidistant from A and G? 2marks

OR

What is the area of Trapezium AFGH?

37. Anita's mother starts a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5pairs in 2nd row, 7 pairs in 3rd row and so on.

On the basis of above information, answer the following questions.

(i) How many shoes are put in 9^{th} row. 1 mark

(ii) How many shoes are put in 10^{th} row. 1 mark

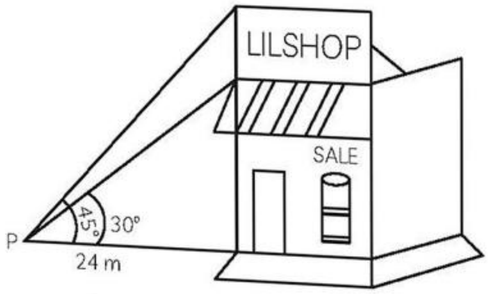
(iii) If she puts a total of 120 pairs of shoes, then find the number of rows required. 2 marks

OR

What is the difference of number of pairs of shoes put in 17th row and 8^{th} th row.

38. Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.

From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.



On the basis of the above information, answer the following questions:

(i) Find the height of the building (without the sign board). (2)

OR

Find the height of the building (with the sign board)

(ii) Find the height of the sign board. (1)

(iii) Find the distance of the point P from the top of the sign board. (1).

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KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

SAMPLE QUESTION PAPER-2023-24

Subject - Mathematics (Basic)

Time Allowed:3 Hours **Maximum Marks:80**

Marking Scheme

Section –Questions 1 to 20 carry 1 mark each

Award 1 mark for each correct answer

Question	Answers	Marks

No		
1	**(b) 6:1**	1
2	**(a)13**	1
3	**(c)1**	1
4	**(a)unique solution**	1
5	**© +12,-12**	1
6	**(d)**$2\sqrt{2}(a^2 + b^2)$units	1
7	(a)$\angle A = \angle Q$	1
8	**(d)** $\sqrt{119}$ cm	1
9	(d)70°	1
10	(d)$\sqrt{2}$	1
11	**(a)**10cm	1
12	(d)15/7	1
13	(c)616m^2	1
14	(c)16:9	1
15	(a)90 cm^2	1
16	(b)18	1
17	(c)24	1
18	(a)5/12	1
19	**(b)** Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A	1
20	a)Both assertion(A) and reason (R) are true and reason(R) is the correct xplanation of assertion (A)	1

SECTION– B

Questions 21 to 25carry 2 marks each.

21.	For unique solution $a_1/a_2 \neq b_1/b_2$ $k/3 \neq 2/6$ $K \neq 1$	1 1
22.	In ΔABC, ∠1 = ∠2 ∴ AB = BD ………………………**(i)** Given, AD/AE = AC/BD Using equation **(i)**, we get AD/AE = AC/AB **……………….(ii)** In ΔBAE and ΔCAD, by equation (ii), AC/AB = AD/AE ∠A= ∠A (common) ∴ ΔBAE ~ ΔCAD [By SAS similarity criterion]	 ½ ½ ½ ½
Or	In ΔABC, DE || AC BD/AD = BE/EC ....(i) (Using BPT) In ΔABE, DF || AE BD/AD = BF/FE ....(ii) (Using BPT) From (i) and (ii) BD/AD = BE/EC = BF/FE Thus, BF/FE =BE/EC	 ½ ½ ½ ½
23	Let ABCD be the rhombus circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points P, Q, R and S respectively. We know that the tangents drawn to a circle from an exterior point are equal in length. ∴ AP = AS………….(1) BP = BQ……………(2) CR = CQ …………...(3) DR = DS……………(4). Adding (1), (2), (3) and (4) we get AP+BP+CR+DR = AS+BQ+CQ+DS	 1 1/2

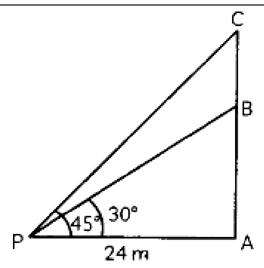
	(AP+BP) + (CR+DR) = (AS+DS) + (BQ+CQ) ∴ AB+CD=AD+BC	1/2
24	Given : sin (A + 2B) = sin 60° ⇒ A + 2B = 60° ...(i) cos (A + B) = cos 60° ⇒ A + B = 60° ...(ii) Subtracting equation (i) and (ii), we get B = 0° Putting the value of B in equation (ii), we get, A = 60° – 0° = 60° So, A = 60° and B = 0°.	 1/2 ½ 1/2 1/2
25.	Here, $\theta = 90^0$ r=35cm Length of arc(l)= (θ/360) × 2 πr =(90/360)×2x(22/7)×35 =55cm	½ ½ ½ ½
OR	The additional area cow can graze = $\pi(R^2-r^{2)}$ $=22/7(23^2-16^2)$ $=858\ m^2$	½ ½ 1
	SECTION – C **Questions 13 to 22 carry 3 marks each**	
26.	Let us assume 5 + 2√3 is rational, then it must be in the form of p/q where p and q are co-prime integers and q ≠0 i.e 5 + 2√3 = p/q So √3 = $p - 5q\ /2q$ ……………………(i) Since p, q, 5 and 2 are integers and q ≠ 0, HS of equation (i) is rational. But LHS of (i) is √3 which is irrational. This is not possible.	1 ½ ½ ½

	This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5 + 2\sqrt{3}$ is irrational**.**	½
27.	$2x^2-x-6=0$ $2x^2-4x+3x-6=0$ X= 2 ; x= - 3/2 For correct verification of sum of Zeroes For correct verification of product of Zeroes	½ ½ 1 1
28.	Let the fixed charge by Rs x and additional charge by Rs y per day Number of days for Mona = 7 = 3 + 4 Hence, Charge x + 4y = 27 x = 27 – 4y ………(1) Number of days for Tanvy = 5 = 3 + 2 Hence, Charge x + 2y = 21 x = 21 – 2y ……. (2) On comparing equation (1) and (2), we get, we get, x=15 y=3 **Therefore, fixed charge = Rs 15 and additional charge = Rs 3 per day**	 ½ ½ ½ ½ ½ ½
or	Let speed of car starting from A =x km/hr Let speed of car starting from B =y km/hr Difference in distance = 100 km. We know that, Distance = Speed × Time. ⇒ 5x − 5y = 100 ⇒ x−y=20.....(i) ⇒ x + y = 100….(ii) Adding equations (i) and (ii), we get, x − y + x + y = 20 +100 ⇒ 2x = 120 ⇒ x = 60	 ½ ½

	Substituting x = 60 in equation (ii), we get, 60 + y = 100 ⇒ y = 40 Therefore, the speed of the first car is 60 km/hr and the speed of the second car is 40 km/hr.	1 1
29	For correct figure For correct given and To prove For correct proof	½ ½ 2
30.	For each correct step Total	½ 3
31.	(i) 2/52=1/26 (ii) 26/52=1/2 (iii) 24/52=6/13	1 1 1
	SECTION – D **Questions 32 to 35 carry 5 marks each**	
32.	Let the usual speed of plane be x km/hr and the reduced speed of the plane be (x – 200) km/hr Distance =600 km [Given] According to the question, (time taken at reduced speed) – (Schedule time) = 30 minutes = 0.5 hours. ⇒ 600/(x-200) - 600 /x =1/2 Which on simplification gives: $x^2 - 200x - 240000 = 0$ ⇒ x2 – 600x + 400x −240000 = 0 ⇒ x(x – 600) + 400(x – 600) = 0 ⇒ (x – 600)(x + 400) =0 ⇒ x = 600 or x = −400 But speed cannot be negative. ∴ The usual speed is 600 km/hr and **the scheduled duration of the flight is 600/600 =1hour**	 ½ 1 1 1 ½ 1

OR	Let uniform speed of train =x km/hr Time taken to travel 360 km = 360/x hr New speed =x+5 km/hr Time taken to travel 360 km = 360/(x+5) hr Acc.to question 360/x - 360/(x+5 = 1 Which on simplification gives: $x^2+ 5x-1800=0$ $X^2+45x-40x-1800=0$ (x+45)(x-40)=0 X=40 ; x= -45 But speed cannot be negative So Speed = 40 km/hr	½ ½ 1 1 1 ½ ½
33.	For correct given and To prove For correct proof For finding EC= 9 cm	1 3 1
34.	Height of cylinder= 2.4 cm Diameter= 1.4 cm Radius= 0.7 cm Slant height = $\sqrt{(0.7)^2 + (2.4)^2}$ = 2.5 cm T.S.A of solid = $2\pi rh+ \pi r^2+\pi rl$ On solving = 17.6 cm^2 =18 cm^2	½ 1 1 2 ½
or	Volume of one conical depression = 1/3 x πr^2h = 1/3 X 22/7X $(0.5)^2$x 1.4 cm^3 = 0.366cm^3	½ ½ ½

	Volume of 4 conical depression = 4 x 0.366 cm3 = 1.464 cm^3 Volume of cuboidal box = L x B x H = 15 x 10 x 3.5 cm3 = 525 cm3 Remaining volume of box = Volume of cuboidal box – Volume of 4 conical depressions **= 525 cm3 − 1.464 cm3 = 523.5 cm3**	½ ½ ½ 1 1
35.	Modal class =60-80 $L=60$, $f_1=61$, $f_0=52, f_2=38$, $h=38$ Correct formula of Mode For correct solution, Mode= 65.625	1 1 1 2
	SECTION – E (Case Study Based Questions) **Questions 36 to 38 carry 4 marks each**	
36.	(i) B(1,2), F(-2,9) $BF^2 = (-2-1)^2 + (9-2)^2$ $= (-3)^2 + (7)^2$ $= 9 + 49$ $= 58$ **So, BF = √58 units** **(ii) h(-4,4) C(3,4)** **Midpoint = (-1,4)** **(iii)** A(-2,2), G(-4,7) Let the point on y-axis be Z(0,y) **$AZ^2 = GZ^2$** $0+2)^2 + (y-2)^2 = (0+4)^2 + (y-7)^2$ $(2)^2 + y^2 + 4 -4y= (4)^2+ y^2 + 49 -14y$ $8-4y = 65-14y$ $10y= 57$	 1 ½ ½ ½ ½ ½ ½ ½ ½

	So, y= 5.7 **i.e. the required point is (0, 5.7)** **OR** Clearly GH = 7-4=3units AF = 9-2=7 units So, height of the trapezium AFGH = 2 units So, area of AFGH = 1/2(AF + GH) x height = 1/2(7+3) x 2 **= 10 sq. units**	½ ½
37.	Number of pairs of shoes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ... **So, it forms an A.P. with first term a = 3, d = 5 – 3 = 2** **(i) a+8d= 19 shoes** **(ii) a+9d= 21 shoes** (iii)Let n be the number of rows required. ∴ Sn = 120 ⇒ (n/2) [2(3) + (n − 1)2] = 120 ⇒ $n^2 + 2n – 120 = 0$ ⇒ $n^2 + 12n – 10n – 120 = 0$ ⇒ (n + 12) (n – 10) = 0 ⇒ n = 10 **So, 10 rows required to put 120 pairs** **OR** No. of pairs in 17th row = a17 = 3 + 16(2) = 35 No. of pairs in 10th row = a10 = 3 + 9(2) = 21 **∴ Required difference = 35 – 21 = 14**	1 1 1 1 1 1
38	 **(i)** In ∆APC tan 30° = AB/AP ⇒ 1/√3 = AB/24 ⇒ AB = 24/√3 m = 13.85 m = 14 m (approx)	

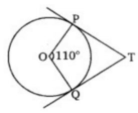
	OR	
	Considering, the diagram in the above question, AC as the new height of the shop including the sign-board.	1
	In ΔAPC,	
	$\tan 45° = AC/AP$	
	$\Rightarrow 1 = AC/24$	1
	$\Rightarrow$ AC = 24 m	
	(ii) From Q (i) and Q (ii).	
	Length of sign board, $BC = AC – AB$	1
	$= 24 – 14$	
	$= 10$ m	1
	(iii) In ΔAPC,	
	$\cos 45° = AP/AC$	
	$\Rightarrow 1/\sqrt{2} = 24/AC$	1
	$\Rightarrow$ PC = $24\sqrt{2}$ m	
		1

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION
SAMPLE QUESTION PAPER -2023-24
Class - X
Subject - Mathematics Basic (UNSOLVED)

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.

6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION - A	
	Section A consists of 20 questions of 1 marks each.	
1	The ratio of LCM and HCF of the least composite and the least prime numbers is (a) 1:2 (b) 2:1 (c) 1:1 (d) 1:3	1
2	The value of k for which the lines 5x+7y=3 and 15x + 21y = k coincide is (a) 9 (b) 5 (c) 7 (d) 18	1
3	A girl walks 200m towards East and then 150m towards North. The distance of the girl from the starting point is (a)350m (b) 250m (c) 300m (d) 225	1
4	The pair of equations y = 0 and y = -7 has: a) one solution b) two solutions c) infinitely many solutions d) no solution	1
5	Value(s) of k for which the quadratic equation $2X^2 - kx + k = 0$ has equal roots is : a) 0 only b) 4 c) 8 only d) 0,8	1
6	The distance of the point(3, 5) from x-axis is k units, then k equals: a) 3 b) 4 c) 5 d) 8	1
7	If in Δ ABC and Δ PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then : a) ΔPQR ~ Δ CAB b) ΔPQR ~ Δ ABC c) ΔCBA ~ Δ PQR d) ΔBCA ~ Δ PQR	1
8	Which of the following is NOT a similarity criterion of traingles? a) AA b) SAS c) AAA d) RHS	1
9	In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to  (a) 60° (b) 70° (c) 80° (d) 90°	1

10	If cos A = $\frac{4}{5}$ then tan A is: a) $\frac{3}{5}$ b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) $\frac{1}{8}$	1
11	If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is ______ a) 30° b) 45° c) 60° d) 90°	1
12	The value of cos1° . cos2° . cos3° . cos4°. . . . . cos90° is (a) 1 (b) 0 (c) – 1 (d) 2	1
13	If the perimeter of a circle is equal to that of a square, then the ratio of their areas is (a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11: 14	1
14	If the radii of two circles are in the ratio of 4 : 3, then their areas are in the ratio of : (a) 4 : 3 (b) 8 : 3 (c) 16 : 9 (d) 9 : 16	1
15	The total surface area of a solid hemisphere of radius 7 cm is : (a) 447π cm^2 (b) 239π cm^2 (c) 174π cm^2 (d) 147π cm^2	1
16	For the following distribution :	1

Class	0 – 55	5 – 10	10- 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

the upper limit of the modal class is

(a) 10 (b) 15 (c) 20 (d) 25

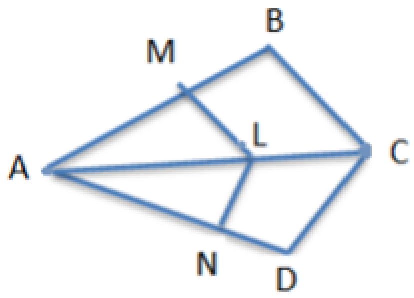
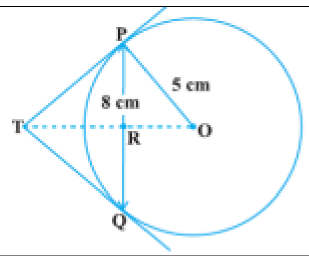
17	If the mean of the following distribution is 2.6, then the value of y is	1

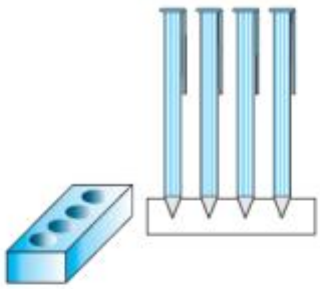
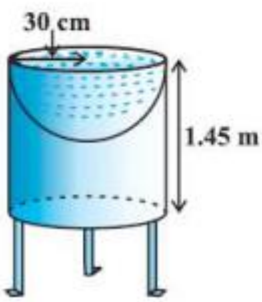
Variable	1	2	3	4	5
Frequency	4	5	Y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

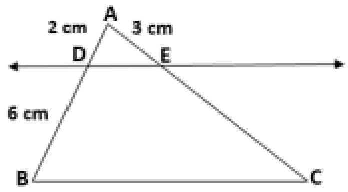
18	A card is selected at random from a well shuffled deck of 52 cards. The probability of its	1

	being a red face card is (a) $\frac{3}{26}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{2}$	
	Direction for questions 19 & 20: In question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.	
19	Assertion: If HCF of 510 and 92 is 2, then the LCM of 510 & 92 is 32460 Reason: as HCF(a,b) x LCM(a,b) = a x b (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	1
20	Assertion (A): The ratio in which the line segment joining (2, -3) and (5, 6) internally divided by x axis is 1:2. Reason (R): as formula for the internal division is $\left(\frac{mx_{2+nx_1}}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21	In the fig. if LM II CB and LN II CD, prove that AM MB = AN ND	2

	 	
22	For what values of k will the following pair of linear equations have infinitely many solutions? kx + 3y – (k – 3) = 0 12x + ky – k = 0	2
23	Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle	2
24	If cot θ = $\frac{7}{8}$, evaluate $\frac{(1+\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)\,(1-\cos\theta)}$.	2
25	Find the perimeter of a quadrant of a circle of radius 14 cm. [OR] Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm.	2
	SECTION C	
	Section C carries 6 questions of 3 marks each.	
26	Prove that v5 is an irrational number.	3
27	Find the zeroes of the quadratic polynomial 6x 2 – 3 – 7x and verify the relationship between the zeroes and the coefficients	3
28	The coach of a cricket team buys 4 bats and 1 ball for Rs. 2050. Later, she buys 3 bats and 2 balls for 1600. Find the cost of each bat and each ball.	3
29	Prove that $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$ **[OR]** If sin θ + cos θ = $\sqrt{3}$, then prove that tan θ + cot θ = 1	3
30	In the figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.  	3

31	A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number. (iii) a number divisible by 5. OR One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) A king of red colour. (ii) A spade (iii) The queen of diamonds	3
	SECTION – D	
	Section D carries 4 questions of 5 marks each.	
32	An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains. [OR] A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.	5
33	A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.  [OR]  Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.	5
34	A survey regarding the heights in (cm) of 51 girls of class X of a school was conducted and the following data was obtained. Find the median height and the mean using the formulae.	5

Height (in cm)	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

35	Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In the figure, find EC if AD DB = AE EC using the above theorem. 	5
	SECTION E	
	CASE BASED QUESTIONS ARE COMPULSARY	
36	Case Study – 1 In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.	4

<table>
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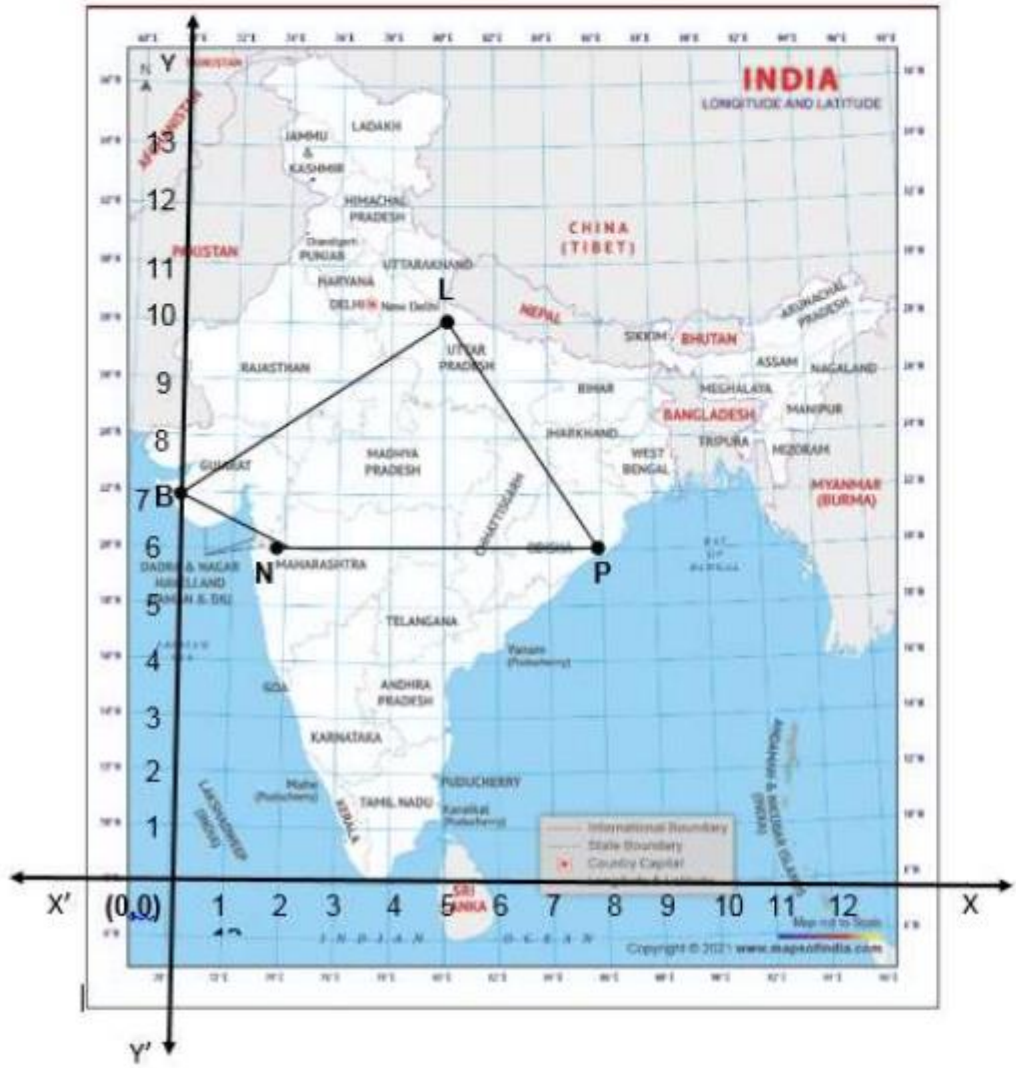

Based on the above information answer the following questions using the coordinate geometry.

I. Find the distance between Lucknow (L) to Bhuj(B).

II. If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

III. Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

[OR]

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).
</td><td></td></tr>
<tr><td>37</td><td>The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.</td><td>4</td></tr>
</table>

	 (i) If the first circular row has 30 seats, how many seats will be there in the 10th row? (ii) For 1500 seats in the auditorium, how many rows need to be there? OR If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row? (iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?	
38	In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year  Based on the above information answer the following questions. I. Find the production in the 1st year. II. Find the production in the 12th year. III. Find the total production in first 10 years. [OR] In how many years will the total production reach 31200 cars?	4