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# KENDRIYA VIDYALAYA SANGATHAN <br> (PATNA REGION) <br> STUDY MATERIAL FOR CLASS - XII <br> SUBJECT: MATHEMATICS (041) 

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## SYLLABUS

Class - XII
Subject - Mathematics (041)
Session: (2023-24)
Max Marks: 80

| S. No. | Units | No. of Periods | Marks |
| :---: | :---: | :---: | :---: |
| I. | Relations and Functions | 30 | 08 |
| II. | Algebra | 50 | 10 |
| III. | Calculus | 80 | 35 |
| IV. | Vectors and Three - Dimensional Geometry | 30 | 14 |
| V. | Linear Programming | - C 20 | 05 |
| VI. | Probability | 30 | 08 |
|  | Total | 240 | 80 |
|  | Internal Assessment |  | 20 |

## Unit - I (Relations and Functions)

1. Relations and Functions: Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.
2. Inverse Trigonometric Functions: Defintion, range, domain, principal value branch. Graphs of inverse trigonometric functions.
Unit - II (Algebra)
3. Matrices: Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and Multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutatitivity of multiplication of matrices and existence of non zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).
4. Determinants: Determinants of a square matrix (up to $3 \times 3$ matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

## Unit - III (Calculus)

1. Continuity and Differentiability: Continuity and Differentiability, chain rule, derivative of inverse trigonometric functions, like $\sin -1 x, \cos -1 x$ and $\tan -1 x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential function. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.
2. Applications of Derivatives: Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).
3. Integrals: Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$
\begin{aligned}
& \int \frac{\mathrm{dx}}{\mathrm{x}^{2} \pm \mathrm{a}^{2},} \int \frac{\mathrm{dx}}{\sqrt{\mathrm{x}^{2} \pm \mathrm{a}^{2}}}, \int \frac{\mathrm{dx}}{\sqrt{\mathrm{a}^{2}-\mathrm{x}^{2}}}, \int \frac{\mathrm{dx}}{\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}}, \int \frac{\mathrm{dx}}{\sqrt{\mathrm{ax}^{2}+b x+c}} \\
& \int \frac{\mathrm{px}+\mathrm{q}}{\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}} \mathrm{dx}, \int \frac{\mathrm{px}+\mathrm{q}}{\sqrt{\mathrm{ax}^{2+} \mathrm{bx}+\mathrm{c}}} \mathrm{dx}, \int \sqrt{\mathrm{a}^{2} \pm \mathrm{x}^{2}} \mathrm{dx}, \int \sqrt{\mathrm{x}^{2}-\mathrm{a}^{2}} \mathrm{dx} \\
& \int \sqrt{a x^{2}+b x+c} d x,
\end{aligned}
$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.
4. Application of the Integrals: Applications in finding the area under simple curves, especially lines, circles / parabolas / ellipses (in standard form only)
5. Differential Equations: Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

## Unit - IV: (Vectors and Three - Dimensional Geometry)

1. Vectors: Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ration. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.
2. Three - dimensional Geometry: Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

## Unit - V: (Linear Programming)

1. Linear Programming: Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

## Unit - VI: (Probability)

1. Probability: Conditional probability, multiplication theorem on probability, independent evens, total probability, Bayes'theorem, Random variable and its probability distribution, mean of random variable.

## Chapter 1 - Relations and Functions

## Definitions:

Let $A$ and $B$ be two non-empty sets, then a function $f$ from set $A$ to set $B$ is a rule which associates each element of A to a unique element of B .

## - Relation

If $(a, b) \in R$, we say that $a$ is related to $b$ under the relation $R$ and we write as a R b $\circ$ Function

It is represented as $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{B}$ and function is also called mapping.

- Real Function
$f: A \rightarrow B$ is called a real function, if $A$ and $B$ are subsets of $R$.
- Domain and Codomain of a Real Function

Domain and codomain of a function $f$ is a set of all real numbers $x$ for which $f(x)$ is a real number. Here, set $A$ is domain and set $B$ is codomain.

- Range of a real function
f is a set of values $\mathrm{f}(\mathrm{x})$ which it attains on the points of its domain


## Types of Relations

- A relation R in a set A is called Empty relation, if no element of A is related to any element of A, i.e., $R=\varphi \subset A \times A$.
- A relation R in a set A is called Universal relation, if each element of A is related to every element of A, i.e., R = A $\times$ A.
- Both the empty relation and the universal relation are sometimes called

Trivial Relations $\circ$ A relation R in a set A is called

- Reflexive
- if $(a, a) \in R$, for every $a \in A$,
- Symmetric
- If $\left(a_{1}, a_{2}\right) \in R$ implies that $\left(a_{2}, a_{1}\right) \in R$, for all $a_{1}, a_{2} \in A$.
- Transitive
- If $\left(a_{1}, a_{2}\right) \in R$ and $\left(a_{2}, a_{3}\right) \in R$ implies that $\left(a_{1}, a_{3}\right) \in R$, for all $a_{1}, a_{2}, a_{3} \in A$.
- A relation $R$ in a set $A$ is said to be an equivalence relation if $R$ is reflexive, symmetric and transitive


## Types of Functions

Consider the functions $f_{1}, f_{2}, f_{3}$ and $f_{4}$ given

- A function $\mathrm{f}: \mathrm{X} \rightarrow \mathrm{Y}$ is defined to be one-one (or injective), if the images of distinct elements of $X$ under $f$ are distinct, i.e., for every $x_{1}, x_{2} \in X, f\left(x_{1}\right)=f\left(x_{2}\right)$ implies $x_{1}=x_{2}$. Otherwise, f is called many-one.

Example

- One- One Function

- Many-One Function

- A function $\mathrm{f}: \mathrm{X} \rightarrow \mathrm{Y}$ is said to be onto (or surjective), if every element of Y is the image of some element of $X$ under f, i.e., for every $y \in Y$, there exists an element $x$ in $X$ such that $f(x)=y$.
$\circ \mathrm{f}: \mathrm{X} \rightarrow \mathrm{Y}$ is onto if and only if Range of $\mathrm{f}=\mathrm{Y}$.
- Eg:

- A function $\mathrm{f}: \mathrm{X} \rightarrow \mathrm{Y}$ is said to be one-one and onto (or bijective), if f is both one-one and onto.
- Eg:



## MCQ

Question 1.The relation $R$ on the set $A=\{1,2,3\}$ given by $R=\{(1,1),(1,2),(2,2),(2,3),(3,3)\}$ is
a) Reflexive b) Symmetric c) Transitive d) Equivalence

Question 2.Let $\mathrm{f}: R \rightarrow R$ be defined as $\mathrm{f}(\mathrm{x})=3 \mathrm{x}-2$. Choose the correct answer. a) f is one-one onto b)f is many one onto c)f is one-one but not onto d)f is neither one-one nor onto

Question 3.Let $R$ be a relation defined on $Z$ as $R=\{(a, b) ; a 2+b 2=25\}$, the domain of $R$ is; (a) $\{3,4,5\}$ (b) $\{0,3,4,5\}$ (c) $\{0,3,4,5,-3,-4,-5\}$ (d) none

Question 4.letR be the relation in the set $N$ given by $R=\{(a, b): a=b-2, b>6\}$.Choose the correct answer. (a) $(2,4) € R(b)(3,8) € R(c)(6,8) € R(d)(8,10) € R$

Question 5.Set $A$ has 3 elements and set $B$ has 4 elements. Then the number of injective functions that can be defined from set $A$ to set $B$ is (a) 144 (b)12 (c)24 (d)64

## SQ

## Question

. Determine whether given relations are reflexive, symmetric and transitive:
(1)Relation $R$ in the set $A=\{1,2,3 \ldots 13,14\}$ defined as
$R=\{(x, y): 3 x-y=0\}$

## Solution:

(i) $A=\{1,2,3 \ldots 13,14\}$
$R=\{(x, y): 3 x-y=0\}$
$\therefore \mathrm{R}=\{(1,3),(2,6),(3,9),(4,12)\}$
$R$ is not reflexive since $(1,1),(2,2) \ldots(14,14) \notin R$.
Also, $R$ is not symmetric as $(1,3) \in R$, but $(3,1) \notin R$. $[3(3)-1 \neq 0]$
Also, $R$ is not transitive as $(1,3),(3,9) \in R$, but $(1,9) \notin R$.
$[3(1)-9 \neq 0]$
Hence, $R$ is neither reflexive, nor symmetric, nor transitive.
(2) Relation $R$ in the set $N$ of natural numbers defined as
$R=\{(x, y): y=x+5$ and $x<4\}$
(3) Relation $R$ in the set $A=\{1,2,3,4,5,6\}$ as
$R=\{(x, y): y$ is divisible by $x\}$
(4) Relation $R$ in the set $Z$ of all integers defined as
$R=\{(x, y): x-y$ is as integer $\}$
(5) Relation R in the set A of human beings in a town at a particular time given by
(a) $R=\{(x, y): x$ and $y$ work at the same place

## LONG TYPE QUESTIONS-

Q1.Let $L$ be the set of all lines in $X Y$ plane and $R$ be the relation in $L$ defined as $R=\left\{\left(L_{1}, L_{2}\right): L_{1}\right.$ is parallel to $\left.L_{2}\right\}$. Show that $R$ is an equivalence relation. Find the set of all lines related to the line $y=2 x+4$.

Solution :
$R=\left\{\left(L_{1}, L_{2}\right): L_{1}\right.$ is parallel to $\left.L_{2}\right\}$
$R$ is reflexive as any line $L_{1}$ is parallel to itself i.e., $\left(L_{1}, L_{1}\right) \in R$.
Now,
Let $\left(L_{1}, L_{2}\right) \in R$.
$\Rightarrow L_{1}$ is parallel to $L_{2}$.
$\Rightarrow L_{2}$ is parallel to $L_{1}$.
$\Rightarrow\left(L_{2}, L_{1}\right) \in R$
$\therefore \mathrm{R}$ is symmetric.
Now,
Let $\left(L_{1}, L_{2}\right),\left(L_{2}, L_{3}\right) \in R$.
$\Rightarrow L_{1}$ is parallel to $L_{2}$. Also, $L_{2}$ is parallel to $L_{3}$.
$\Rightarrow L_{1}$ is parallel to $L_{3}$.
$\therefore \mathrm{R}$ is transitive.
Hence, $R$ is an equivalence relation.
The set of all lines related to the line $y=2 x+4$ is the set of all lines that are parallel to the line $y=2 x+4$.

Slope of line $y=2 x+4$ is $m=2$
It is known that parallel lines have the same slopes.
The line parallel to the given line is of the form $y=2 x+c$, where $c \in R$.
Hence, the set of all lines related to the given line is given by $y=2 x+c$, where $c \in$ R.

Q2.Show that the relation $R$ defined in the set $A$ of all polygons as $R=\left\{\left(P_{1}, P_{2}\right)\right.$ : $P_{1}$ and $P_{2}$ have same number of sides \}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle $T$ with sides 3, 4 and 5 ?

Q3. Show that the Modulus Function $f: R \rightarrow R$, given by $f(x)=|x|$ is neither one-one nor onto, where is $|x|$ if $x$ is positive or 0 and $|-x|$ is $-x$ if $x$ is negative.

Q4 rove that the Greatest Integer Function $f: R \rightarrow R$ given by $f(x)=[x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to $x$.

Q5. Show that function $f: R \rightarrow\{x \in R:-1<x<1\}$ defined by $f(x)=x / 1+|x| x \in R$ is one-one and onto function.

## BASIC CONCEPTS

Function Domain Range 1. $\mathrm{y}=\sin ^{-1} \mathrm{x}$

| Function Name | Notation | Definition | Domain of $x$ | Range |
| :---: | :---: | :---: | :---: | :---: |
| Arcsine or inverse sine | $\begin{gathered} y=\sin - \\ 1(x) \end{gathered}$ | $x=\sin y$ | $-1 \leq x \leq 1$ | - $-\pi / 2 \leq y \leq \pi / 2$ <br> - $-90^{\circ} \leq \mathrm{y} \leq 90^{\circ}$ |
| Arccosine or inverse cosine | $\begin{gathered} y=\text { cos }- \\ 1(x) \end{gathered}$ | $x=\cos y$ | $-1 \leq x \leq 1$ | $\begin{gathered} \cdot 0 \leq y \leq \pi \\ \cdot \quad 0^{\circ} \leq y \leq 180^{\circ} \end{gathered}$ |
| Arctangent or inverse tangen | $y=\tan -1(x)$ | $x=\tan y$ | For all real numbers | - $-\pi / 2<y<\pi / 2$ <br> - $-90^{\circ}<y<90^{\circ}$ |
| Arccotangent or inverse cot | $y=\cot -1(x)$ | $x=\cot y$ | For all real numbers | $\begin{array}{r} \quad 0<y<\pi \\ \cdot \quad 0^{\circ}<y<180^{\circ} \end{array}$ |
| Arcsecant or inverse secant | $\begin{gathered} y=\text { sec- } \\ 1(x) \end{gathered}$ | $x=\sec y$ | $\begin{gathered} x \leq-1 \text { or } \\ 1 \leq x \end{gathered}$ | - $0 \leq y<\pi / 2$ or $\pi / 2<y \leq \pi$ <br> - $0^{\circ} \leq y<90^{\circ}$ or $90^{\circ}<y \leq 180^{\circ}$ |
| Arccosecant | $\begin{gathered} y=c s c- \\ 1(x) \end{gathered}$ | $x=\csc y$ | $\begin{gathered} x \leq-1 \text { or } \\ 1 \leq x \end{gathered}$ | - $-\pi / 2 \leq y<0$ or $0<y \leq \pi / 2$ <br> - $-90^{\circ} \leq y<0^{\circ}$ or $0^{\circ}<y \leq 90^{\circ}$ |

we have also learnt in chapter1bthat if $f: X$ tends to $y$ that $f(x)=y$ is one one and onto then we can define a unique function $g$ : $Y$ tends $X$ xuch that $g(y)=X$, where $x$ belongs to $X$ and $y$ belongs to $Y$. here domain of $g=$ range of $f$ and range of $f$ and range of $g$ $=0$ main of $\mathbf{f}$
since domain of sine function the et of allral nos. and range is the closed inerval[$1,1]$, if we retrict its domain in $[-\Pi / 2,) \Pi / 2]$ then it becomes one-one and onto with range[$\mathbf{1 , 1}$ ] this principal value branch for sin inverse function.
for $\cos ^{-1}$ it is $[0, \Pi]$,
for $\operatorname{cosec}^{-1}$ it is $[-\Pi / 2, \Pi / 2]-0$
for $\sec ^{-1}$ itis [ $\left.0, ~ \Pi /\right]$,
for $\tan ^{-1}$ it is $(-\Pi / / 2, \Pi / / 2)$
and for $\cot ^{-1}$ it is $(0, \Pi /)$
Note-(i) $\operatorname{Sin}^{-1} x \& \tan ^{-1} x$ are increasing functions in their domain.
(ii) $\operatorname{Cos}^{-1} x \& \cot ^{-1} x$ are decreasing functions in over domain.
(i) $\sin \left(\sin ^{-1} x\right)=x$, for all $x[-1,1]$
(ii) $\cos \left(\cos ^{-1} x\right)=x$, for all $x[-1,1]$
(iii) $\tan \left(\tan ^{-1} x\right)=x$, for all $x R$
(iv) $\operatorname{cosec}\left(\operatorname{cosec}^{-1} x\right)=x$, for all $x(-,-1][1),(v) \sec \left(\sec ^{-1} x\right)=x$, for all $x(-,-1][1),(v i) \cot \left(\cot ^{-1} x\right)=x$, for all $x R$
now answer the following question
MCQ-
Q1.sec-1 $x+\operatorname{cosec}-1 x=$ $\qquad$
(a) $\pi / 2$ (b) $2 \pi / 2$ (c) $\pi / 4$ (d) $\pi / 3$

Ans-a
Q2. $\tan -1 \mathrm{x}+\cot -1 \mathrm{x}=$ $\qquad$
(a) $\pi / 3$ (b) $2 \pi / 2$ (c) $\pi / 4$ (d) $\pi / 2$

Q3. Principal value of $\sin -1(-1 / 2)$ is
(a) $-\pi / 6$ (b) $-\pi / 5$ (c) $-\pi / 2$ (d) $-5 \pi / 3$

Q4. Principal value of $\sin -1(-1 \sqrt{ } 2)$ is
(a) $-\pi / 6$ (b) $-\pi / 4$ (c) $-\pi / 2$ (d) $-5 \pi / 3$

Q5. Principal value of cos-1 $(-1 / 2)$ is $\qquad$
a) $2 \pi / 3$ (b) $\pi / 4$
(c) $3 \pi / 2$
(d) $5 \pi / 3$
b) VSQ
.1.can we find the inverse of trigonometric functions?
ANS. NO In their natural domain we can not find inverse trigonometric functions are defined for restricted domains.
2.Why are all T.functions periodic?
3. $\sec ^{-1}(2 / \sqrt{3})=$ ?

4 find the value of $\cos 1(\cos 13 \Pi / 6)$
5. . $\cot ^{-1}(\sqrt{ } 3)=$ ?

SQ
1.draw the graph of $\tan$ invese function.
2.why the secant function is not defined between in $(-1,1)$ of its range?

3Draw the graph of cot inverse function.
4.Draw the graph of sec inverse function.
5.Draw the graph of sin inverse function.

LONG TYPE QUESTIONS-
.Find the value of the followings

1. $\sin ^{-1}(\sin 3 \pi / 5)$

Ans- $\sin 3 \pi / 5=\sin (\pi-3 \pi / 5)=\sin 2 \pi / 5$
Now, $\sin ^{-1}(\sin 2 \pi / 5)=2 \pi / 5$, and $2 \pi / 5 €[-\pi / 2, \pi / 2]$
2. $\tan ^{-1}(\tan 3 \pi / 4)$
3. $\cos ^{-1}(\cos 7 \pi / 6)$

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4.sin(\pi/3- 齐-1-1/2)
5.tan}\mp@subsup{}{}{-1}\sqrt{}{3}-\mp@subsup{\operatorname{cot}}{}{-1}(-\sqrt{}{}3
```


## CHAPTER-3 Matrices

A matrix is a rectangular arrangement of numbers (real or complex) which may be represented as
matrix is enclosed by [ ] or () or | | ||
Compact form the above matrix is represented by $\left[\mathrm{a}_{\mathrm{ij}}\right]_{\mathrm{mxn}}$ or $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right]$.

1. Element of a Matrix The numbers $a_{11}, a_{12} \ldots$ etc., in the above matrix are known as the element of the matrix, generally represented as $\mathrm{a}_{\mathrm{ij}}$, which denotes element in ith row and jth column.
2. Order of a Matrix $\operatorname{In}$ above matrix has $m$ rows and $n$ columns, then $A$ is of order $m x$ n.

## Types of Matrices

1. Row Matrix A matrix having only one row and any number of columns is called a row matrix.
2. Column Matrix A matrix having only one column and any number of rows is called column matrix.
3. Rectangular Matrix A matrix of order $m \times n$, such that $m \neq n$, is called rectangular matrix.
4. Horizontal Matrix A matrix in which the number of rows is less than the number of columns, is called a horizontal matrix.
5. Vertical Matrix A matrix in which the number of rows is greater than the number of columns, is called a vertical matrix.
6. Null/Zero Matrix A matrix of any order, having all its elements are zero, is called a null/zero matrix. i.e., $\mathrm{a}_{\mathrm{ij}}=0, \forall \mathrm{i}, \mathrm{j}$
7. Square Matrix A matrix of order $m \times n$, such that $m=n$, is called square matrix.
8. Diagonal Matrix $A$ square matrix $A=\left[a_{i j}\right]_{\mathrm{mxn}}$, is called a diagonal matrix, if all the elements except those in the leading diagonals are zero, i.e., $a_{i j}=0$ for $i \neq j$. It can be represented as
$A=\operatorname{diag}\left[a_{11} a_{22} \ldots a_{n n}\right]$
9. Scalar Matrix A square matrix in which every non-diagonal element is zero and all diagonal elements are equal, is called scalar matrix. i.e., in scalar matrix $\mathrm{a}_{\mathrm{ij}}=0$, for $\mathrm{i} \neq \mathrm{j}$ and $\mathrm{a}_{\mathrm{ij}}=\mathrm{k}$, for $\mathrm{i}=\mathrm{j}$
10. Unit/Identity Matrix A square matrix, in which every non-diagonal element is zero and every diagonal element is 1 , is called, unit matrix or an identity matrix.

MCQ

A: zero matrixB: diagonal matrixC: column matrixD: row matrix ANS-C

Q2. . Total number of possible matrices of order $3 \times 3$ with each entry 2 or 0 is (a) 9(b) 27(c) 81 (d) 512

Correct option: (d) 512
Q3. If $A$ and $B$ are two matrices of the order $3 \times m$ and $3 \times n$, respectively, and $m=n$, then the order of matrix $(5 A-2 B)$ is
(a) $m \times 3(b) 3 \times 3(c) m \times n(d) 3 \times n$
(b) Correct option: (d) $3 \times n$

Q4 If $A$ and $B$ are symmetric matrices of the same order, then ( $A B^{\prime}-B A^{\prime}$ ) is a
(a) Skew symmetric matrix(b) Null matrix(c) Symmetric matrix(d) None of these

Correct option: (a) Skew symmetric matrix

Q5. If $\mathbf{A}$ is a skew-symmetric matrix, then $A^{2}$ is a
(a) Skew symmetric matrix(b) Symmetric matrix(c) Null matrix(d) Cannot be determined

Correct option: (b) Symmetric matrix

SQ-

1. can we have transpose of any matrix?
2.can we get symmetric matrix of any matrix?

3, isit possible to get skew symmetric matrix from square natrix?
4, what is diagonal matrix?
5.can we express any square matrix as a sum of symmeric matrx and skew symmetric matrix?

LONG TYPE QUESTIONS-
Q1.IF If $A$ and $B$ are symmetric matrices, prove that $A B-B A$ is a skew symmetric matrix.

## Solution:

$A$ and $B$ are symmetric matrices.
$A^{\prime}=A$ and $B^{\prime}=B$
Now, $(A B-B A)^{\prime}=(A B)^{\prime}-(B A)^{\prime} \quad \therefore(A B-B A)^{\prime}=B^{\prime} A^{\prime}-A^{\prime} B^{\prime}$ [Reversal law]
$\therefore(A B-B A)^{\prime}=B A-A B \quad[$ Using eq. (i)]
$\therefore(A B-B A)^{\prime}=-(A B-B A)$
Therefore, $(A B-B A)$ is a skew symmetric.
Q2. Show that the matrix $B^{\prime} A B$ is symmetric or skew symmetric according as $A$ is symmetric or skew symmetric.

Q3. A manufacturer produces three products, $x, y, z$ which he sells in two markets. Annual sales are indicated below:

MARKETS PRODUCTS
$\begin{array}{llll}I & 10.000 & 2,000 & 18,000\end{array}$
II $6,000 \quad 20,000 \quad 8,000$
(a) If unit sales prices of $x, y$ and $z$ are `2.50 ,` 1.50 and `1.00 respectively, find the total revenue in each market with the help of matrix algebra. (b) If the unit costs of the above three commodities are` $2.00, ` 1.00$ and 50 paise respectively. Find the gross profit.

Q4. If $A=\left[\begin{array}{ll}3 & 1 \\ -1 & 2 \\ -1\end{array}\right]$
show that $A^{2}-\overline{5 A}+7 I=0$.
Q5 Find $x$, if $\left[\begin{array}{lll}x & -5 & -1\end{array}\right]\left[\begin{array}{ccc}1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3\end{array}\right]\left[\begin{array}{l}x \\ 4 \\ 1\end{array}\right]=0$

## Determinant

Determinant: Determinant is the numerical value of the square matrix. So, to every square matrix $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right]$ of order n , we can associate a number (real or complex) called determinant of the square matrix $A$. It is denoted by $\operatorname{det} A$ or $|A|$.
Note
(i) Read $|\mathrm{A}|$ as determinant $A$ not absolute value of $A$.
(ii) Determinant gives numerical value but matrix do not give numerical value.
(iii) A determinant always has an equal number of rows and columns, i.e. only square matrix have determinants.

For calculations of determinant, we shall expand the determinant along that row or column which contains the maximum number of zeroes.
(ii) While expanding, instead of multiplying by $(-1)^{i+j}$, we can multiply by +1 or -1 according to as $(i+j)$ is even or odd.

Let $A$ be a matrix of order $n$ and let $|A|=x$. Then, $|k A|=k^{n}|A|=k^{n} x$, where $n=1,2$, $3, \ldots$

Minor: Minor of an element ay of a determinant, is a determinant obtained by deleting the ith row and jth column in which element ay lies. Minor of an element $\mathrm{a}_{\mathrm{ij}}$ is denoted by $\mathrm{M}_{\mathrm{ij}}$.
Note: Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order ( $n-1$ ).

Cofactor: Cofactor of an element $\mathrm{a}_{\mathrm{ij}}$ of a determinant, denoted by $\mathrm{A}_{\mathrm{ij}}$ or $\mathrm{C}_{\mathrm{ij}}$ is defined as $A_{i j}=(-1)^{i+j} M_{i j}$, where $M_{i j}$ is a minor of an element $a_{i j}$.
IF $\operatorname{det}(A)=\left|\begin{array}{ll}\text { a11 } & \text { a12 } \\ \text { a21 } & \text { a22 }\end{array}\right|$

Then value of determinant will be a11a22- a21a12

Singular and non-singular Matrix: If the value of determinant corresponding to a square matrix is zero, then the matrix is said to be a singular matrix, otherwise it is non-singular matrix, i.e. for a square matrix $A$, if $|A| \neq 0$, then it is said to be a nonsingular matrix and of $|A|=0$, then it is said to be a singular matrix.
Theorems
(i) If $A$ and $B$ are non-singular matrices of the same order, then $A B$ and $B A$ are also non-singular matrices of the same order.
(ii) The determinant of the product of matrices is equal to the product of their respective determinants, i.e. $|A B|=|A||B|$, where $A$ and $B$ are a square matrix of the same order.

Adjoint of a Matrix: The adjoint of a square matrix ' $A$ ' is the transpose of the matrix which obtained by cofactors of each element of a determinant corresponding to that given matrix. It is denoted by $\operatorname{adj}(A)$.
In general, adjoint of a matrix $A=\left[a_{i j}\right]_{n \times n}$ is a matrix $\left[A_{j i}\right]_{n \times n}$, where $A_{j i}$ is a cofactor of element $\mathrm{a}_{\mathrm{j} \text {. }}$.

MCQ-

Q1. If $A$ is a square matrix of order 3 and $|A|=5$, then the value of $\left|2 A^{\prime}\right|$ is
(a) -10(b) 10(c) -40(d) 40

Correct option: (d) 40

Q2The area of a triangle with vertices $(-3,0),(3,0)$ and $(0, k)$ is 9 sq. units. The value of $k$ will be
(c) $9(b) 3(c)-9 d) 6$
(d) Correct option: (b) 3

Q3. Given that $A$ is a square matrix of order 3 and $|A|=-4$, then $|\operatorname{adj} A|$ is equal to
(a) -4(b) 4(c) -16(d) 16

Correct option: (d) 16
Q4.find value of $k$ for which $A=\left[\begin{array}{ll}K & 8 \\ 4 & 2 K\end{array}\right]$ is a singular matrix
(a) 4 (b) $-4(\mathrm{c}) \pm 4$ (d) 0

Correct option: (c) $\pm 4$
Q5. Let $A$ is a singular matrix of order $3 \times 3$, then $\operatorname{adj} A$ |s equall to

$$
\text { b. }|A| \text { b. }\left|A^{3}\right| d A^{3} \mid \text { d| } 3 A^{A} \mid
$$

VSQ-

## Evaluate the determinants .

## Question1.

$\left|\begin{array}{ll}2 & 4 \\ 5 & 1\end{array}\right|$

Q2
$\left|\begin{array}{cc}\cos A & -\sin A \\ \sin A & \cos A\end{array}\right|$

Q3.Find area of triangle with vertices at the points
$(1,0),(6,0),(4,3)$

Q4 wite minor of $\left|\begin{array}{cc}2 & -4 \\ 0 & 3\end{array}\right|$

Q5.write co-factor of


SQ
Q1.
Using cofactors of elements of third column, evaluate:
$\Delta=\left\lvert\, \begin{array}{cc}1 \mathrm{x} & \mathrm{yz} \\ 1 \mathrm{y} & \mathrm{zx} \\ 1 \mathrm{z} & \mathrm{xy}\end{array}\right.$

Q2. Using cofactors of elements of second row, evaluate
$\Delta=\left|\begin{array}{lll}5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3\end{array}\right|$

Q3.CHECK whether the determinant $\left|\begin{array}{ll}1 & 2 \\ 1 & 2\end{array}\right|$ is singular or not?
Q4.check, given determinant is non -singular or singular?
$A=\left\lvert\, \begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right.$
Q5.What is condition of being a determinant non - singular?
LONG QUESTIONS.
Q1. Prove that the determinant $\left\lvert\, \begin{array}{lll}x & \sin A & \cos A \\ \sin A & -x & 1 \\ \cos A & 1 & x\end{array}\right.$
is independent of $\boldsymbol{A}$.
Q2. Solve the system of the following equations: (Using matrices):
$2 / X+3 / Y+10 / Z=4$
$4 / X-6 / Y+5 / Z=1$
6/X+9/Y-20/Z=2

Q3. $3 x-y-2 z=2$

$$
2 y-z=-1
$$

$3 x-5 y=3$
Ans. The given system of equations is:
$3 x-y-2 z=2$
$0 x+2 y-z=-1$
$3 x-5 y+0 z=3$
The given system of equations can be written in the form of $A X=B$, where
Now $|A|=3(-5)+3(5)=0$
$\therefore \mathrm{A}$ is a singular matrix and hence $\mathrm{A}-1$ doesn't exist.
So the system of equations will be inconsistent.
Q4.For matrix $A=1\left[\begin{array}{ll}1 & 1 \\ 2 & -3 \\ -1 & 3\end{array}\right]$
Show that $A=A^{2}-6 A^{2}+5 A+111=0$ Hence find $A^{-1}$
Q5.Find inverse of


## CCT TYPES QUESTIONS

Q1.Relation and Function CASE STUDY 1: A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about $67 \%$, the highest ever Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation ' R ' is defined on I as follows: $\mathrm{R}=$ $\{(V 1, V 2): V 1, V 2 \in I$ and both use their voting right in general election - 2019\}

1. Two neighbors $X$ and $Y \in I . X$ exercised his voting right while $Y$ did not cast her vote in general election - 2019. Which of the following is true?
a. $(X, Y) \in R$ b. $(Y, X) \in R$ c. $(X, X) \notin R$ d. $(X, Y) \notin R$
2. Mr.' $X$ ' and his wife ' $W$ 'both exercised their voting right in general election -2019,

Which of the following is true? a. both $(\mathrm{X}, \mathrm{W})$ and $(\mathrm{W}, \mathrm{X}) \in \mathrm{Rb} .(\mathrm{X}, \mathrm{W}) \in \mathrm{R}$ but $(\mathrm{W}, \mathrm{X}) \notin \mathrm{Rc}$. both $(\mathrm{X}, \mathrm{W})$ and $(\mathrm{W}, \mathrm{X}) \notin \mathrm{R}$ d. $(\mathrm{W}, \mathrm{X}) \in \mathrm{R}$ but $(\mathrm{X}, \mathrm{W}) \notin \mathrm{R}$
3. Three friends F1, F2 and F3 exercised their voting right in general election-2019, then which of the following is true?
a. $(F 1, F 2) \in R,(F 2, F 3) \in R$ and $(F 1, F 3) \in R$ b. $(F 1, F 2) \in R,(F 2, F 3) \in R$ and $(F 1, F 3) \notin R c$. $(F 1, F 2) \in R,(F 2, F 2) \in R$ but $(F 3, F 3) \notin R d .(F 1, F 2) \notin R,(F 2, F 3) \notin R$ and $(F 1, F 3) \notin R$ ONE NATION ONE - ELECTION FESTIVAL OF DEMOCRACY GENERAL ELECTION 2019
4. The above defined relation R is $\qquad$ a. Symmetric and transitive but not reflexive
b. Universal relation c. Equivalence relation d. Reflexive but not symmetric and transitive
5. Mr. Shyam exercised his voting right in General Election - 2019, then Mr. Shyam is related to which of the following? a. All those eligible voters who cast their votes b. Family members of Mr.Shyam c. All citizens of India d. Eligible voters of India

ANSWERS 1. (d) (X,Y) $\notin \mathrm{R}$
2. (a) both $(X, W)$ and $(W, X) \in R$
3. (a) $(F 1, F 2) \in R,(F 2, F 3) \in R$ and $(F 1, F 3) \in R$
4. (c) Equivalence relation
5. (a) All those eligible voters who cast their votes

## CASE STUDY 2

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1,2,3,4,5,6\}$. Let $A$ be the set of players while $B$ be the set of all possible outcomes. $A=\{S, D\}, B=\{1,2,3,4,5,6\}$

1. Let $R: B \rightarrow B$ be defined by $\mathrm{R}=\{(x, y): y$ is divisible by $x\}$ is a. Reflexive and transitive but not symmetric b. Reflexive and symmetric and not transitive c. Not reflexive but symmetric and transitive d. Equivalence
2. Raji wants to know the number of functions from $A$ to $B$. How many number of functions are possible? a. 62 b. 26 c. 6 ! d. 212
3. Let R be a relation on B defined by $\mathrm{R}=\{(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)\}$. Then $R$ is $a$. Symmetric $b$. Reflexive $c$. Transitive d. None of these three
4. Raji wants to know the number of relations possible from A to B . How many numbers of relations are possible? a. 62 b. 26 c. 6! d. 212
5. Let $R: B \rightarrow B$ be defined by $\mathrm{R}=\{(1,1),(1,2),(2,2),(3,3),(4,4),(5,5),(6,6)\}$, then R is a. Symmetric b. Reflexive and Transitive c. Transitive and symmetric d. Equivalence

## Inverse Trigonometric Function:

## CASE STUDY1:

Two men on either side of a temple of 30 meters high observe its top at the angles of elevation $\alpha$ and $\beta$ respectively. (as shown in the figure above). The distance between the two men is $40 \sqrt{ } 3$ meters and the distance between the first person A and the temple is $30 \sqrt{ } 3$ meters. Based on the above information answer the following:


1. $\angle C A B=\alpha=$ a. $\sin -1(2 \sqrt{ } 3)$ b. $\sin -1(12)$ c. $\sin -1(2)$ d. $\sin -1(\sqrt{ } 32)$
2. $\angle C A B=\alpha=$ a. $\cos ^{-1}(15)$ b. $\cos ^{-1}(25) \mathrm{c} \cdot \cos ^{-1}(\sqrt{ } 32)$ d. $\cos ^{-1}(45)$
3. $\angle B C A=\beta=\mathrm{a} \cdot \tan -1(12)$ b. $\tan -1(2) \mathrm{c} \cdot \tan -1(1 \sqrt{ } 3)$ d. $\tan -1(\sqrt{3})$
4. $\angle A B C=$ a. $\pi 4$ b. $\pi 6$ c. $\pi 2$ d. $\pi 3$
5. Domain and Range of $\cos -1 x=$ a. $(-1,1),(0, \pi)$ b. $[-1,1],(0, \pi) \mathrm{c} .[-1,1],[0, \pi]$ d.
$(-1,1),[-\pi 2, \pi 2]$

## MATRICES

## CASE STUDY1:

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below Market Products (in numbers) Pencil Eraser Sharpener A 10,000 2000 18,000 B 6000 20,000 8,000 If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then, Based on the above information answer the following:

1. Total revenue of market A a. Rs. 64,000 b. Rs. 60,400 c. Rs. 46,000 d. Rs. 40600
2. Total revenue of market B a. Rs. 35,000 b. Rs. 53,000 c. Rs. 50,300 d. Rs. 30,500
3. Cost incurred in market A a. Rs. 13,000 b. Rs.30,100 c. Rs. 10,300 d. Rs. 31,000
4. Profit in market A and B respectively are a. (Rs. 15,000 , Rs. 17,000 ) b. (Rs. 17,000, Rs. 15,000 ) c. (Rs. 51,000 , Rs. 71,000) d. (Rs. 10,000 , Rs. 20,000)
5. Gross profit in both market a. Rs. 23,000 b. Rs. 20,300 c. Rs. 32,000 d. Rs. 30,200 ANSWERS 1. Rs. 46,000 2. Rs. 53,000 3. RS.31,000 4. (Rs.15, 000, Rs.17, 000) 5. Rs. 32,000

## Determinants

## CASE STUDY 1:

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m , then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m , then its area will decrease by 5300 m 2


Based on the information given above, answer the following questions:

1. The equations in terms of $X$ and $Y$ are $a . x-y=50,2 x-y=550 b . x-y=50,2 x+y=550 c . x+y$ $=50,2 \mathrm{x}+\mathrm{y}=550 \mathrm{~d} . \mathrm{x}+\mathrm{y}=50,2 \mathrm{x}+\mathrm{y}=550$
2. The value of x (length of rectangular field) is a. $150 \mathrm{~m} \mathrm{~b} .400 \mathrm{~m} \mathrm{c}$.200 m y x d. 320 m
3. The value of $y$ (breadth of rectangular field) is a. 150 m. b. 200 m . c. 430 m . d. 350 m
4. How much is the area of rectangular field? a. 60000 Sq.m. b. 30000 Sq.m. c. 30000 m d . 3000m

ANSWERS 1. b) $\mathrm{x}-\mathrm{y}=50,2 \mathrm{x}+\mathrm{y}=550$

* A function f is said to be continuous at $\mathrm{x}=\mathrm{a}$ if

Left hand limit $=$ Right hand limit $=$ value of the function at $x=a$

$$
\text { i.e. } \lim _{x \rightarrow a^{-}} f(x)=\lim _{x \rightarrow a^{+}} f(x)=f(a)
$$

i.e. $\lim _{h \rightarrow 0} f(a-h)=\lim _{h \rightarrow 0} f(a+h)=f(a)$.

* A function is said to be differentiable at $x=a$
if $\operatorname{Lf}^{\prime}(a)=\operatorname{Rf}^{\prime}(a)$ i.e $\lim _{h \rightarrow 0} \frac{f(a-h)-f(a)}{-h}=\lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
(i) $\frac{\mathrm{d}}{\mathrm{dx}}\left(\mathrm{x}^{\mathrm{n}}\right)=\mathrm{nx}^{\mathrm{n}-1}, \frac{\mathrm{~d}}{\mathrm{dx}}\left(\frac{1}{\mathrm{x}^{\mathrm{n}}}\right)=-\frac{\mathrm{n}}{\mathrm{x}^{\mathrm{n}+1}}, \quad \frac{\mathrm{~d}}{\mathrm{dx}}(\sqrt{\mathrm{x}})=-\frac{1}{2 \sqrt{\mathrm{x}}}$
(ii) $\frac{d}{d x}(x)=1$
(iii) $\frac{\mathrm{d}}{\mathrm{dx}}$ (c) $=0, \forall \mathrm{c} \in \mathrm{R}$
(iv) $\frac{d}{d x}\left(a^{x}\right)=a^{x} \log a, a>0, a \neq 1$.
(v) $\frac{d}{d x}\left(e^{x}\right)=e^{x}$.
(vi) $\frac{d}{d x}\left(\log _{a} x\right)=, \frac{1}{x \log a} a>0, a \neq 1, x$
(vii) $\frac{d}{d x}(\log x)=\frac{1}{x}, x>0$
(viii) $\frac{d}{d x}\left(\log _{a}|x|\right)=\frac{1}{x \log a}, a>0, a \neq 1, x \neq 0$
(ix) $\frac{d}{d x}(\log |x|)=\frac{1}{x}, x \neq 0$
(x) $\frac{d}{d x}(\sin x)=\cos x, \forall x \in R$.
(xi) $\frac{d}{d x}(\cos x)=-\sin x, \forall x \in R$.
(xii) $\frac{d}{d x}(\tan x)=\sec ^{2} x, \forall x \in R$.
(xiii) $\frac{d}{d x}(\cot x)=-\operatorname{cosec}^{2} x, \forall x \in R$.
(xiv) $\frac{d}{d x}(\sec x)=\sec x \tan x, \forall x \in R$.
$(x v) \frac{d}{d x}(\operatorname{cosec} x)=-\operatorname{cosec} x \cot x, \forall x \in R$.
(xvi) $\frac{d}{d x}\left(\sin ^{-1} x\right)=\frac{1}{\sqrt{1-x^{2}}}$.
(xvii) $\frac{d}{d x}\left(\cos ^{-1} x\right)=\frac{-1}{\sqrt{1-x^{2}}}$.
(xviii) $\frac{\mathrm{d}}{\mathrm{dx}}\left(\tan ^{-1} \mathrm{x}\right)=\frac{1}{1+\mathrm{x}^{2}}, \forall \mathrm{x} \in \mathrm{R}$
(xix) $\frac{d}{d x}\left(\cot ^{-1} x\right)=-\frac{1}{1+x^{2}}, \forall x \in R$.
(xx) $\frac{d}{d x}\left(\sec ^{-1} x\right)=\frac{1}{|x| \sqrt{x^{2}-1}}$.
(xxi) $\frac{d}{d x}\left(\operatorname{cosec}^{-1} x\right)=-\frac{1}{|x| \sqrt{x^{2}-1}}$.
(xxii) $\frac{d}{d x}(|x|)=\frac{x}{|x|}, x \neq 0$
(xxiii) $\frac{d}{d x}(k u)=k \frac{d u}{d x}$
(xxiv) $\frac{d}{d x}(u \pm v)=\frac{d u}{d x} \pm \frac{d v}{d x}$
$(x x v) \frac{d}{d x}(u . v)=u \frac{d v}{d x}+v \frac{d u}{d x}$
$(x x v i) \frac{d}{d x}\left(\frac{u}{v}\right)=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}$


## SOME ILLUSTRATIONS:

## MCQ

1. If $y=\log \tan \sqrt{x}$ then the value of $\frac{d y}{d x}$ is
( a ) $\frac{1}{2 \sqrt{x}}$
(b) $\frac{\sec 2 \sqrt{x}}{\sqrt{x} \tan x}$
(c ) $2 \sec ^{2} \sqrt{x}$
(d) $\frac{\sec 2 \sqrt{x}}{2 \sqrt{x} \tan x}$

Ans: (d)
2. If $y=\left(\cos x^{2}\right)^{2}$ then $\frac{d y}{d x}$ is
( a ) $-4 x \sin 2 x^{2}$
(b) $-x \sin x^{2}$
(c) $-2 x \sin 2 x^{2}$
(d) $-x \cos 2 x^{2}$

Ans: (c)
3. If $y=\cot ^{-1}\left(x^{2}\right)$ then the value of $\frac{d y}{d x}$ is
(a) $\frac{2 x}{1+x 4}$
(b) $\frac{2 x}{\sqrt{1+4 x}}$
( c ) $\frac{-2 \mathrm{x}}{1+\mathrm{x} 4}$
(d) $\frac{-2 x}{\sqrt{1+x^{2}}}$

Ans: (c)

## SHORT ANSWER TYPE QUESTIONS

**Q. 1. If $f(x)=\left\{\begin{array}{lll}3 a x+b, & \text { if } & x>1 \\ 11 & \text { if } & x=1 \\ 5 a x-2 b & \text { if } & x<1\end{array}\right.$, continuous at $x=1$, find the values of $a$ and $b$.
Sol. $\lim _{x \rightarrow 1^{-}} f(x)=\lim _{x \rightarrow 1^{+}} f(x)=f(1)$
$\lim _{x \rightarrow 1^{-}} f(x)=\lim _{h \rightarrow 0} f(1-h)=\lim _{h \rightarrow 0}[5 a(1-h)-2 b]=5 a-2 b$
$\lim _{x \rightarrow 1^{+}} f(x)=\lim _{h \rightarrow 0} f(1+h)=\lim _{h \rightarrow 0}[3 a(1+h)+b]=3 a+b$
$\mathrm{f}(1)=11$
From (i) $3 \mathrm{a}+\mathrm{b}=5 \mathrm{a}-2 \mathrm{~b}=11$ and solution is $\mathrm{a}=3, \mathrm{~b}=2$
$* * * *$ Q. 2. If $y=\sin \left(m \sin ^{-1} x\right)$, prove that $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}+m^{2} y=0$
Sol. $y=\sin \left(m \sin ^{-1} x\right) \Rightarrow \frac{d y}{d x}=\cos \left(m \sin ^{-1} x\right) \cdot \frac{m}{\sqrt{1-x^{2}}}$
$\Rightarrow \sqrt{1-x^{2}} \frac{d y}{d x}=m \cos \left(m \sin ^{-1} x\right)$
Again diff .w.r.t. $x, \sqrt{1-x^{2}} \frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}\left(\frac{-2 x}{\sqrt{1-x^{2}}}\right)=-m \sin \left(m \sin ^{-1} x\right) \cdot \frac{m}{\sqrt{1-x^{2}}}$
$\Rightarrow\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}=-m^{2} \sin \left(m \sin ^{-1} x\right)=-m^{2} y$
$\Rightarrow\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}+m^{2} y=0$

## LONG ANSWER TYPE QUESTIONS

**If $y=\left(\log _{e} x\right)^{x}+x^{\log _{e} x}$ find $\frac{d y}{d x}$.
Sol. $y=\left(\log _{e} x\right)^{x}+x^{\log _{e} x}=e^{\log ^{2}\left\{\left(\log _{e} x\right)^{x}\right\}}+e^{\log \left\{x^{\log _{e} x}\right\}}$

$$
=\mathrm{e}^{\mathrm{x} \log \left\{\left(\log _{\mathrm{e}} \mathrm{x}\right)\right\}}+\mathrm{e}^{\log _{\mathrm{e}} \mathrm{x} \cdot \log _{\mathrm{e}} \mathrm{x}}
$$

$\frac{d y}{d x}=e^{x \log ^{( }\left(\log _{c} x\right)}\left\{\left[x \cdot \frac{1}{\log x} \cdot \frac{1}{x}+\log (\log x) \cdot 1\right]+e^{\log _{c} x \cdot \log _{c} x}\left[\frac{\log x}{x}+\frac{\log x}{x}\right]\right.$
$=\left(\log _{e} x\right)^{x}\left[\frac{1}{\log x}+\log (\log x)\right]+x^{\log x}\left[2 \frac{\log x}{x}\right]$

## QUESTIONS FOR PRACTICE

Q 1. If $f(x)=\left\{\begin{array}{ccccccc}3 x-5 \quad x \leq 5 \\ 2 k & x>5\end{array}\right.$ is continuous at $x=5$ then $k$ is (a )5 (b) $10 \quad$ (c ) 15 (d) $\frac{-2}{7}$
Q 2. The function $f(x)=[x]$ is continuous at (a) 4
(b) -2 (c) 1
(d) 1.5
Q 3. If $x=t^{2}$ and $y=t^{3}$ then $\frac{d^{2} y}{d x^{2}}$ is equal to
(a) $\frac{3}{2}$
(b) $\frac{3}{4 t}$
(c) $\frac{3}{2 t}$
(d) $\frac{3 t}{2}$

Q 4. Derivative of $x^{2}$ w.r.t $x^{3}$ is (a) $\frac{1}{x}$ (b) $\frac{2}{3 x}$
Q 5. Assertion : $f(x)=[x]$ not differentiable at $x=2$

Reason : $\mathrm{f}(\mathrm{x})=[\mathrm{x}]$ not continuous at $\mathrm{x}=2$
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true and R is not the correct explanation of A
(c) $A$ is true but $R$ is false
(d) A is false but $R$ is true

## ANSWERS

1. 

(a)
2. (d)
3. (b)
4. (b)
5. (a)

## SHORT ANSWER TYPE QUESTIONS

Q 1. . Find the number of points at which the function $f(x)=\frac{9-x^{2}}{9 x-x^{3}}$ is discontinuous.
Q 2. . Find the value of $k$ for which $f(x)=\left\{\begin{array}{lll}\frac{\sin 2 x}{5 x} & , x \neq 0 \\ k & , & x=0\end{array}\right.$ is continuous at $x=0$.
Q 3. .Discuss the differentiability of the function $f(x)=|x-2|$ at $x=2$.
Q 4. . Find $: \frac{d}{d x}\left[\log _{e} \tan \left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$
Q 5. Find : $\frac{d}{d x}\left[\tan ^{-1}\left(\sqrt{\frac{1+\sin \mathrm{x}}{1-\sin \mathrm{x}}}\right)\right]$, where $0<\mathrm{x}<\frac{\pi}{4}$
Q 6. . Find : $\frac{\mathrm{d}}{\mathrm{dx}}\left[\mathrm{x}^{\sin \mathrm{x}}\right]$
Q 7. Find the relationship between a and b so that the function defined by
$f(x)=\left\{\begin{array}{ll}a x+1 & , \text { if } x \leq 3 \\ b x+3 & \text {,if } x>3\end{array}\right.$ is continuous at $x=3$.
Q 8. If $x=a(\cos t+t \sin t)$ and $y=a(\sin t-t \cos t)$ find $\frac{d^{2} y}{d x^{2}}$
Q 9. If $x=\sqrt{a^{\sin ^{-1} t}}, y=\sqrt{a^{\cos ^{-1} t}}$, show that $\frac{d y}{d x}=-\frac{y}{x}$.
Q 10. Find : $\frac{d}{d x}\left[\log \sin \sqrt{x^{2}+1}\right]$

## ANSWERS

1. Exactly three points $(0,3$ and -3$) \quad$ 2. $\frac{2}{5}$ 3. not differentiable at $x=2$
2. $\operatorname{Sec} x$
3. $\frac{1}{2}$
4. $x^{\sin x}\left(\cos x \cdot \log _{e} x+\frac{\sin x}{x}\right)$
5. 

$\mathbf{3 a}-\mathbf{3 b}=\mathbf{2}$ is the relation between a and b
8. $\frac{\sec ^{3} \mathrm{t}}{\mathrm{at}} \quad$ 10. $\frac{\mathrm{x} \cos \sqrt{\mathrm{x}^{2}+1}}{\sqrt{\mathrm{x}^{2}+1} \cdot \sin \sqrt{\mathrm{x}^{2}+1}}$

## APPLICATION OF DERIVATIVE

** Whenever one quantity $y$ varies with another quantity $x$, satisfying some rule $y=f(x)$, then $\frac{d y}{d x}$ (or $f$ $\left.{ }^{\prime}(x)\right)$ represents the rate of change of $y$ with respect to $x$ and $\left[\frac{d y}{d x}\right]_{x=x_{0}}\left(\operatorname{or~}^{\prime}\left(x_{0}\right)\right)$ represents the rate of change of y with respect to x at $\mathrm{x}=\mathrm{x}_{0}$.
** Let I be an open interval contained in the domain of a real valued function $f$. Then $f$ is said to be
(i) increasing on I if $x_{1}<x_{2}$ in $I \Rightarrow f\left(x_{1}\right) \leq f\left(x_{2}\right)$ for all $x_{1}, x_{2} \in I$.
(ii) strictly increasing on $I$ if $x_{1}<x_{2}$ in $I \Rightarrow f\left(x_{1}\right)<f\left(x_{2}\right)$ for all $x_{1}, x_{2} \in I$.
(iii) decreasing on $I$ if $x_{1}<x_{2}$ in $I \Rightarrow f\left(x_{1}\right) \geq f\left(x_{2}\right)$ for all $x_{1}, x_{2} \in I$.
(iv) strictly decreasing on I if $x_{1}<x_{2}$ in $I \Rightarrow f\left(x_{1}\right)>f\left(x_{2}\right)$ for all $x_{1}, x_{2} \in I$.
** (i) $f$ is strictly increasing in (a,b) if $f^{\prime}(x)>0$ for each $x \in(a, b)$
(ii) $f$ is strictly decreasing in (a,b) if $f^{\prime}(x)<0$ for each $x \in(a, b)$
(iii) A function will be increasing (decreasing) in $\mathbf{R}$ if it is so in every interval of $\mathbf{R}$.
** Slope of the tangent to the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})$ at the point $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ is given by $\left[\frac{\mathrm{dy}}{\mathrm{dx}}\right]_{\left(\mathrm{x}_{0}, y_{0}\right)}\left(=\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)\right)$.
** The equation of the tangent at $\left(\mathbf{x}_{\mathbf{0}}, \mathbf{y}_{0}\right)$ to the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})$ is given by $\mathrm{y}-\mathrm{y}_{0}=\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)\left(\mathrm{x}-\mathrm{x}_{0}\right)$.
** Slope of the normal to the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})$ at $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ is $-\frac{1}{\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)}$.
** The equation of the normal at $\left(\mathbf{x}_{\mathbf{0}}, \mathrm{y}_{\mathbf{0}}\right)$ to the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})$ is given by $\mathrm{y}-\mathrm{y}_{0}=-\frac{1}{\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)}\left(\mathrm{x}-\mathrm{x}_{0}\right)$.
** If slope of the tangent line is zero, then $\tan \theta=0$ and so $\theta=0$ which means the tangent line is parallel to the $x$-axis. In this case, the equation of the tangent at the point ( $x_{0}, y_{0}$ ) is given by $y=y_{0}$.
** If $\theta \rightarrow \frac{\pi}{2}$, then $\tan \theta \rightarrow \infty$, which means the tangent line is perpendicular to the $x$-axis, i.e., parallel to the y -axis. In this case, the equation of the tangent at $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ is given by $\mathrm{x}=\mathrm{x}_{0}$.
** Let f be a function defined on an interval I. Then
(a) f is said to have a maximum value in I , if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$.

The number $f(c)$ is called the maximum value of $f$ in $I$ and the point $c$ is called a point of maximum value of $f$ in $I$.
(b) f is said to have a minimum value in I , if there exists a point c in I such that $\mathrm{f}(\mathrm{c}) \leq \mathrm{f}(\mathrm{x})$, for all $\mathrm{x} \in \mathrm{I}$.

The number f ( c ), in this case, is called the minimum value of f in I and the point c , in this case, is called a point of minimum value of $f$ in $I$.
(c) f is said to have an extreme value in I if there exists a point c in I such that $\mathrm{f}(\mathrm{c})$ is either a maximum value or a minimum value of $f$ in $I$.

The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

*     * Absolute maxima and minima

Let $f$ be a function defined on the interval $I$ and $c \in I$. Then
(a) $f(c)$ is absolute minimum if $f(x) \geq f(c)$ for all $x \in I$.
(b) $f(c)$ is absolute maximum if $f(x) \leq f(c)$ for all $x \in I$.
(c) $\mathrm{c} \in \mathrm{I}$ is called the critical point off if $\mathrm{f}^{\prime}(\mathrm{c})=0$
(d) Absolute maximum or minimum value of a continuous function $f$ on $[a, b]$ occurs at a or b or at critical points off (i.e. at the points where $f$ 'is zero)

If $\mathrm{c}_{1}, \mathrm{c}_{2}, \ldots, \mathrm{c}_{\mathrm{n}}$ are the critical points lying in $[\mathrm{a}, \mathrm{b}]$, then absolute maximum value of $\mathrm{f}=\max \left\{\mathrm{f}(\mathrm{a}), \mathrm{f}\left(\mathrm{c}_{1}\right), \mathrm{f}\left(\mathrm{c}_{2}\right), \ldots, \mathrm{f}\left(\mathrm{c}_{\mathrm{n}}\right), \mathrm{f}(\mathrm{b})\right\}$ and absolute minimum value of $\mathrm{f}=\min \left\{\mathrm{f}(\mathrm{a}), \mathrm{f}\left(\mathrm{c}_{1}\right), \mathrm{f}\left(\mathrm{c}_{2}\right), \ldots, \mathrm{f}\left(\mathrm{c}_{\mathrm{n}}\right), \mathrm{f}(\mathrm{b})\right\}$.

## ** Local maxima and minima

(a)A function $f$ is said to have a local maxima or simply a maximum value at $x=a$ if $f(a \pm h) \leq f(a)$ for sufficiently small $h$
(b)A function $f$ is said to have a local minima or simply a minimum value at $x=a$ if $f(a \pm h) \geq f(a)$.
** First derivative test :A function f has a maximum at a point $\mathrm{x}=\mathrm{a}$ if
(i) $f^{\prime}(a)=0$, and
(ii) $f^{\prime}(x)$ changes sign from + ve to $-v e$ in the neighbourhood of 'a' (points taken from left to right).

However, f has a minimum at $\mathrm{x}=\mathrm{a}$, if
(i) $f^{\prime}(a)=0$, and
(ii) $f^{\prime}(x)$ changes sign from $-v e$ to $+v e$ in the neighbourhood of ' $a$ '.

If $f^{\prime}(a)=0$ and $f^{\prime}(x)$ does not change sign, then $f(x)$ has neither maximum nor minimum and the point ' $a$ ' is called point of inflation.
The points where $\mathrm{f}^{\prime}(\mathrm{x})=0$ are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.
** Second derivative test
(i) a function has a maxima at $\mathrm{x}=\mathrm{a}$ if $\mathrm{f}^{\prime}(\mathrm{x}) 3.4 . \mathrm{b}=0$ and $\mathrm{f}^{\prime \prime}(\mathrm{a})<0$
(ii) a function has a minima at $\mathrm{x}=\mathrm{a}$ if $\mathrm{f}^{\prime}(\mathrm{x})=0$ and $\mathrm{f}^{\prime \prime}(\mathrm{a})>0$.

## MULTIPLE CHOICE OUESTIONS

1.The rate of change of the area of a circle with respect to its radius $r$ at $r=6 \mathrm{~cm}$ is
(a) $10 \pi$
(b) $12 \pi$
(c) $8 \pi$
(d) $11 \pi$
2. On which of the following is the function $f$ given by $f(x)=x^{100}+\sin x+1$ strictly decreasing
(a) $(0,1)$
(b) $\left(\frac{\pi}{2}, \pi\right)$
(c) $\left(0, \frac{\pi}{2}\right)$
(d) None of these
3. Let the function $f: R \rightarrow R$ be defined by $f(x)=2 x+\cos x$. Then $f(x)$
(a) has a maximum at $x=\pi$
(b) has a maximum at $\mathrm{x}=0$
(c) is a decreasing function
(d) is an increasing function
4. Which of the following functions is decreasing in $\left(0, \frac{\pi}{2}\right)$
(a) $\quad \sin 2 x$
(b) $\tan x$
(c) $\cos \mathrm{X}$
(d) $\cos 3 x$

## ASSERTION REASON BASED QUESTIONS

The following question contains STATEMENT-1 (Assertion) and STATEMENT-2(Reason) and has the following four choices, only one of which is correct answer. Mark the correct choice.
(a) Statement-1 and Statement-2 are true.Statement-2 is a correct explanation for statement-1.
(b) Statement-1 and Statement-2 are true.Statement-2 is not a correct explanation for statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false Statement-2 is true.
Q. STATEMENT-1(Assertion) : The function $f(x)=x^{3}-3 x^{2}+6 x-10$ is strictly increasing on $R$.

STATEMENT-2(Reason): A strictly increasing function is an injective map.

## SHORT ANSWER TYPE QUESTIONS

1. A stone is dropped into a quiet lake and waves move in circles at the speed of $5 \mathrm{~cm} / \mathrm{sec}$. At the instant when the radius of circular wave is 8 cm ,how fast is the enclosed area increasing ?
2. A balloon , which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm .
3. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
4. Sand is pouring from a pipe at the rate of $12 \mathrm{~cm}^{3} / \mathrm{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand of the cone increasing when the height is 4 cm ?
5. Find the intervals in which the following functions are strictly increasing or decreasing
(a) $-2 x^{3}-9 x^{2}-12 x+1$

$$
\text { (b) }(x+1)^{3}(x-3)^{3}
$$

6. Prove that $y=\frac{4 \sin \theta}{2+\cos \theta}-\theta$ is an increasing function of $\theta$ in $\left[0, \frac{\pi}{2}\right]$.
7. Twenty meters of wire is available for fencing of a flower bed in the form of a circular sector. Find the maximum area of the flower bed.
8. The sides of an equilateral triangle is increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. Find the rate at which the area increases when the side is 10 cm .
9. Find the rate of change of volume of a sphere with respect to it's surface area when the radius of the sphere is 12 cm .
10. Let $x$ and $y$ be the radius two circles such that $y=x^{2}+1$. Find the rate of change of circumference of the second circle with respect to the circumference of first circle.

## CASE BASED QUESTIONS

Q.1) An architect designs a building for a multinational company the floor consist of rectangular region with semicircular ends having a perimeter of 200 m


Building
(i) If $x$ and $y$ represent the length and breadth of rectangular region then find the relationship between the variables
(ii) Find the expression for the area of rectangular region.
(iii) Find the maximum area of the region.
Q.2) A telephone company in Gurgaon has 500 subscribers on its list and collect fixed charge of ₹ 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of ₹ 1 one subscriber will discontinue the service based on the above information answer the following

(i) if the annual subscription is increased by ₹x per subscriber then find the total revenue are of the company .
(ii) Find average revenue of the company when annual subscription is increased by ₹x per subscriber
(iii) Find the maximum annual revenue of the company.
Q.3) Sonam wants to repair sweet box for Diwali at home for making lower part of box she takes a square piece of cardboard of side 18 cm . Based on the above information answer the following questions

(i) If x cm be the length of each side of a square cardboard which is to be cut off from each corner of the square piece of side 18 cm then find the interval in which x lies.
(ii) Find the expression for volume of the open box.
(iii) Find the maximum volume of the box.

## ANSWERS

MCQ's- 1. B 2. D 3.A 4.C 5.B

## Short Answer Type Questions

1. $\quad 80 \pi 2.1 / \pi 3.8 / 3 \mathrm{~cm} / \mathrm{sec} 4.1 / 48 \pi 5$. (a)-(-2,-1) increasing (b) $x<-2, x>-1$ Decreasing 6.Proof $7.25 \mathrm{~m}^{2} \quad 8.10 \sqrt{3} \mathrm{~cm}^{2} / \mathrm{sec} 9.6 \mathrm{~cm}^{3} / \mathrm{cm}^{2} \quad 10.2 \mathrm{x}$
CASE BASED ANSWERS
Q. 1 (i) $2 x+\pi y=200$
(ii) $2\left(100 x-x^{2}\right) / \pi$
( iii ) 5000/ $\pi$
Q. 2 (i) $(500-x)(300+x)$
(ii) $[(500 / x)-1][300+x]$
(iii) $\mathbf{1 6 0 , 0 0 0}$
Q. 3 (i) (0,9)
(ii) $\mathbf{x}(\mathbf{1 8 - 2 x})(18-2 x)$
(iii) $432 \mathrm{~cm}^{3}$
2. 1.The rate of change of the area of a circle with respect to its radius $r$ at $r=6 \mathrm{~cm}$ is
(a) $10 \pi$
(b) $12 \pi$
(c) $8 \pi$
(d) $11 \pi$
1Marks
2.A balloon, which always remains spherical on inflationis being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm .

3Marks
3.A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of $2 \mathrm{~cm} / \mathrm{sec}$.How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ? 3Marks
4.Sand is pouring from a pipe at the rate of $12 \mathrm{~cm}^{3} / \mathrm{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand of the cone increasing when the height is 4 cm ?

3Marks
5.Find the intervals in which the following functions are strictly increasing or decreasing
(a) $-2 x^{3}-9 x^{2}-12 x+1$
(b) $(x+1)^{3}(x-3)^{3}$
3Marks
6.Prove that $y=\frac{4 \sin \theta}{2+\cos \theta}-\theta$ is an increasing function of $\theta$ in $\left[0, \frac{\pi}{2}\right]$. 3Marks
Q.7. An architect designs a building for a multinational company the floor consist of rectangular region with semicircular ends having a perimeter of 200 m .


Building
(i) If x and y represent the length and breadth of rectangular region then find the relationship between the variables
(ii) Find the expression for the area of rectangular region.
(iii) Find the maximum area of the region. 4 Marks

## ANSWERS

1.B $2.1 / \pi$
3. $8 / 3 \mathrm{~cm} / \mathrm{sec}$
4. $1 / 48 \pi$
5. (a)-(-2,-1) increasing (b) $x<-2, x>-1$
Decreasing
Q.7. (i) $2 x+\pi y=200$
(ii) $2\left(100 x-x^{2}\right) / \pi$
(iii) 5000/ $\pi$

## TEST PAPER-1(30 Marks)

11. A stone is dropped into a quiet lake and waves move in circles at the speed of $5 \mathrm{~cm} / \mathrm{sec}$. At the instant when the radius of circular wave is 8 cm , how fast is the enclosed area increasing ? 3Marks 12. A balloon, which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm.
12. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
13. Sand is pouring from a pipe at the rate of $12 \mathrm{~cm}^{3} / \mathrm{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand of the cone increasing when the height is 4 cm ?

3Marks
15. Find the intervals in which the following functions are strictly increasing or decreasing
(b) $\quad-2 x^{3}-9 x^{2}-12 x+1$
(b) $(x+1)^{3}(x-3)^{3}$

3Marks
16. Prove that $y=\frac{4 \sin \theta}{2+\cos \theta}-\theta$ is an increasing function of $\theta$ in $\left[0, \frac{\pi}{2}\right]$.
17. Twenty meters of wire is available for fencing of a flower bed in the form of a circular sector. Find the maximum area of the flower bed.

3Marks
18. The sides of an equilateral triangle is increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. Find the rate at which the area increases when the side is 10 cm .
19. Find the rate of change of volume of a sphere with respect to its surface area when the radius of the sphere is 12 cm .

3Marks
20. Let x and y be the radius two circles such that $\mathrm{y}=\mathrm{x}^{2}+1$. Find the rate of change of circumference of the second circle with respect to the circumference of first circle.

## Answers

$1.80 \pi 2.1 / \pi 3.8 / 3 \mathrm{~cm} / \mathrm{sec} 4.1 / 48 \pi 5$. (a)-(-2,-1) increasing (b) $x<-2, x>-1$ Decreasing 6.Proof $7.25 \mathrm{~m}^{2}$ $8.10 \sqrt{3} \mathrm{~cm}^{2} / \mathrm{sec} 9.6 \mathrm{~cm}^{3} / \mathrm{cm}^{2}$ 10.2x

## INDEFINITE INTEGRALS

## SOME IMPORTANT RESULTS/CONCEPTS

* $\int x^{n} d x=\frac{x^{n+1}}{n+1}+C$
* $\int 1 . d x=x+C$
* $\int \frac{1}{x^{n}} d x=-\frac{1}{x^{n}}+C$
* $\int \frac{1}{\sqrt{\mathrm{x}}}=2 \sqrt{\mathrm{x}}+\mathrm{C}$
* $\int \frac{1}{x} d x=\log _{e} x+C$
* $\int e^{x} d x=e^{x}+C$
* $\int a^{x} d x=\frac{a^{x}}{\log _{e} a}+C$
* $\int \sin x d x=-\cos x+C$
* $\int \cos x d x=\sin x+C$
* $\int \sec ^{2} x d x=\tan x+C$
* $\int \operatorname{cosec}^{2} x d x=-\cot x+C$
* $\int \sec x \cdot \tan x d x=\sec x+C$
* $\int \operatorname{cosec} x \cdot \cot x d x=-\operatorname{cosec} x+C$
* $\int \tan x d x=-\log |\cos x|+C=\log |\sec x|+C$
* $\int \cot \mathrm{xdx}=\log |\sin \mathrm{x}|+\mathrm{C}$
* $\int \sec x d x=\log |\sec x+\tan x|+C$
$=\log \left|\tan \left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C$
* $\int \operatorname{cosec} \mathrm{xdx}=\log |\operatorname{cosec} \mathrm{x}-\cot \mathrm{x}|+C$
$=-\log |\operatorname{cosec} x+\cot x|+C=\log \left|\tan \frac{x}{2}\right|+c$

$$
\begin{aligned}
& * \int \frac{d x}{x^{2}-a^{2}}=\frac{1}{2 a} \log \left|\frac{x-a}{x+a}\right|+C, \text { if } x>a \\
& * \int \frac{d x}{a^{2}-x^{2}}=\frac{1}{2 a} \log \left|\frac{a+x}{a-x}\right|+C, \text { if } x>a \\
& * \int \frac{d x}{x^{2}+a^{2}}=\frac{1}{a} \tan ^{-1} \frac{x}{a}+C,=-\frac{1}{a} \cot ^{-1} \frac{x}{a}+C \\
& * \int \frac{1}{\sqrt{a^{2}-x^{2}}} d x=\sin ^{-1} \frac{x}{a}+c=-\cos ^{-1} \frac{x}{a}+C \\
& * \int \frac{d x}{\sqrt{a^{2}+x^{2}}}=\log \left|x+\sqrt{x^{2}+a^{2}}\right|+C \\
& * \int \frac{d x}{\sqrt{x^{2}-a^{2}}}=\log \left|x+\sqrt{x^{2}-a^{2}}\right|+C \\
& * \int \sqrt{x^{2}+a^{2}} d x=\frac{x}{2} \sqrt{x^{2}+a^{2}}+\frac{a^{2}}{2} \log \left|x+\sqrt{x^{2}+a^{2}}\right|+C \\
& * \int \sqrt{x^{2}-a^{2}} d x=\frac{x}{2} \sqrt{x^{2}-a^{2}}-\frac{a^{2}}{2} \log \left|x+\sqrt{x^{2}-a^{2}}\right|+C \\
& * \int \sqrt{a^{2}-x^{2}} d x=\frac{x}{2} \sqrt{a^{2}-x^{2}}+\frac{a^{2}}{2} \sin { }^{-1} \frac{x}{a}+C \\
& * \int\left\{f_{1}(x) \pm f_{2}(x) \pm \ldots \ldots . . f_{n}(x)\right\} d x \\
& * \int f_{1}(x) d x \pm \int f_{2}(x) d x \pm \ldots \ldots \ldots . \pm \int f_{n}(x) d x \\
& * \int \lambda f(x) d x=\lambda \int f(x) d x+C \\
& * \int u . v d x=u \cdot \int v \cdot d x-\int\left[\int v \cdot d x\right] \frac{d u}{d x} \cdot d x
\end{aligned}
$$

## II Some Illustration / Examples

MCQ based questions with solutions
Q1. $\int \frac{1}{1+\mathrm{x}^{2}} \mathrm{dx}$ is equal to
(a) $\tan ^{-1} \mathrm{x}+\mathrm{c}$
(b) $\sin ^{-1} x+c$
(c) $\cos ^{-1} \mathrm{x}+\mathrm{c}$
(d) $\sec ^{-1} \mathrm{X}+\mathrm{c}$

Answer: (a) $\tan ^{-1} \mathbf{x}+\mathbf{c}$
Q2. $\int \frac{\mathrm{e}^{\mathrm{x}}}{\mathrm{e}^{\mathrm{x}}+1} \mathrm{dx}$ is equal to
(a) $x+\log \left(x^{2}+1\right)(b) e^{x}$
(c) $\mathrm{e}^{\mathrm{x}}+1$
(d) $\log \left(e^{x}+1\right)$

Answer: (d) $\log \left(\mathrm{e}^{\mathrm{x}}+1\right)$
Q3. $\int \frac{1}{x \log x} d x$ is equal to
(a) $\quad \log x+c$
(b) $\log (\log x+c)$
(c) $\log (\log (\log x))+c$
(d) None of these

## Answer: $\log (\log x)+c$

Q4. $\int \mathrm{e}^{\mathrm{x}} \sin \mathrm{x} d \mathrm{x}$ is equal to
(a) $\frac{1}{2} e^{x}(\sin x+\cos x)$
(b) $\frac{1}{2} e^{x}(\sin x-\cos x)(c) e^{x}(\sin x-\cos x)$
(d) None of these

Answer: (b) $\frac{1}{2} \mathrm{e}^{\mathrm{x}}(\sin x-\cos x)$
ii) Case based study Question

## Results:

(i) $\int \sec ^{2} x d x=\tan x+c$
(ii) $\int \sec x \tan x d x=\sec x+c$
(iii) $\int \frac{1}{\mathrm{ax}+\mathrm{b}} \mathrm{dx}=\frac{\log (\mathrm{ax}+\mathrm{b})}{\mathrm{a}}+\mathrm{c} \quad ; \mathrm{ax}+\mathrm{b}>0$
(iv) $\int \frac{f^{x}(x)}{f(x)} d x=\log [f(x)]+c$
(v) $\int[\mathrm{f}(\mathrm{x})]^{\mathrm{n}} \mathrm{f}^{\prime}(\mathrm{x}) \mathrm{dx}=\frac{[\mathrm{ff}(\mathrm{x})]^{\mathrm{n}+1}}{\mathrm{n}+1}+\mathrm{c} ; \mathrm{n} \neq-1$
on the above information answer the following
(1) $\int \frac{1}{1+\sin x} d x$ is equal to
(a) $\tan x+\sec x+c$
(b) $\tan x-\sec x+c$
(c) $\tan ^{2} x-\sec x+c$
(d) $\tan \mathrm{x}-\sec ^{2} \mathrm{x}+\mathrm{c}$
(2) $\int \frac{\cos 2 x+2 \sin ^{2} x}{\cos ^{2} x} d x$ is equal to
(a) $\tan x+c$ (b) $-\tan x+c$
(c) $-\sin x+c$
(d) $\cos x+c$
(3) $\int \frac{x+3}{x^{2}+4 x+3} d x$ is equal to
(a) $\log \mathrm{I} x-1 I+c$
(b) $-\log \mathrm{I}+1 \mathrm{I}+\mathrm{c}$
(c) $\log \mathrm{Ix}+2 \mathrm{I}+\mathrm{c}$
(d) $\log \mathrm{I} x+1 \mathrm{I}+\mathrm{c}$
(4) $\int \frac{2 \mathrm{x}}{1+\mathrm{x}^{2}} \mathrm{dx}$ is equal to
(a) $\log \left(1-x^{2}\right)+c$ (b) $\log \left(2+x^{2}\right)+c$
(c) $\log \left(1+x^{2}\right)+c$
(d) $\log \left(3+x^{2}\right)+c$

| Answer: Q1. (b) | Q2. (a) | Q3. (d) |
| :--- | :--- | :--- |
| (iii)Short Answer type questions with solutions:- |  |  |

Q1. Evaluate: $\int \frac{1}{\sqrt{9+\mathrm{x}^{2}}} \mathrm{dx}$
Sol. $\int \frac{1}{\sqrt{9+x^{2}}} d x=\int \frac{1}{\sqrt{3^{2}+x^{2}}} d x=\log I \frac{x+\sqrt{3^{2}+x^{2}}}{3} I+c$
Q2. Find $\int \sin ^{5}\left(\frac{x}{2}\right) \cdot \cos \left(\frac{x}{2}\right) d x$
Sol. Put $\sin \frac{x}{2}=y$
$\Rightarrow \quad \frac{1}{2} \cos \left(\frac{x}{2}\right) \mathrm{dx}=\mathrm{dy}$
$\Rightarrow \quad \cos \left(\frac{x}{2}\right) d x=2 d y$
so, $I=2 \int y^{5} d y=2 \frac{y^{6}}{6}=\frac{1}{3} y^{6}+c=\frac{1}{3} \sin ^{6}\left(\frac{x}{2}\right)+c$
Q3. Integrate: $\int \frac{\mathrm{e}^{\tan ^{-1} \mathrm{x}}}{1+\mathrm{x}^{2}} \mathrm{dx}$
Sol. Put $\tan ^{-1} \mathrm{x}=\mathrm{y}$
$\Rightarrow \quad \frac{1}{1+x^{2}} d x=d y$
$\Rightarrow \quad I=e^{y} d y=e^{y}+c=e^{\tan ^{-1} x}+c$

## (iv)Long Answer type questions:

Q1. Find $\int \frac{2 x}{\left(x^{2}+1\right)\left(x^{2}+2\right)} d x$
Sol. Put $x^{2}=y=>2 x d x=d y$

$$
\text { So, } \begin{aligned}
& \mathrm{I}=\int \frac{\mathrm{dy}}{(\mathrm{y}+1)(\mathrm{y}+2)}=\int\left[\frac{1}{(\mathrm{y}+1)}-\frac{1}{(\mathrm{y}+2)}\right] \mathrm{dy} \\
& =\log \mathrm{Iy}+1 \mathrm{I}-\log \mathrm{y}+2 \mathrm{I}+\mathrm{c} \\
& =\log \left[\frac{\mathrm{y}+1}{\mathrm{y}+2} \mathrm{I}+\mathrm{c}=\log \frac{\mathrm{x}^{2}+1}{\mathrm{x}^{2}+2} \mathrm{I}+\mathrm{c}\right.
\end{aligned}
$$

Q2. Evaluate: $\int \frac{\mathrm{dx}}{\sqrt{3 \mathrm{x}^{2}+6 \mathrm{x}+12}}$
Sol. $I=\frac{1}{\sqrt{3}} \int \frac{d x}{\sqrt{\mathrm{x}^{2}+2 \mathrm{x}+4}}=\frac{1}{\sqrt{3}} \int \frac{\mathrm{dx}}{\sqrt{(\mathrm{x}+1)^{2}+(\sqrt{3})^{2}}}=\frac{1}{\sqrt{3}} \int \frac{\mathrm{dx}}{\sqrt{(\mathrm{t})^{2}+(\sqrt{3})^{2}}} \quad($ put $\mathrm{x}+1=\mathrm{t} \Rightarrow \mathrm{dx}=\mathrm{dt})$

$$
=\frac{1}{\sqrt{3}} \log \left|t+\sqrt{t^{2}+3}\right|+c \quad=\frac{1}{\sqrt{3}} \log \left|(x+1)+\sqrt{x^{2}+2 x+4}\right|+c
$$

## III Practice Questons

(I) MCQ Questions for practices

1. Given $\int 2^{x} d x=f(x)+c$ then $f(x)=$
(a) $\quad 2^{x}$
(b) $2^{x} \log e^{2}$
(c) $\frac{2^{x}}{\log 2}$
(d) $\frac{2^{\mathrm{x}+1}}{\mathrm{x}+1}$
2. Given $\int \frac{1}{\sin ^{2} \operatorname{xcos}^{2} x} d x$ is equal to
(a) $\sin ^{2} \mathrm{x}-\cos ^{2} \mathrm{x}+\mathrm{c}$
(b) -1
(c) $\tan \mathrm{x}+\cot \mathrm{x}+\mathrm{c}$
(d) $\tan \mathrm{x}-\cot \mathrm{x}+\mathrm{c}$
3. $\int \frac{\cos 2 x-\cos 2 \theta}{\cos x-\cos \theta} d x$ is equal to
(a) $2(\sin x+x \cos \theta)+c$
(b) $2(\sin x-x \cos \theta)+c$
(c) $2(\sin x+2 x \cos \theta)+c$
(d) $2(\sin x-\sin \theta)+c$
4. $\quad \int \cot ^{2} \mathrm{xdx}$ equals to
(a) $\quad \operatorname{Cot} \mathrm{x}-\mathrm{x}+\mathrm{c}$
(b) $-\cot x+x+c$
(c) $\cot \mathrm{x}+\mathrm{x}+\mathrm{c}$
(d) $-\cot x-x+c$

## ASSERTION REASON TYPE QUESTION

5.The following question consists of two statements - Assertion (A) and Reason (R ). Answer the question selecting the appropriate option given below:
(a) Both A and R are true and R is the correct explanation of A .
(b) Both A and R are true but R is not the correct explanation of A .
(c) A is true but R is false
(d) A is false but R is true

Assertion: Geometrically, derivative of a function is the slope of the tangent to the corresponding curve at a point.
Reason: Geometrically, indefinite integral of a function represents a family of curves parallel go each other.
MCQ's 1-(c), 2-(d) 3-(a) 4-(d) 5 . Assertion Reason type Question: (b)
SHORT ANSWER TYPE QUESTIONS

1. Find $\int \frac{3+3 \cos x}{x+\sin x} d x$
2. Find $\int \frac{d x}{\sqrt{5-4 x-x^{2}}} d x$
3. Find $\int \frac{x^{3}-1}{x^{2}} d x$
4. Find $\int \frac{\sin ^{2} x-\cos ^{2} x}{\sin x \cos x} d x$
5. Find $\int \frac{d x}{x^{2}+16} d x$
6. Find $\int \frac{\cos 2 x+2 \sin ^{2} x}{\cos ^{2} x} d x$
7. Find $\int \frac{\sec ^{2} x}{\sqrt{\tan ^{2} x+4}} d x$
8. Find $\int \sqrt{1-\sin 2 x} d x$
9. Find $\int \frac{\left(x^{2}+\sin ^{2} x\right) \sec ^{2} \mathrm{x}}{1+\mathrm{x}^{2}} d x$
10. Find $\int e^{x} \frac{x-3}{(x-1)^{3}} d x$
11. Find $\int \sin ^{-1}(2 x) d x$
12. Find $\int \frac{3-5 \sin x}{\cos ^{2} x} d x$
13. Find $\int \frac{\tan ^{2} x \cdot \sec ^{2} x}{1-\tan ^{6} x} d x$
14. Find $\int \sin x \log (\cos x) d x$
15. Find $\int \frac{\sin ^{6} x+\cos ^{6} x}{\sin ^{2} x \cdot \cos ^{2} x} d x$

LONG ANSWER TYPE QUESTIONS

1. Find $\int \frac{6 x+8}{3 x^{2}+6 x+2} d x$
2. Find $\int \frac{1-\tan ^{2} \mathrm{x}}{1+\tan ^{2} \mathrm{x}} \mathrm{dx}$
3. Find $\int \frac{x^{4}}{1+x^{10}} d x$
4. Find $\int \frac{x^{2} \tan ^{-1} x^{3}}{1+x^{6}} d x$
5. Find $\int \frac{\sin 8 \mathrm{x}}{\sqrt{1-\cos ^{4} 4 \mathrm{x}}} \mathrm{dx}$

## Answer (Short answer type Questions):

1. $\quad 3 \log (x+\sin x)+c$
2. $\sin ^{-1}\left(\frac{x+2}{3}\right)+c$
3. $\frac{x^{2}}{2}+\frac{1}{x}+c$
4.     - logI cosxsinxI + c
5. $\frac{1}{4} \tan ^{-1}\left(\frac{x}{4}\right)+c$
6. $\tan x+c$
7. $\quad \log \left|\tan x+\sqrt{\tan ^{2} x+4}\right|+c$
8. $\operatorname{Sin} x+\cos x+c$
9. $\quad \boldsymbol{t a n} x-\tan ^{-1} x+c$
10. $\frac{\mathrm{e}^{\mathrm{x}}}{(\mathrm{x}-1)^{2}}+\mathrm{c}$
11. $x \sin ^{-1} 2 x+\frac{1}{2} \sqrt{1-4 x^{2}}+c$
12. $3 \tan x-5 \sec x+c$
13. $\frac{1}{6} \log I \frac{1+\tan ^{3} x}{1-\tan ^{3} x} I+c$
14. $\cos x[1-\log \cos x]+c$
15. $\tan x-\cot x-3 x+c$

Answer (Long answer type Questions):

1. $\quad \log \left(3 x^{2}+6 x+2\right)+\frac{2}{3} \log I \frac{x+1-\frac{1}{\sqrt{2}}}{x+1+\frac{1}{\sqrt{2}}} I+c$
2. $\frac{\sin 2 \mathrm{x}}{2}+\mathrm{c}$
3. $\frac{1}{5} \boldsymbol{\operatorname { t a n }}^{-1} \mathrm{x}^{5}+\mathrm{c}$
4. $\quad \frac{1}{6}\left(\tan ^{-1} \mathrm{X}^{3}\right)^{2}+\mathrm{c}$
5. $-\frac{1}{8} \sin ^{-1}\left(\cos ^{2} 4 x\right)+c$

## V. Class Test

1. $\int x^{2}\left(1-\frac{1}{x^{2}}\right) d x$ is equal to
(a) $\frac{x^{3}}{3}-x+c(b) \frac{x^{3}}{3}+x+c$
(c) $x+1$
(d) $-\frac{x^{3}}{3}+x+c$
2. $\quad \int 3^{x} d x$ is equal to ( 1 M )
(a) $\quad 3^{x}(b) 3^{x}+c$
(c) $\frac{3^{x}}{\log 3}+c$
(d) none of these
3. $\int \frac{x^{3}}{x+1} d x$ is equal to
(a) $x+\frac{x^{2}}{2}+\frac{x^{3}}{3}-\log |1-x|+c$
(b) $x+\frac{x^{2}}{2}-\frac{x^{3}}{3}-\log |1-x|+c$
(c) $\quad x-\frac{x^{2}}{2}-\frac{x^{3}}{3}-\log |1+x|+c$
(d) $\quad x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\log |1+x|+c$
4. $\int \mathrm{e}^{\mathrm{x}}\left(\frac{1-\mathrm{x}}{1+\mathrm{x}}\right)^{2} \mathrm{dx}$ is equal to
(a) $\frac{\mathrm{e}^{\mathrm{x}}}{1+\mathrm{x}^{2}}+\mathrm{c}$
(b) $\frac{-\mathrm{e}^{\mathrm{x}}}{1+\mathrm{x}^{2}}+\mathrm{c}$
(c) $\frac{\mathrm{e}^{\mathrm{x}}}{\left(1+\mathrm{x}^{2}\right)^{2}}+\mathrm{C}$
(d) $\frac{-\mathrm{e}^{\mathrm{x}}}{\left(1+\mathrm{x}^{2}\right)^{2}}+\mathrm{c}$
5.Evaluate $\int \frac{\sec ^{2}(\log x)}{x} d x$
5. Evaluate $\int \frac{x^{2}+1}{(x-1)^{2}(x+3)} d x$
6. Evaluate $\int \cos ^{2} 3 x d x$
7. Evaluate $\int \frac{5 x+3}{\sqrt{x^{2}+4 x+10}} d x$

9 Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} d x$

## DEFINITE INTEGRALS SOME IMPORTANT RESULTS/CONCEPTS

$* \int_{a}^{b} f(x) d x=F(b)-F(a)$, where $F(x)=\int f(x) d x$
$* \int_{a}^{b} f(x) d x=\int_{a}^{b} f(t) d x$
$* \int_{a}^{b} f(x) d x=-\int_{a}^{b} f(x) d x$
$* \int_{a}^{b} f(x) d x=\int_{a}^{c} f(x) d x+\int_{c}^{b} f(x) d x$
$* \int_{a}^{b} f(x) d x=\int_{a}^{b} f(a+b-x) d x$
$* \int_{0}^{a} f(x) d x=\int_{0}^{a} f(a-x) d x$
$* \int_{-a}^{a} f(x) d x= \begin{cases}2 \int_{0}^{a} f(x) d x, & \text { if } f(x) \text { is an even function of } x . \\ 0 & \text { if } f(x) \text { is an odd function of } x\end{cases}$
$* \int_{0}^{2 a} f(x) d x=\left\{\begin{array}{ll}2 \int_{0}^{a} f(x) d x, & \text { if } f(2 a-x)=f(x) . \\ 0 & \text { if } f(2 a-x)=-f(x)\end{array}\right.$.

MCQ's

| Qno | Question | Mark | Correct Response |
| :--- | :--- | :--- | :--- |
| 1 | $\int_{-\pi / 4}^{\pi / 4} \operatorname{Sec}^{2} x d x($ a) -1 (b) 0 (c) 1 (d) 2 | 1 | d |


| 2 | $\int_{1 / 3}^{1} \frac{\left(x-x^{3}\right)^{1 / 3}}{\mathrm{x}^{4}} \mathrm{dx}$ is (a) $6(\mathrm{~b}) 0(\mathrm{c}) 1(\mathrm{~d}) 4$ | 1 | a |
| :--- | :--- | :--- | :--- |
| 3 | $\int_{0}^{2 / 3} \frac{\mathrm{dx}}{4+9 \mathrm{x}^{2}}$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}(\mathrm{c}) \pi / 24$ (d) $\pi / 4$ | 1 | c |
| 4 | $\int_{0}^{1} \frac{\mathrm{dx}}{1+\mathrm{x}^{2}}$ is (a) $0(\mathrm{~b}) \pi / 4 \quad$ (c) $\pi / 12 \quad$ (d) $\pi / 6$ | 1 | b |
| 5 | $\int_{-1}^{1} \mathrm{x}^{17}+\mathrm{x}^{71} \mathrm{dx}$ is (a) 1 (b) 0 (c) 2 (d) 4 | 1 | b |
| 6 | $\int_{0}^{\pi / 4} \tan ^{2} \mathrm{xdx}$ is (a) $1-\pi / 4$ (b) $1+\pi / 4(\mathrm{c}) 1-\pi / 2$ (d) $1+\pi / 2$ | 1 | a |

## Problems for Practice

All the questions carry 3 marks
1 Evaluate $\int_{0}^{1} \frac{\sin x}{1+\sin x} d x$
2 Evaluate $\int_{0}^{1} \cot ^{-1}\left(1-x-x^{2}\right) d x$
3 Evaluate $\int_{0}^{\pi / 2}(\sqrt{\tan x}+\sqrt{\cot x}) d x$
4 Evaluate $\int_{-1}^{3 / 2}|x \sin \pi x| d x$
5 Evaluate $\int_{0}^{1} \frac{x d x}{1+x^{2}}$
6 Evaluate $\int_{1}^{3}|2 x-1| d x$
7 Evaluate $\int_{0}^{2 \pi} \frac{1}{1+\mathrm{e}^{\sin x}} \mathrm{dx}$
8 Evaluate $\int_{-2}^{2} \frac{x^{2} d x}{1+5^{x}}$
9 Evaluate $\int_{\pi / 6}^{\pi / 3} \frac{\mathrm{dx}}{1+\sqrt{\tan x}}$
10 Evaluate $\left.\int_{0}^{\pi / 2( } 2 \log \cos x-\log \sin 2 x\right) d x$
All the questions carry 5 marks
1 Evaluate $\int_{-1}^{2}\left|x^{3}-x\right| d x$
2 Evaluate $\int_{-6}^{6}|x+3| d x$
3 Evaluate $\int_{0}^{\pi / 2} \frac{x \sin x \cos x}{\sin ^{4} x+\cos ^{4 x}} d x$
4 Evaluate $\int_{0}^{\pi} \frac{x d x}{a^{2} \cos ^{2} x+b^{2} \sin ^{2} x} d x$
5 Evaluate $\int_{0}^{\pi} \frac{x \tan x}{\tan x+\sec x} d x$
MCQ
$1 \quad \int_{0}^{2}\left(x^{2}+3\right) d x$ is
(a)8
(b) $25 / 3$
(c) $26 / 3$
(d) 9
$2 \quad \int_{0}^{\pi} \sin ^{2} x d x$ is
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\pi$
$3 \int_{0}^{\pi} \frac{\mathrm{dx}}{1+\sin \mathrm{x}}$ is
(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $e^{\pi / 2}$
(d) 2
$4 \int_{0}^{1} \frac{1-\mathrm{x}}{1+\mathrm{x}} \mathrm{dx}$
(a) $\frac{\log 2}{2}$
(b) $\frac{\log 2}{2}-1$
(c) $2 \log 2-1$
(d) $2 \log 2+1$
$5 \int_{0}^{\pi / 6} \cos x \cos 2 x d x$
(a) $1 / 4$
(b) $5 / 12$
(c) $1 / 3$
(d) $-1 / 12$
$6 \int_{0}^{1} \frac{d x}{e^{x}+e^{-x}}$
(a) $1-\pi / 4$
(b) $\tan ^{-1} \mathrm{e}$
(c) ) $\tan ^{-1} e+\pi / 4$ (d) $\tan ^{-1} e-\pi / 4$
$7 \int_{0}^{1} \sqrt{\frac{1-\mathrm{x}}{1+\mathrm{x}}} \mathrm{dx}$
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{2}-1$
(c) $\pi / 2+1$
(d) 0
$8 \quad \int_{0}^{\pi / 2} \log \left(\frac{4+3 \sin x}{4+3 \cos x}\right) d x$
(a) 2
(b) $3 / 4$
(c) 0
(d) -2
$9 \int_{0}^{1} \tan ^{-1} \frac{2 x-1}{1+x-x^{2}} d x$
(a) 1
(b) 0
(c) -1
(d) $\pi / 4$
$10 \int_{0}^{\pi / 2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} d x$ is
(a) $\frac{\pi}{2}$
(b) $\pi / 3$
(c) $\pi / 4$
(d) $\pi$
$11 \int_{0}^{\pi / 2} \frac{\mathrm{dx}}{1+\tan \mathrm{x}}=$
(a) $\frac{\pi}{2}$
(b) $\pi / 3$
(c) $\pi / 4$
(d) $\pi$
$12 \int_{-1}^{1} \sin ^{3} \mathrm{x} \cos ^{2} \mathrm{x} d \mathrm{x}$
(a) 0
(b) 1
(c) 2
(d) 3

In the following questions a statement of assertion Statement 1 is followed by statement of reason Statement 2 Mark the correct choice as
(a) If statement 1 and statement 2 is true and statement 2 is the correct explanation of 1
(b) If statement 1 and statement 2 is true and statement 2 is not the correct explanation of 1
(c) If statement 1 is true and statement 2 is false
(d) If statement 1 is false and statement 2 is true

Now answer the following
1 Statement I $\int_{0}^{\pi / 2} \sin ^{2} x d x=\pi / 4 \quad$ Statement $\quad$ II $\int_{0}^{a} f(x) d x=\int_{0}^{a} f(a-x) d x$
2Statement I $\int_{2}^{3} \frac{\sqrt{x} d x}{\sqrt{x}+\sqrt{5-x}}=1 / 2$
Statement II $\int_{0}^{2 a} f(x) d x=2 \int_{0}^{a} f(x) d x$ if $f(x)=f(2 a-x)$
ANSWERS

| SHORT ANSWER 3 MARKS |  |  |  |
| :--- | :---: | :--- | :--- |
| 1 | $\pi-2$ | 6 | 6 |
| 2 | $\frac{\pi}{2}-\log 2$ | 7 |  |
| 3 | $\pi / 2 \sqrt{2}$ | 8 | $8 / 3$ |
| 4 | $3 / \pi+1 / \pi^{2}$ | 9 |  |
| 5 | $\log 2 / 2$ | 10 | $\pi / 12$ |


| LONG ANSWER 5MARKS |  |  |
| :--- | :--- | :---: |
| 1 | $11 / 4$ |  |
| 2 | 45 |  |
| 3 | $\frac{\pi^{2}}{16}$ |  |
| 4 | $\frac{\pi^{2}}{2 \mathrm{ab}}$ |  |
| 5 | $\frac{\pi(\pi-2)}{2}$ |  |


| S NO | ANSWER | S No | Correct answer |
| :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | c $26 / 3$ | 7 | (b) $\frac{\pi}{2}-1$ |
| 2 | A $\frac{\pi}{2}$ | 8 | (c) 0 |
| 3 | D $^{2} 2$ | 9 | (b) 0 |
| 4 | C $2 \log 2-1$ | 10 | (a) $\frac{\pi}{2}$ |
| 5 | (d)-1/12 | 11 | (a) $\frac{\pi}{2}$ |
| 6 | (d) $\tan ^{-1}$ e $-\pi / 4$ | 12 | (a) 0 |

## ASSERTION AND REASONING

## 1 Choice (a) is correct

2 Choice (c) is correct
QUESTION PAPER OF DEFINITE INTEGRAL
Marks 20

| Q No | SECTION A | Marks |
| :--- | :--- | :--- |
| 1 | $\int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\operatorname{cosx}}}{\sqrt{\operatorname{Cosx}}+\sqrt{\operatorname{Sinx}}} \mathrm{dx}$ value is (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$ | 1 |
| 2 | $\int_{2}^{6} \frac{\sqrt{8-\mathrm{x}}}{\sqrt{x}+\sqrt{8-x}} \mathrm{dx}$ is (a) 0 (b) 1 (c) 2 (d) 3 | 1 |
| 3 | $\int_{1}^{3}(\mathrm{x}-1)(\mathrm{x}-2)(\mathrm{x}-3) \mathrm{dx}$ is (a) 3 (b) 1 (c) 2 (d) 0 | 1 |
|  | SECTION B <br> The question is response based 1 mark is awarded for writing correct answer and 2 <br> marks is awarded for writing the correct explanation |  |


| 4 | $\int_{1}^{2} \mathrm{e}^{\mathrm{x}}\left(\frac{1}{\mathrm{x}}\right.$ - $\left.-\frac{1}{\mathrm{x}^{2}}\right) \mathrm{dx} \quad$ (a)e(e-2)/2 (b)e(e-1) (c) 0 (d) 1 | 2 |
| :---: | :---: | :---: |
|  | SECTION CThe question is case based |  |
| 5 | For a function $f(x)$ if $f(-x)=f(x)$ it is called even function and $f(-x)=-f(x)$ is called an odd function Again we have $\int_{-a}^{a} f(x) d x=\left\{\begin{array}{c} \int_{0}^{a} 2 f(x) d x \text { if } f(x) \text { is even } \\ 0 \text { if } f(x) \text { is odd } \end{array}\right.$ <br> (i)What is the nature of $f(x)=x^{7} \operatorname{Sin} x$ (ii) $\int_{-\pi}^{\pi} x^{7} \operatorname{Sin} x d x$ is (iii) $\int_{-\pi}^{\pi} x \operatorname{Sin} x d x$ is | 1 1 2 |
|  | SECTION DBoth questions are 3 marks each Evaluate $\int_{0}^{2 \pi} \frac{\operatorname{Cos} x}{\sqrt{4+3 \operatorname{Sinx}}} \mathrm{dx}$ | 3 |
| 7 | Evaluate $\int_{-1}^{1} 5 \mathrm{x}^{4}\left(\sqrt{\mathrm{x}^{5}}+1\right) \mathrm{dx}$ | 3 |
|  | SECTION EThe question is of 5 marks |  |
| 8 | $\int_{0}^{\frac{\pi}{4}} \frac{\operatorname{Sin} x \operatorname{Cos} x d x}{\operatorname{Cos}^{2} \mathrm{x}+\operatorname{Sin}^{4} \mathrm{x}}$ | 5 |

## QUESTION PAPER OF DEFINITE INTEGRAL

Marks 30

| Q No | SECTION A | Marks |
| :---: | :---: | :---: |
| 1 | $\int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\operatorname{Cos} x}+\sqrt{\operatorname{Sin} x}} d x \text { value is (a) } \frac{\pi}{2} \text { (b) } \frac{\pi}{4} \text { (c) } \frac{\pi}{6} \text { (d) } \frac{\pi}{12}$ | 1 |
| 2 | Evaluate $\int_{-1}^{1} \sin ^{3} \mathrm{x} \cos ^{2} \mathrm{xdx}$ (a) 0 (b)1 (c)2(d) 3 | 1 |
| 3 | $\int_{-1}^{1} \frac{\sin x-x^{2}}{3-\|x\|} d x \text { is (a) } 0 \text { (b) } 1 \text { (c) } 3 \text { (d) } 5$ | 1 |
|  | SECTION BThe question is response based 1 mark is awarded for writing correct answer and 2 marks is awarded for writing the correct explanation |  |
| 4 | $\int_{0}^{2 a} \frac{f(x)}{f(x)+f(2 a-x)} d x \quad$ (a) a (b) -a (c) 1 (d) 0 | 2 |
| 5 | $\int_{0}^{\pi / 2} \frac{(\sin x+\cos x)^{2}}{\sqrt{1+\sin 2 \mathrm{x}}} \mathrm{dx}$ (a) 0 (b) 1 (c) 2 (d) 3 | 2 |
| 6 | $\int_{0}^{2 / 3} \frac{d x}{4+9 \mathrm{x}^{2}}$ (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{24}$ (c) $\pi / 8 \quad$ (d) $\pi / 2$ | 2 |
| 7 | $\int_{0}^{\pi / 2} \operatorname{cossx}^{\sin x} \mathrm{dx}$ (a) e+1 (b0 e (c) e-1 (d) 1 | 2 |
| 8 | $\int_{-\pi}^{\pi} \frac{\cos ^{2} x}{1+a^{x}} d x \quad$ (a) $a \pi$ (b) $\pi / 2$ (c) $a \pi / 4$ (d) |  |
|  | SECTION CAll the questions carry 3 marks each |  |
| 9 | Evaluate the integral $\int_{0}^{1} \mathrm{xe}^{\mathrm{x}^{2}} \mathrm{dx}$ | 3 |
| 10 | Evaluate $\int_{0}^{4}\|\mathrm{x}-1\| \mathrm{dx}$ | 3 |
| 11 | Evaluate $\int_{0}^{2} x \sqrt{2-x} d x$ | 3 |
|  | SECTION DThe question are of 5 marks |  |
| 12 | Evaluate $\int_{0}^{\pi} \frac{x \tan x}{\sec x+\cos x} d x$ | 5 |
| 13 | $\int_{0}^{\frac{\pi}{2}} \frac{x d x}{\sin x+\cos x}$ | 5 |

## Application of integrals

## SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region $\mathrm{PQRSP}=\int_{a}^{b} d A=\int_{a}^{b} y d x=\int_{a}^{b} f(x) d x$.
** The area $A$ of the region bounded by the curve $x=g(y), y$-axis and
 the lines $\mathrm{y}=\mathrm{c}, \mathrm{y}=\mathrm{d}$ is given by $\mathrm{A}=\int_{c}^{d} x d y=\int_{c}^{d} g(y) d y$


## MCQ(1-Mark)

1.) Area enclosed by the curve $x^{2}+y^{2}=4$ is
(a) 4 sq. units
(b) $2 \pi$ sq. units
(c) $4 \pi$ sq.units
(d) $16 \pi$ sq. units

Solution:- (c)
2.) The area of the region bounded by $y^{2}=4 x, y$-axis and the line $y=3$
(a) $\sqrt{3}$ sq.units
(b) $\frac{4}{3}$ sq.units
(c) $\frac{9}{4}$ sq.units
(d) 2 sq.unit

Solution:- (c)

## ASSERTION - REASON TYPE QUESTIONS :

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct
3.) Assertion : The area bounded by the curve $y=\cos x$ in I quadrant with the coordinate axes is 1 sq. unit.
Reason : $\int_{0}^{\frac{\pi}{2}} \cos x d x=1$
Ans (a) $\int_{0}^{\frac{\pi}{2}} \cos x d x=[\sin x]_{0}^{\pi / 2}=\sin \frac{\pi}{2}-0=1$
4.) Assertion : The area bounded by the curves $y^{2}=4 a^{2}(x-1)$ and lines $x=1$ and $y=$ 4 a is 16 a 3 sq. units.

Reason: The area enclosed between the parabola $y^{2}=x^{2}-x+2$ and the line $y=x+2$ is 83 sq. units
Ans. (c)
Very short answer type questions(2-Marks)
5.) Find the area enclosed by $y=x^{3}$ and the lines $x=0, y=1, y=8$.

Solution:- Given curve, $\mathrm{y}=\mathrm{x}^{3}$ or $\mathrm{x}=\mathrm{y}^{1 / 3}$.
Hence, the required area, $A={ }_{1} \int^{8} \mathrm{y}^{1 / 3}$ dy
$\mathrm{A}=\left[\left(\mathrm{y}^{4 / 3}\right) /(4 / 3)\right]_{1}{ }^{8}$
Now, apply the limits, we get
$\mathrm{A}=(3 / 4)(16-1)$
$\mathrm{A}=(3 / 4)(15)=45 / 4$.

## Short answer type questions(3-Marks)

6.) Find the area bounded by the line $y=x, x-a x i s$ and lines $x=-1$ to $x=2$.

Solution:- We have, $y=x$, a line
Required Area $=$ Area of shaded region
$=\left|\int_{-1}^{0} x d x\right|+\left|\int_{0}^{2} x d x\right|=\left|\frac{x^{2}}{2}\right|_{-1}^{0}+\left|\frac{x^{2}}{2}\right|_{0}^{2}$
$=\left|-\frac{1}{2}\right|+\left|\frac{2}{1}\right|=2+\frac{1}{2}=\frac{5}{2}$ sq. units


Long Answer Type questions (5-Marks)
7.) Find the area of the region enclosed by the parabola $x^{2}=y$, the line $y=x+2$ and the x -axis,
Solution:- From the given equation
$x^{2}=y$ and $y=x+2$
$\Rightarrow \mathrm{x}^{2}=\mathrm{x}+2$
$\Rightarrow \mathrm{x}^{2}-\mathrm{x}-2=0$
$\Rightarrow(\mathrm{x}-2)(\mathrm{x}+1)=0$
$\Rightarrow \mathrm{x}=2, \mathrm{x}=-1$


So the Required area $=\int_{-1}^{2}(x+2) d x-\int_{-1}^{2} x^{2} d x$
$=\left[\frac{(x+2)^{2}}{2}\right]_{-1}^{2}-\left[\frac{x^{3}}{2}\right]_{-1}^{2}$
$=\frac{1}{2}[16-1]\left[\frac{8}{3}+\frac{1}{3}\right]=\frac{15}{3}-3=\frac{9}{2}$ sq. units

## Case -Based Questions (4 marks)

## 8.)



The cable of bridge connects two pillars 100 feet apart. The cable on the bridge is in a parabolic form. The height of pillars on the bridge is 10 feet above the road as seen in fgure. Based on the information given above, answer the following questions
(a) Find the equation of parabola as shown in figure.
(b) Find the area formed by the parabola, $x$-axis , $y=0$ and $y=10$ is

Solution:- a) required equation of parabola is $x^{2}=250 y$
b) required area $=\int_{-50}^{50} \frac{x^{2}}{250} d x$

## Differential equations

## SOME IMPORTANT RESULTS/CONCEPTS

* Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.
**Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives are of the given differential equation is called the degree of the differential equatio $=\frac{f_{1}(x, y)}{f_{2}(x, y)}$, where $f_{1}(x, y) \& f_{2}(x, y)$ be the homogeneous function of same degree.
** Linear Differential Equation :
i. $\frac{\mathrm{dy}}{\mathrm{dx}}+p y=q$, wherep \&qbethefunctionofxorconstant.

Solutionofthe equationis: $y . e^{\int p d x}$

$$
=\int e^{\int p d x} . q d x \text {, where } e^{\int p d x} \text { is Integrating Factor (I.F.) }
$$

ii. $\frac{\mathrm{dx}}{\mathrm{dy}}+p x=q$, wherep\&qbethefunctionofyorconstant.

Solutionofthe equationis: $x . e^{\int p d y}$

$$
=\int e^{\int p d y} . q d y \text {, where } e^{\int p d y} \text { is Integrating Factor (I.F.) }
$$

## $\underline{\operatorname{MCQ}(1-M a r k)}$

1.) The order and degree of the differential equation $\frac{d y}{d x}+\sin \left(\frac{d y}{d x}\right)=0$ is
(a) Order 1, degree not defined (b) order 1 , degree 1 (c) order 2 degree 1 (d) order 2 degree 2
Solution:- (a)
2.) General solution of the differential equation $\log \left(\frac{d y}{d x}\right)=2 x+y$ is
(a) $e^{-y}=\frac{1}{2} e^{2 x}+c$
(b) $e^{2 y}=\frac{1}{2} e^{2 x}+c$
(c) $-e^{-y}=\frac{1}{2} e^{2 x}+c$
(d) $e^{y}=\frac{1}{2} e^{2 x}+c$

## Solution:- (c)

## ASSERTION - REASON TYPE QUESTIONS :

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct
3.) Assertion (A): The Integrating Factor of the differential equation $\frac{d y}{d x}-y=x^{2}$ is $e^{-x}$
Reasoning ( $\mathbf{R}$ ) : A function of the form $f(x, y)=\lambda^{n} g\left(\frac{y}{x}\right)$ is called homogeneous function.
Solution:- ©
4.) Assertion (A ) : The differential equation $\frac{d y}{d x}=\frac{x^{2}+x y}{y^{2}}$ is a homogeneous differential equation.
Reasoning ( $\mathbf{R}$ ) : A function $F(x, y)$ is said to be homogeneous function if $F(\lambda x, \lambda y)$ $=F(x, y)$.
Solution:- (a)

## Very short answer type questions(2-Marks)

5.) Find the general solution of the differential equation $\frac{d y}{d x}=\frac{x+1}{2-y},(y \neq 2)$

Solution:- $\frac{d y}{d x}=\frac{x+1}{2-y}$,

$$
\text { Or, }(2-y) d y=(x+1) d x
$$

Integrating both sides ,

$$
\begin{gathered}
\int(2-y) d y=\int(x+1) d x \\
2 y-\frac{y^{2}}{2}=\frac{x^{2}}{2}+x+c
\end{gathered}
$$

6.) Find the integrating factor for the differential equation $\cos x \frac{d y}{d x}+y \sin x=1$.

Solution :- $\cos x \frac{d y}{d x}+y \sin x=1$.
Dividing both sides by $\cos \mathrm{x}$

$$
\begin{aligned}
& \frac{d y}{d x}+y \tan x=\sec x \\
& \text { Here } \mathrm{P}=\tan \mathrm{x}, \mathrm{Q}=\sec \mathrm{x} \\
& \mathrm{I} . \mathrm{F}=e^{\int \tan x d x}=e^{\log \sec x}=\sec x
\end{aligned}
$$

Short answer type questions(3-Marks)
7.) Solve $(1+x+y)=\frac{d x}{d y}$

Solution :- $x+y+1=\frac{d x}{d y}$
$\frac{d x}{d y}-x=y+1$
This is like Type 2 linear differential equation as $\frac{d x}{d y}+P x=Q$ where $P$ and $Q$ are functions of y.

Here $\mathrm{P}=-1$ and $\mathrm{Q}=1+\mathrm{y}$
Integrating factor is $\mathrm{e}^{\int-1 \mathrm{dy}}=\mathrm{e}^{-\mathrm{y}}$
Solution is
$x . e^{-y}=\int(1+y) e^{-y} d y+c$

$$
=\int e^{-y} d y+\int y \cdot e^{-y} d y+c=-e^{-y}-y \cdot e^{-y}-e^{-y}+c
$$

$\mathrm{x}=-\mathrm{y}-2+\mathrm{c} \mathrm{e}^{\mathrm{y}}$
8.) Solve $e^{x} \tan y d x+\left(1-e^{x}\right) \sec ^{2} y d y=0$

Solution : The equation is $e^{x} \operatorname{tany} d x+\left(1-e^{x}\right) \sec ^{2} y d y=0$

$$
\begin{aligned}
\frac{d y}{d x} & =-\frac{e^{x} \tan y}{\left(1-e^{x}\right) \sec ^{2} y} \\
\frac{d y}{d x} & =-\left(\frac{e^{x}}{1-e^{x}}\right)\left(\frac{\tan y}{\sec ^{2} y}\right)
\end{aligned}
$$

By using variable separating method

$$
\frac{\sec ^{2} y}{\tan y} d y=\frac{e^{x}}{e^{x}-1} d x
$$

Integrating both sides $\int \frac{\sec ^{2} y}{\operatorname{tany}} d y=\int \frac{e^{x}}{e^{x}-1} d x$

$$
\log (\tan y)=\log \left(e^{x}-1\right)+\log c
$$

$$
\log (\tan y)=\log \left(e^{x}-1\right) c \quad \text { by using }(\log a+\log b=\log a b)
$$

$\tan \mathrm{y}=\left(\mathrm{e}^{\mathrm{x}}-1\right) \mathrm{c}$.

## Long Answer Type questions (5-Marks)

9.) Solve the following Homogeneous differential equation ;

$$
2 x y d y=\left(x^{2}+y^{2}\right) d x
$$

Solution : write the given equation as $\frac{d y}{d x}=\frac{x^{2}+y^{2}}{2 x y}$
Since it is Homogeneous differential equation
Put $y=v x \Rightarrow \frac{d y}{d x}=v+x \frac{d v}{d x}$ in the given equation
So,$v+x \frac{d v}{d x}=\frac{1+v^{2}}{2 v}$
$\Rightarrow \mathrm{x} \frac{\mathrm{dv}}{\mathrm{dx}}=\frac{1+\mathrm{v}^{2}}{2 \mathrm{v}}-\mathrm{v} \Rightarrow \mathrm{x} \frac{\mathrm{dv}}{\mathrm{dx}}=\frac{-\left(\mathrm{v}^{2}-1\right)}{2 \mathrm{v}}$
Separating the variables and writing the equation as
$\frac{2 v}{v^{2}-1} d v=-\frac{d x}{x}$
Integrating both sides and getting
$\int \frac{2 v}{v^{2}-1} d v=-\int \frac{d x}{x} \Rightarrow \log \left|\left(v^{2}-1\right)\right|=-\log |x|+\log |c|$
$\Rightarrow \quad \log \left|x\left(v^{2}-1\right)\right|=\log c \Rightarrow x\left(v^{2}-1\right)= \pm c=c_{1}$
Now replace $v$ by $\frac{y}{x}$ to get $x^{2}-y^{2}=c x$

## Case -Based Questions (4 marks)

10.) Polio drops are delivered to 50 K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3 rd week can be estimated using the solution to the differential equation $\frac{d y}{d x}=k(50-y)$ where x denotes the number of weeks and y the number of children who have been given the drops.
1.State the order of the above given differential equation.
2. Which method of solving a differential equation can be used to solve $\frac{d y}{d x}=k(50-y)$
3. The solution of the differential equation $\frac{d y}{d x}=k(50-y)$ is ?
4. The value of constant c in the particular solution given that $\mathrm{y}(0)=0$ and $\mathrm{k}=0.049$ is.

Solution:- 1. First order
2.variable seperable
3. Applying variable seperable,$\frac{d y}{(50-y)}=k d x$

Integrating both sides , $-\log (50-y)=k x+c$
4. $\mathrm{c}=-\log 50$

# KENDRIYA VIDYALAYA SANGATHAN PATNA REGION <br> CLASS: XII <br> CHAPTER : VECTOR ALGEBRA <br> <br> SOME IMPORTANT RESULTS/CONCEPTS 

 <br> <br> SOME IMPORTANT RESULTS/CONCEPTS}

* Position vector of point $A(x, y, z)=\overrightarrow{O A}=x \hat{i}+y \hat{j}+z \hat{k}$
* If $A\left(x_{1}, y_{1}, z_{1}\right)$ and point $B\left(x_{2}, y_{2}, z_{2}\right)$ then $\overrightarrow{A B}=\left(x_{2}-x_{1}\right) \hat{i}+\left(y_{2}-y_{1}\right) \hat{j}+\left(z_{2}-z_{1}\right) \hat{k}$
* If $\vec{a}=x \hat{i}+y \hat{j}+z \hat{k} \quad ;|\vec{a}|=\sqrt{x^{2}+y^{2}+z^{2}}$
*Unit vector parallel to $\vec{a}=\frac{\vec{a}}{|\vec{a}|}$
* Scalar Product (dot product) between two vectors : $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}=|\overrightarrow{\mathrm{a}}| \overrightarrow{\mathrm{b}} \mid \cos \theta ; \theta$ is angle between the vectors
$* \cos \theta=\frac{\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}}{|\overrightarrow{\mathrm{a}} \| \overrightarrow{\mathrm{b}}|}$
* If $\vec{a}=a_{1} \hat{i}+b_{1} \hat{j}+c_{1} \hat{k}$ and $\vec{b}=a_{2} \hat{i}+b_{2} \hat{j}+c_{2} \hat{k} \quad$ then $\vec{a} \cdot \vec{b}=a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}$
* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a} \cdot \vec{b}=0$
$* \overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{a}}=|\overrightarrow{\mathrm{a}}|^{2}$
* Projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
* Vector product between two vectors:
$\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}=|\overrightarrow{\mathrm{a}} \| \overrightarrow{\mathrm{b}}| \sin \theta \hat{\mathrm{n}} ; \hat{\mathrm{n}}$ is the normal unit vector
which is perpendicular to both $\vec{a} \& \vec{b}$
* $\hat{n}=\frac{\vec{a} \times \vec{b}}{|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|}$
* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a} \times \vec{b}=0$
* Area of triangle (whose sides are given by $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ ) $=\frac{1}{2}|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$
* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$ ) $=|\vec{a} \times \vec{b}|$
* Area of parallelogram (whose diagonals are given by $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ ) $=\frac{1}{2}|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$


## SOLVED EXAMPLES

1.The position vector of a point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2 \vec{a}-\vec{b}$ . in the ratio $1: 2$ internally is (a) $\frac{3 \bar{a}+2 \bar{b}}{3}$ (b) $\bar{a}$ (c) $\frac{5 \bar{a}-\bar{b}}{3}$ (d) $\frac{4 \bar{a}+\bar{b}}{3}$
ANS: (d), as position vector $=\frac{2(\bar{a}+\bar{b})+1(\overline{2 a}-\bar{b})}{1+2}=\frac{4 \bar{a}+\bar{b}}{3}$
2. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c}=\vec{b}$ and $\vec{a} \cdot \vec{c}=3$
answer. Let $\vec{c}=x \hat{i}+y \hat{j}+z \hat{k}$
$\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{c}}=\overrightarrow{\mathrm{b}} \Rightarrow\left|\begin{array}{ccc}\hat{\mathrm{i}} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \mathrm{x} & y & z\end{array}\right|=\hat{\mathrm{j}}-\hat{\mathrm{k}} \Rightarrow \hat{\mathrm{i}}(z-y)+\hat{j}(x-z)+\hat{\mathrm{k}}(y-x)=\hat{\mathrm{j}}-\hat{\mathrm{k}}$
$\Rightarrow \mathrm{z}-\mathrm{y}=0, \mathrm{x}-\mathrm{z}=1, \mathrm{y}-\mathrm{x}=-1$
Also $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{c}}=3 \Rightarrow \mathrm{x}+\mathrm{y}+\mathrm{z}=3$
$\Rightarrow \mathrm{x}+\mathrm{z}+\mathrm{z}=3 \Rightarrow \mathrm{x}+2 \mathrm{z}=3$
and $\mathrm{x}-\mathrm{z}=1 \Rightarrow \mathrm{z}=\frac{2}{3}, \mathrm{x}=\frac{5}{3}, \mathrm{y}=\frac{2}{3}$
$\therefore \vec{c}=\frac{5}{3} \hat{i}+\frac{2}{3} \hat{j}+\frac{2}{3} \hat{k}$ or $\frac{1}{3}(5 \hat{i}+2 \hat{j}+2 \hat{k})$
3. . If $\overrightarrow{\mathrm{a}}+\overrightarrow{\mathrm{b}}+\overrightarrow{\mathrm{c}}=0$ and $|\overrightarrow{\mathrm{a}}|=3,|\overrightarrow{\mathrm{~b}}|=5$ and $|\overrightarrow{\mathrm{c}}|=7$, show that angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is $60^{\circ}$.
answer. $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a}+\vec{b}=-\vec{c} \Rightarrow(\vec{a}+\vec{b}) \cdot(\vec{a}+\vec{b}) \cdot=(-\vec{c}) \cdot(-\vec{c})$
$\Rightarrow|\overrightarrow{\mathrm{a}}|^{2}+|\overrightarrow{\mathrm{b}}|^{2}+2 \overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}=|\overrightarrow{\mathrm{c}}|^{2} \Rightarrow 9+25+2 \overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}=49$
$\Rightarrow 2 \overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}=15 \Rightarrow 2|\overrightarrow{\mathrm{a}}||\overrightarrow{\mathrm{b}}| \cos \theta=15 \Rightarrow 30 \cos \theta=15$
$\Rightarrow \cos \theta=1 / 2 \Rightarrow \theta=60^{\circ}$

III .Questions for Practice: Number of questions should be as mentioned in the table:

1. If $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$.are unit vectors, then what is the angle between $\overrightarrow{\mathrm{a}}$ and $\vec{b}$ for $\sqrt{3} \overrightarrow{\mathrm{a}}-\overrightarrow{\mathrm{b}}$ to be a unit vector?
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) none of these
2. The value of $\lambda$ for which vectors $2 \hat{i}+\hat{j}+3 \hat{k}$ and $\hat{i}-\lambda \hat{j}+4 \hat{k}$ are orthogonal is
(A) 12
(B) 14
(C) 16
(D) none of these
3.If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|=$, then angle between $\bar{a}$ and $\bar{b}$ is
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$
3. If $|\overrightarrow{\mathrm{a}}|=5,|\bar{b}|=13$ and $\left|\bar{a}_{X} \bar{b}\right|=25$, then $\bar{a} \cdot \bar{b}$ is equal to
(a) 12
(b) 5
(c) 13
(d) 60
4. Assertion- Reason question

Assertion: (a)Two vectors are said to be like vectors if they have the same direction but different magnitude.
Reason: (b)Vector quantities do not have a specific direction.
(A) Both (a) and (b) are correct and (b)is the correct explanation of (a)
(B) Both (a) and (b) are correct and (b)is not the correct explanation of (a)
(C) (a)is correct but (b)is false
(D) (a)is false but (b)is correct
6. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$
7. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+2 \hat{j}+2 \hat{k}$.
8. For given vectors, $\vec{a}=3 \hat{i}-\hat{j}+2 \hat{k}$ and $\vec{b}=-2 \hat{i}+3 \hat{j}-\hat{k}$, find the unit vector in the direction of the vector $\vec{a}+\vec{b}$.
9. Find a vector in the direction of vector $3 \hat{i}-4 \hat{j}+5 \hat{k}$ which has magnitude 7 units.
10. Find the value of $x$ for which $x(\hat{i}+2 \hat{j}+3 \hat{k})$ is a unit vector.
11. Find the value of $\lambda$ for which the vectors $2 \hat{i}-3 \hat{j}+4 \hat{k}$ and $-4 \hat{i}+6 \hat{j}-\lambda \hat{k}$ are collinear.
12. Find the projection of the vector $\hat{i}+3 \hat{j}+7 \hat{k}$ on the vector $2 \hat{i}-3 \hat{j}+6 \hat{k}$
13. Write the projection of vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.
14. Write the value of $p$ for which $\vec{a}=3 \hat{i}+2 \hat{j}+9 \hat{k}$ and $\vec{b}=\hat{i}+p \hat{j}+3 \hat{k}$ are parallel vectors.
15.Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} \cdot \vec{b}=8$ and $\vec{b}=2 \hat{i}+6 \hat{j}+3 \hat{k}$.

ANSWERS: 1. (a) $30^{\circ} \quad$ 2. (B) 14 3.(d) $90^{\circ}$ 4. (d) 60 5.: (C) (a)is correct but (b)is false
6. $\cos ^{-1}\left(-\frac{1}{3}\right)$;7. ${ }^{\frac{1}{3}(\hat{\mathrm{i}}+2 \hat{\mathrm{j}}+2 \hat{\mathrm{k}})}$; 8. $\frac{1}{\sqrt{6}}(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}})$; 9. $\frac{7}{5 \sqrt{2}}(3 \hat{\mathrm{i}}-4 \hat{\mathrm{j}}+5 \hat{\mathrm{k}})$
10. $\pm \frac{1}{\sqrt{14}} \quad$; $11 . \lambda=8$;
12.5; 13.1;14. $\mathrm{p}=\frac{2}{3} \quad 15 . \frac{8}{7}$

CLASS TEST 20 MARKS (two marks each)
1 Write two different vectors having same magnitude.
2. Write two different vectors having same direction.
3. Write down a unit vector in XY-plane, making an angle of $30^{\circ}$ with the positive direction of x -axis.
4. Find the scalar and vector components of the vector with initial point $(2,1,3)$ and terminal point $(-5,7,7)$.
5. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+2 \hat{j}+2 \hat{k}$.

6 . Find the unit vector in the direction of vector $\overrightarrow{\mathrm{PQ}}$, where P and Q are the points $(2,3,4)$ and $(5,6,7)$, respectively.
7. For given vectors, $\vec{a}=3 \hat{i}-\hat{j}+2 \hat{k}$ and $\vec{b}=-2 \hat{i}+3 \hat{j}-\hat{k}$, find the unit vector in the direction of the vector $\vec{a}+\vec{b}$.
8. Find a vector of magnitude 4 units, and parallel to the resultant of the vectors $\vec{a}=3 \hat{i}+2 \hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2 \hat{j}+3 \hat{k}$.
9. Find a vector in the direction of vector $3 \hat{i}-4 \hat{j}+5 \hat{k}$ which has magnitude 7 units.
10. Find the value of $x$ for which $x(\hat{i}+2 \hat{j}+3 \hat{k})$ is a unit vector

ANSWERS

1. $2 \hat{i}+3 \hat{j}$ and $3 \hat{i}+2 \hat{j}$
2. $\hat{i}+\hat{j}+\hat{k}$ and $2 \hat{i}+2 \hat{j}+2 \hat{k}$
3. $\frac{\sqrt{3}}{2} \hat{i}+\frac{1}{2} \hat{j}$
4. Scalar components : $-7,6$ and 4 , Vector components : $-7 \hat{i}, 6 \hat{j}$, and $4 \hat{k}$.
5. $\frac{1}{3}(\hat{i}+2 \hat{j}+2 \hat{k})$
6. $\frac{1}{\sqrt{3}}(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}})$
7. $\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+\hat{k})$
8. $\pm \frac{2}{3}(4 \hat{\mathrm{i}}+4 \hat{\mathrm{j}}+2 \hat{\mathrm{k}})$
9. $\frac{7}{5 \sqrt{2}}(3 \hat{i}-4 \hat{\mathrm{j}}+5 \hat{\mathrm{k}})$
10. $\pm \frac{1}{\sqrt{14}}$

## CASE STUDY :

Teams A, B, C went for playing a tug of war game.
Teams $\mathrm{A}, \mathrm{B}, \mathrm{C}$ have attached a rope to a metal ring and are trying to
pull the ring into their own area (team areas shown below).
Team A pulls with force $\mathrm{F} 1=4 \hat{\imath}+0 \jmath \mathrm{KN}$
Team $\mathrm{B} \rightarrow \mathrm{F} 2=-2 \hat{\imath}+4 \jmath \mathrm{KN}$
Team $\mathrm{C} \rightarrow \mathrm{F} 3=-3 \hat{i}-3 j \mathrm{KN}$


Based on the above information, answer the following questions.
(I) What is the magnitude of the force of Team B?
(a) $2 \sqrt{ } 5 \mathrm{KN}$
(b) 6 KN
(c) 2 KN
(d) $6 \sqrt{ } 6 \mathrm{KN}$
(II) How many KN force is applied by Team A?
(a) 5 KN
(b) 4 KN
(c) 2 KN
(d) 16 KN
(III) what is the total force applied on the ring?
(a) $\hat{\imath}+j \mathrm{KN}$
(b) $-\hat{i}+j \mathrm{KN}$
(c) $-\hat{i}-j \mathrm{KN}$
(d) $\hat{i}-j \mathrm{KN}$
(IV) Which team will win the game?
(a) Team B
(b) Team A
(c) Team C
(d) No one
(V) What is the magnitude of the teams combined force ?
(a) 7 KN
(b) 1.4 KN
(c) 1.5 KN
(d) 2 KN
ANSWER KEY
I. a
I. B
III .b
IV.a
V.b

## CLASS TEST 30 MARKS (two marks each)

1. Write the value of $p$ for which $\vec{a}=3 \hat{i}+2 \hat{j}+9 \hat{k}$ and $\vec{b}=\hat{i}+p \hat{j}+3 \hat{k}$ are parallel vectors.
2. If $\theta$ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then write the values of $\theta$ for which $\vec{a} \cdot \vec{b} \geq 0$.
3. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} \cdot \vec{b}=8$ and $\vec{b}=2 \hat{i}+6 \hat{j}+3 \hat{k}$.
4. If $|\vec{a}|=\sqrt{3},|\vec{b}|=2$ and $\vec{a} \cdot \vec{b}=\sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.
5. If $|\overrightarrow{\mathrm{a}}|=\sqrt{3},|\overrightarrow{\mathrm{~b}}|=2$ and the angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is $60^{\circ}$, find $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}$.
6. For what value of $\lambda$ are the vectors $\vec{a}=2 \hat{i}+\lambda \hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2 \hat{j}+3 \hat{k}$ perpendicular to each other?
7.If $\vec{a} \cdot \vec{a}=0$ and $\vec{a} \cdot \vec{b}=0$, then what can be concluded about the vector $\vec{b}$ ?
7. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}) \cdot(\vec{x}+\vec{a})=80$, then find $|\vec{x}|$.
8. If $\vec{a}$ is a unit vector and $(2 \vec{x}-3 \vec{a}) \cdot(2 \vec{x}+3 \vec{a})=91$, then write the value of $|\vec{x}|$.
9. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$, find the value of $|\vec{b}|$.
10. Write the value of $(\hat{i} \times \hat{j}) \cdot \hat{k}+\hat{i} \cdot \hat{j}$.
11. Write the value of $(\hat{k} \times \hat{j}) \cdot \hat{i}+\hat{j} \cdot \hat{k}$.
12. Write the value of $(\hat{i} \times \hat{j}) . \hat{k}+(\hat{j} \times \hat{k}) . \hat{i}$.
14.If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}|=|\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$ ?
13. Write a unit vector perpendicular to both the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$

## ANSWERS

1. $\mathrm{p}=\frac{2}{3}$
2. $\frac{8}{7}$
3. $\sqrt{3}$
4. $\vec{b}$ may be any vector.
5. 5
11.1
13.2
6. $-\frac{\hat{\mathrm{i}}}{\sqrt{2}}+\frac{\hat{\mathrm{j}}}{\sqrt{2}}$
$2.0 \leq \theta \leq \frac{\pi}{2}$
7. $\frac{\pi}{3}$
8. $\lambda=\frac{5}{2}$

Any three numbers proportional to direction cosines are direction ratios denoted by a, b, c

$$
\frac{1}{a}=\frac{m}{b}=\frac{n}{c} \quad 1= \pm \frac{a}{\sqrt{a^{2}+b^{2}+c^{2}}}, \quad m= \pm \frac{b}{\sqrt{a^{2}+b^{2}+c^{2}}}, \quad n= \pm \frac{c}{\sqrt{a^{2}+b^{2}+c^{2}}},
$$

* Direction ratios of a line segment joining $\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ and $\mathrm{Q}\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$ may be taken as $\mathrm{x}_{2}-\mathrm{x}_{1}, \mathrm{y}_{2}-\mathrm{y}_{1}, \mathrm{z}_{2}-\mathrm{z}_{1}$
* Angle between two lines whose direction cosines are $\mathrm{l}_{1}, \mathrm{~m}_{1}, \mathrm{n}_{1}$ and $\mathrm{l}_{2}, \mathrm{~m}_{2}, \mathrm{n}_{2}$ is given by

$$
\cos \theta=l_{1} 1_{2}+m_{1} m_{2}+n_{1} n_{2}=\frac{a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}}{\sqrt{\left(a_{1}{ }^{2}+b_{1}{ }^{2}+c_{1}{ }^{2}\right)\left(\mathrm{a}_{2}{ }^{2}+\mathrm{b}_{2}{ }^{2}+\mathrm{c}_{2}{ }^{2}\right)}}
$$

* For parallellines $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \quad$ and
for perpendicularlines $\mathrm{a}_{1} \mathrm{a}_{2}+\mathrm{b}_{1} \mathrm{~b}_{2}+\mathrm{c}_{1} \mathrm{c}_{2}=0$ or $\mathrm{l}_{1} \mathrm{l}_{2}+\mathrm{m}_{1} \mathrm{~m}_{2}+\mathrm{n}_{1} \mathrm{n}_{2}=0$


## STRAIGHT LINE:-

* Line passing through point A whose position vector is $\vec{a}$ and parallel to $\vec{m}$ in vector

$$
\text { form is } \quad \vec{r}=\vec{a}+\lambda \vec{m} .
$$

Cartesian form: A $\left(x_{1}, y_{1}, z_{1}\right) \mathrm{a}, \mathrm{b}, \mathrm{c}$ be Direction Ratios of vector parallel to line

$$
\frac{x-x_{1}}{a}=\frac{y-y_{1}}{b}=\frac{z-z_{1}}{c}
$$

* A general point on the line is $\mathrm{P}\left(a \lambda+x_{1}, \mathrm{~b} \lambda+y_{1}, \mathrm{c} \lambda+z_{1}\right)$
* Line passing through two points A whose position vector is $\vec{a}$ \& B whose position vector is $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$
Cartesian form: For points $\mathrm{A}\left(x_{1}, y_{1}, z_{1}\right)$ and $\mathrm{B}\left(x_{2}, y_{2}, z_{2}\right)$ is

$$
\frac{x-x 1}{x 2-x 1}=\frac{y-y 1}{y 2-y 1}=\frac{z-z 1}{z 2-z 1}
$$

* For lines having angle $\theta$ between them

$$
\begin{gathered}
\overrightarrow{r_{1}}=\overrightarrow{a_{1}}+\lambda m_{1}, \overrightarrow{r_{2}}=\overrightarrow{a_{2}}+\mu m_{2} \text { or } \frac{x-x 1}{a}=\frac{y-y 1}{b}=\frac{z-z 1}{c} \& \frac{x-x 2}{a}=\frac{y-y 2}{b}=\frac{z-z 2}{c} \\
\cos \theta=\frac{a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}}{\sqrt{a_{1}{ }^{2}+b_{1}{ }^{2}+c_{1}^{2}} \sqrt{a_{2^{2}+b_{2}{ }^{2}+c_{2}^{2}}{ }^{2}}}=\frac{\overrightarrow{m_{1} \cdot \overrightarrow{m_{2}}}}{\left|\overrightarrow{m_{1}} \| \overrightarrow{m_{2}}\right|}
\end{gathered}
$$

For perpendicular lines $\overrightarrow{m_{1}} \cdot \overrightarrow{m_{2}}=a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}=0$
For parallel lines $\overrightarrow{m_{1}}=\mu m_{2}$ i.e. $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$

* Skew lines in space are lines which are neither parallel nor intersecting.
* Shortest distance between two skew lines.

For two lines $\overrightarrow{r_{1}}=\overrightarrow{a_{1}}+\lambda m_{1}, \quad \overrightarrow{r_{2}}=\overrightarrow{a_{2}}+\mu m_{2}$
S. $\mathrm{D}=\frac{\left(\overrightarrow{a_{2}}-\overrightarrow{a_{1}}\right) \cdot\left(\overrightarrow{m_{1}} \times \overrightarrow{m_{2}}\right)}{\left|\overrightarrow{m_{1}} \times m_{2}\right|}$

Condition for two lines to intersect $\left[\left(\overrightarrow{a_{2}}-\overrightarrow{a_{1}}\right) \cdot\left(\overrightarrow{m_{1}} \times \overrightarrow{m_{2}}\right)\right]=0$
For parallel line $\overrightarrow{r_{1}}=\overrightarrow{a_{1}}+\lambda m ; \quad \overrightarrow{r_{2}}=\overrightarrow{a_{2}}+\mu m$ S. $\mathrm{D}=\frac{\left(\overrightarrow{a_{2}}-\overrightarrow{a_{1}}\right) \cdot\left(\overrightarrow{1_{1}} \times \overrightarrow{m_{2}}\right)}{\left|\overrightarrow{m_{1}} \times m m_{2}\right|}$

## OR

## ** STRAIGHTLINE :

* Equation of line passin $g$ through a point $\left(x_{1}, y_{1}, z_{1}\right)$ with direction cosines $a, b, c: \frac{x-x_{1}}{a}=\frac{y-y_{1}}{b}=\frac{z-z_{1}}{c}$
* Equation of line passing through a point $\left(x_{1}, y_{1}, z_{1}\right)$ and parallel to the line: $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ is $\frac{\mathrm{x}-\mathrm{x}_{1}}{\mathrm{a}}=\frac{\mathrm{y}-\mathrm{y}_{1}}{\mathrm{~b}}=\frac{\mathrm{z}-\mathrm{z}_{1}}{\mathrm{c}}$
* Equation of line passing through two point $\left(x_{1}, y_{1}, z_{1}\right)$ and $\left(x_{2}, y_{2}, z_{2}\right)$ is $\frac{x-x_{1}}{x_{2}-x_{1}}=\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{z-z_{1}}{z_{2}-z_{1}}$
* Equation of line (Vector form)

Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda \vec{b}$

* Equation of line passing through two points $\vec{a} \& \vec{b}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$
* Shortest distance between two skew lines: if lines are $\vec{r}=\overrightarrow{a_{1}}+\lambda \overrightarrow{b_{1}} \vec{r}=\overrightarrow{a_{2}}+\lambda \overrightarrow{b_{2}}$

$$
\begin{aligned}
& \text { then Shortest distance }= \frac{\left(\overrightarrow{a_{2}}-\overrightarrow{a_{1}}\right) \cdot\left(\overrightarrow{b_{1}} \times \overrightarrow{b_{2}}\right)}{\left|\overrightarrow{\mathrm{b}_{1}} \times \overrightarrow{\mathrm{b}_{2}}\right|} ; \overrightarrow{\mathrm{b}_{1}} \times \overrightarrow{\mathrm{b}_{2}} \neq 0 \\
& \frac{\left|\left(\overrightarrow{\mathrm{a}_{2}}-\overrightarrow{\mathrm{a}_{1}}\right) \times \overrightarrow{\mathrm{b}_{1}}\right|}{\left|\overrightarrow{\mathrm{b}_{1}}\right|} ; \overrightarrow{\mathrm{b}_{1}} \times \overrightarrow{\mathrm{b}_{2}}=0
\end{aligned}
$$

## SOME ILUSTRATIONS :

Q 1. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is
(a) $\vec{r}=(3 i+7 j+2 k)+\mu(5 i+4 j-6 k)$
(b) $\vec{r}=(5 i+4 j-6 k)+\mu(3 i+7 j+2 k)$
(c) $\vec{r}=(5 i-4 j-6 k)+\mu(3 i-7 j-2 k)$
(d) $\vec{r}=(5 i-4 j+6 k)+\mu(3 i+7 j+2 k)$

Q 2. The angle between a line whose direction ratios are in the ratio $2: 2: 1$ and a line joining $(3,1,4)$ to $(7,2,12)$ is
(a) $\cos ^{-1}(2 / 3)$
(b) $\cos ^{-1}(-2 / 3)$
(c) $\tan ^{-1}(2 / 3)$
(d) None of these

Q3. If a line makes $60^{\circ}$ and $45^{\circ}$ angles with the positive direction of x -axis and z -axis respectively ,then find the angle that it makes with positive direction of $y$-axis .Hence, write the direction cosines of the line.
Sol : Let , $\beta, \gamma$ be the angles which line makes with axes.
Hence the direction cosines of the lines are
$\therefore \cos 60^{\circ}, \cos \beta, \cos 45^{\circ}$ by taking $\alpha=60^{\circ}$ and $\gamma=45^{\circ}$
$\therefore \frac{1}{4}+\cos ^{2} \beta+\frac{1}{2}=1$
$\therefore \cos \beta= \pm \frac{1}{2}$
$\therefore \beta=60^{\circ}$ or $120^{\circ}$
So the direction cosines of the lines are ( $\frac{1,1}{2}, \frac{1}{2}$ 数 $)$ or $\left(\frac{1}{2}-\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
Q4. Find the direction cosines of a line which makes equal angles with the axes. How many such lines are there?
Sol: :Let $\alpha$ be the angle which the line makes with all axes.
$\therefore$ Its direction cosines are $\cos \alpha, \cos \alpha, \cos \alpha$
$\therefore \cos ^{2} \alpha+\cos ^{2} \alpha+\cos ^{2} \alpha=1 \quad \therefore \cos ^{2} \alpha=\frac{1}{3}$
$\therefore \cos \alpha= \pm \frac{1}{\sqrt{3}}$
$\therefore$ The required direction cosines are $\left( \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$ There are four distinct lines.
Q5. Show that the points $(2,3,4),(-!,-2,1)$ and $(5,8,7)$ are collinear.

Sol: Let $\mathrm{P}(2,3,4), \mathrm{Q}(-!,-2,1)$ and $\mathrm{R}(5,8,7)$ be the given points.
He direction ratios of PQ are(-1-2,-2-3,1-4) direction ratios of QR are
$(5+1,8+2,7-1)$ ie $(-3,-5,-3)$ and $(6,10,6)$ so $\frac{-3}{6}=\frac{-5}{10}=\frac{-3}{6}$
the lines are collinear.
Q6. Find the shortest distance between the following lines :

$$
\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text { and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}
$$

Sol. Given lines are $\vec{r}=(3 \hat{i}+5 \hat{j}+7 \hat{k})+\lambda(\hat{i}-2 \hat{j}+\hat{k})$, and

$$
\begin{aligned}
& \vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\mu(7 \hat{i}-6 \hat{j}+\hat{k}) \vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\mu(7 \hat{i}-6 \hat{j}+\hat{k}) \\
& \overrightarrow{a_{1}}=3 \hat{i}+5 \hat{j}+7 \hat{k}, \quad \overrightarrow{b_{1}}=\hat{\mathrm{i}}-2 \hat{j}+\hat{k} ; \\
& \overrightarrow{a_{2}}=-\hat{i}-\hat{j}-\hat{k}, \quad \vec{b}_{2}=7 \hat{i}-6 \hat{j}+\hat{k} \\
& \overrightarrow{a_{2}}-\overrightarrow{a_{1}}=-4 \hat{i}-6 \hat{j}-8 \hat{k}, \quad \overrightarrow{b_{1}} \times \overrightarrow{b_{2}}=\left|\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
1 & -2 & 1 \\
7 & -6 & 1
\end{array}\right|=4 \hat{i}+6 \hat{j}+8 \hat{k} \\
& \text { S.D. }=\frac{\left|\left(\overrightarrow{a_{2}}-\overrightarrow{a_{1}}\right) \cdot\left(\overrightarrow{b_{1}} \times \vec{b}_{2}\right)\right|}{\left|\overrightarrow{\mathrm{b}}_{1} \times \overrightarrow{\mathrm{b}}_{2}\right|}=\frac{|(4 \hat{\mathrm{i}}-6 \hat{\mathrm{j}}-8 \hat{\mathrm{k}}) \cdot(4 \hat{\mathrm{i}}+6 \hat{\mathrm{j}}+8 \hat{\mathrm{k}})|}{|4 \hat{\mathrm{i}}+6 \hat{j}+8 \hat{k}|} \\
& =\frac{|-16-36-64|}{\sqrt{16+36+64}}=\frac{116}{\sqrt{116}}=\sqrt{116}=2 \sqrt{29}
\end{aligned}
$$

Q7. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Find their point of intersection.
Sol. Any point on $\frac{\mathrm{x}+1}{3}=\frac{\mathrm{y}+3}{5}=\frac{\mathrm{z}+5}{7}=\lambda$ is $(3 \lambda-1,5 \lambda-3,7 \lambda-5)$
Any point on $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$ is $(\mu+2,3 \mu+4,5 \mu+6)$
If the lines intersect than for some $\lambda \& \mu$

$$
\begin{array}{lr}
3 \lambda-1=\mu+2 & \Rightarrow 3 \lambda-\mu=3 \ldots \ldots . \text { (i) } \\
5 \lambda-3=3 \mu+4 & \Rightarrow 5 \lambda-3 \mu=7 \ldots \ldots .(\text { ii }) \\
7 \lambda-5=5 \mu+6 & \Rightarrow 7 \lambda-5 \mu=11 . \tag{iii}
\end{array}
$$

From (i) \& (ii) $\lambda=\frac{1}{2}, \mu=-\frac{3}{2}$ which satisfies (iii)
$\Rightarrow$ given lines intersect and point of intersection is $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$

## Case study

The equation of motion of a rocket are : $x=2 t, y=-4 t, z=4 t$, where the time ' $t$ ' is given in seconds, and the distance measured is in kilometres


Based on the above information, answer the following questions
1.What is the path of the rocket?
a. Straight line
b. Circle
c. Parabola
d. none of these
2. Which of the following points lie on the path of the rocket?
a. $(0,1,2)$
b. $(1,-2,2)$
c. $(2,-2,2)$
d. none of these
3. At what distance will the rocket be from the starting point $(0,0,0)$ in 5 seconds?
(a) $\sqrt{550} \mathrm{~km}$
(b) $\sqrt{450} \mathrm{~km}$
(c) $\sqrt{650} \mathrm{~km}$
(d) $\sqrt{750} \mathrm{~km}$
4. If the position of rocket at certain instant of time is $(3,-8,10)$, then what will be the height of the rocket from the ground, which is along the xy plane?
(a) 10 km
(b)
20 km
(c) 12 km
(d) 11 km
ANSWERS 1. (a) Straight line
2. (c) $(2,-2,2)$
3. (c) $\sqrt{650} \mathrm{~km}$
4. (a) 10 km

## QUESTIONS FOR PRACTICE

## SHORT ANSWER TYPE QUESTIONS

1. Find the direction cosines of the line passing through the two points $(1,-2,4)$ and $(-1,1,-2)$.
2. Find the direction cosines of $x, y$ and $z$-axis.
3. If a line makes angles $90^{\circ}, 135^{\circ}, 45^{\circ}$ with the $\mathrm{x}, \mathrm{y}$ and z axes respectively, find its direction cosines.
4. Find the acute angle which the line with direction-cosines $\left\langle\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, n\right\rangle$ makes with positive direction of $z$ axis.
5. Find the length of the perpendicular drawn from the point $(4,-7,3)$ on the $y$-axis.
6. Find the coordinates of the foot of the perpendicular drawn from the point $(2,-3,4)$ on the $y$-axis.
7. Find the coordinates of the foot of the perpendicular drawn from the point $(-2,8,7)$ on the XZ-plane.
8. Find the image of the point $(2,-1,4)$ in the YZ-plane.
9. Find the vector and Cartesian equations for the line passing through the points $(1,2,-1)$ and $(2,1,1)$.
10. Find the vector equation of a line passing through the point $(-2,3,2)$ and parallel to the line $\vec{r}=(-2 \hat{i}+3 \hat{j})+\lambda(2 \hat{i}-3 \hat{j}+6 \hat{k})$.
11. Find the angle between the lines $\vec{r}=(2 \hat{j}-3 \hat{k})+\lambda(\hat{i}+2 \hat{j}+2 \hat{k})$ and $\vec{r}=(2 \hat{i}+6 \hat{j}+3 \hat{k})+\lambda(2 \hat{i}+3 \hat{j}-6 \hat{k})$.
12. The two lines $x=a y+b, z=c y+d$; and $x=a^{\prime} y+b^{\prime}, z=c^{\prime} y+d^{\prime}$ are perpendicular to each other, find the relation involving $\mathrm{a}, \mathrm{a}^{\prime}, \mathrm{c}$ and $\mathrm{c}^{\prime}$.
13. If the two lines $L_{1}: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}, L_{2}: x=2, \frac{y}{-1}=\frac{z}{2-\alpha}$ are perpendicular, then find value of $\alpha$.
14. Find the vector equation of the line passing through the point $(-1,5,4)$ and perpendicular to the plane $\mathrm{z}=0$.
ANSWERS
15. $\left(-\frac{2}{7}, \frac{3}{7},-\frac{6}{7}\right)$
16. $(1,0,0) ;(0,1,0)$ and $(0,0,1)$
17. $\left(0,-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
18. $\frac{\pi}{4}$
19. 5 units
20. $(0,-3,0)$
21. $(-2,0,7)$
22. $(-2,-1,4)$
23. $\vec{r}=(\hat{i}+2 \hat{j}-\hat{k})+\mu(\hat{i}-\hat{j}+2 \hat{k}) ; \frac{x-1}{1}=\frac{x-2}{-1}=\frac{z+1}{2}$
24. $\vec{r}=(-2 \hat{i}+3 \hat{j}+2 \hat{k})+\mu(2 \hat{i}-3 \hat{j}+6 \hat{k})$
$\begin{array}{llll}11 \cos ^{-1} \frac{4}{21} & \text { 12. } \mathbf{a a}^{\prime}+\mathbf{c c}^{\prime}=-\mathbf{1} & \text { 13. } \frac{7}{3} & \text { 14. } \overrightarrow{\mathrm{r}}=-\hat{\mathrm{i}}+5 \hat{\mathrm{j}}+(4+\lambda) \hat{\mathrm{k}}\end{array}$

## LONG ANSWER TYPE QUESTIONS

1. Find the shortest distance between the lines $\vec{r}=(\hat{i}+2 \hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2 \hat{i}-\hat{j}-\hat{k})+\mu(2 \hat{i}+\hat{j}+2 \hat{k})$
2. Find the shortest distance between the following lines : $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$
3. Find the equation of a line parallel to
$\vec{r}=(\hat{i}+2 \hat{j}+3 \hat{k})+\lambda(2 \hat{i}+3 \hat{j}+4 \hat{k})$ and passing through $2 \hat{i}+4 \hat{j}+5 \hat{k}$. Also find the S.D. between these lines.
4. Find the equation of the line passing through $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2),(1,-1,0)$ and $(1,2,-1),(2,2,1)$.
5. Find the value of $\lambda$ so that the lines $\frac{1-x}{3}=\frac{y-2}{2 \lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3 \lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular to each other.
6. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Find their point of intersection.
7. Find the image of the point $(1,6,3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
8. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance 5 units from the point $P(1,3,3)$.
9. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not. $\frac{x-1}{2}=\frac{y-1}{3}=z, \frac{x+1}{5}=\frac{y-2}{1}, z=2$.

## ANSWERS

1. $\frac{3 \sqrt{2}}{2}$ units $\quad 2.2 \sqrt{29}$ units
2. $\overrightarrow{\boldsymbol{r}}=(\mathbf{2} \hat{\imath}+4 \hat{\jmath}+5 \widehat{\boldsymbol{k}})+\lambda(2 \hat{\imath}+4 \hat{\jmath}+5 \widehat{\boldsymbol{k}})$ and $\frac{\sqrt{5}}{\sqrt{29}}$ or $\frac{\sqrt{145}}{29}$ units
3. $\frac{x-1}{-2}=\frac{y+1}{1}=\frac{z-1}{-1}$
4. $\lambda=-2$
5. $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$
6. $(-2,-1,3)$ or $(4,3,7)$
7. $(1,0,7)$
$9 . \frac{19}{\sqrt{195}}$, not intersecting

## TEST-1

20 MARKS
30 MINUTES

## SECTION A

Q 1 The line which passes through the origin and intersect the two lines $\frac{x-1}{2}=\frac{y+3}{4}=\frac{z-5}{3}$, $\frac{x-4}{2}=\frac{y+3}{3}=\frac{z-14}{4}$ is,
(a) $\frac{x}{1}=\frac{y}{-3}=\frac{z}{5}$ (b) $\frac{x}{-1}=\frac{y}{3}=\frac{z}{5}$
(c) $\frac{x}{1}=\frac{y}{3}=\frac{z}{-5}$
(d) $\frac{x}{1}=\frac{y}{4}=\frac{z}{-5}$

## SECTION B

Q2. Study the two statements labelled as assertion (A) reason (R).
Point out if :
(A) Both Assertion and reason are true and reason is correct explanation of assertion.
(B) Assertion and reason both are true but reason is not the correct explanation of assertion.
(C) Assertion is true, reason is false.
(D) Assertion is false, reason is true.

Assertion: Equation of line Passing through the point $(1,2,3)$ and $(2,-1,5)$ is $(x-1) / 1=(y-2 /(-3)=$ ( $\mathrm{z}-3$ )/ 2
Reason : Equation of line passing through the point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$ is
$\left(x-x_{1}\right) /\left(x_{2}-x_{1}\right)=\left(y-y_{1}\right) /\left(y_{2}-y_{1}\right)=\left(z-z_{1}\right) /\left(z_{2}-z_{1}\right)$
Q3. If the two lines $L_{1}: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}, L_{2}: x=2, \frac{y}{-1}=\frac{\mathrm{z}}{2-\alpha}$ are perpendicular, then find value of $\alpha$.

## SECTION C

4. Find the equation of the line passing through $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2),(1,-1,0)$ and $(1,2,-1),(2,2,1)$.
5. Find the value of $\lambda$ so that the lines $\frac{1-x}{3}=\frac{y-2}{2 \lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3 \lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular to each other.

## SECTION D

7. Find the shortest distance between the following lines :

$$
\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text { and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}
$$

## SECTION-E (CASE BASED)

Based on the above information ,answer the following questions
(i) What is the Cartesian equation of line along EA ?
(ii) Findthevectorequationof The vector $\overrightarrow{E D}$

## ANSWERS

1.(a) 2.(a) $3.7 / 34$. $\quad$ 5. $\lambda=-26.2 \sqrt{29}$ units

7 (i) $\frac{x}{-4}=\frac{y}{3}=\frac{z-24}{12}$ (ii) $-8 \hat{\imath}-6 \hat{\jmath}-24 \hat{k}$


## SECTION A

Q 1. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is
(a) $\vec{r}=(3 i+7 j+2 k)+\mu(5 i+4 j-6 k)$
(b) $\vec{r}=(5 i+4 j-6 k)+\mu(3 i+7 j+2 k)$
(c) $\vec{r}=(5 i-4 j-6 k)+\mu(3 i-7 j-2 k)$
(d) $\vec{r}=(5 i-4 j+6 k)+\mu(3 i+7 j+2 k)$

## SECTION B

Q2. Find the angle between the lines $\vec{r}=(2 \hat{j}-3 \hat{k})+\lambda(\hat{i}+2 \hat{j}+2 \hat{k})$ and $\vec{r}=(2 \hat{i}+6 \hat{j}+3 \hat{k})+\lambda(2 \hat{i}+3 \hat{j}-6 \hat{k})$.
Q3. Find the vector equation of the line passing through the point $(-1,5,4)$ and perpendicular to the plane $z$ $=0$.
Q4. Find the direction cosines of the line passing through the following points:
$(-2,4,-5),(1,2,3)$.

## SECTION-C

Q5 Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point $\quad \mathrm{P}(1,3,3)$
Q6 Find the coordinates of the foot of the perpendicular drawn from the point $\mathrm{A}(1,8,4)$ to the line joining the points $\mathrm{B}(0,-1,3)$ and $\mathrm{C}(2,-3,-1)$.
Q7 The Cartesian equation of a line AB is $\frac{2 x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

## SECTION-D

Q. 8 Find the image of the point $\mathrm{P}(5,9,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

Q9 Find the equation of the line passing through the points $\mathrm{P}(-1,3,-2)$ and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

## SECTION-E


(i) Find the equation of line along AD .
(ii) Find the length of DC.

## ANSWERS

1(d) $\quad 2 . \cos ^{-1}\left(\frac{4}{\sqrt{21}}\right) \quad 3 \overrightarrow{\mathrm{r}}=-\hat{\mathrm{i}}+5 \hat{\mathrm{j}}+(4+\lambda) \hat{\mathrm{k}} \quad 4 \frac{3}{\sqrt{77}} \quad \mathbf{5 ( - 2 , - 1 , 3 )}$ or (4,3,7)$\quad 6\left(\frac{5}{3}, \frac{-8}{3}, \frac{-1}{3}\right)$
$7(\sqrt{3} / \sqrt{55}, 4 / \sqrt{55}, 6 / \sqrt{55}) \quad 8(1,1,11) \quad 9 \frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4} \quad 10$ (i) $\frac{x}{4}=\frac{y}{5}=\frac{z-30}{-15} \quad$ (ii) $5 \sqrt{61}$

## TOPICS

## 1.LINEAR PROGRAMMING PROBLEM

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

## 2. PROBABILITY

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable

## LINEAR PROGRAMMING

** An Optimisation Problem A problem which seeks to maximise or minimise a function is called an optimisation problem. An optimisation problem may involve maximisation of profit, production etc or minimisation of cost, from available resources etc.
** Linear Programming Problem (LPP)
A linear programming problem deals with the optimisation (maximisation/minimisation) of a linear function of two variables (say $x$ and $y$ ) known as objective function subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints). A linear programming problem is a special type of optimisation problem.
** Objective Function Linear Function $\mathrm{Z}=a x+b y$, where a and b are constants, which has to be
maximised or minimised is called a linear objective function.
** Decision Variables In the objective function $\mathrm{Z}=a x+b y, \mathrm{x}$ and y are called decision variables.
** Constraints The linear inequalities or restrictions on the variables of an LPP are called constraints. The conditions $\mathrm{x} \geq 0, \mathrm{y} \geq 0$ are called non-negative constraints.
** Feasible Region The common region determined by all the constraints including nonnegative
constraints $x \geq 0, y \geq 0$ of an LPP is called the feasible region for the problem.
** Feasible Solutions Points within and on the boundary of the feasible region for an LPP represent feasible solutions.
** Infeasible Solutions Any Point outside feasible region is called an infeasible solution.
** Optimal (feasible) Solution Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
** Let R be the feasible region (convex polygon) for an LPP and let $\mathrm{Z}=\mathrm{ax}+$ by be the objective
function. When Z has an optimal value (maximum or minimum), where x and y are subject to
constraints described by linear inequalities, this optimal value must occur at a corner point (vertex)
of the feasible region.
** Let R be the feasible region for a LPP and let $\mathrm{Z}=a x+b y$ be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occur at a corner point of $R$. If the feasible region $R$ is unbounded, then a maximum or a minimum value of the objective function may or may not exist. However, if it exits, it must occur at a corner point of R .

## ILLUSTRATIONS

1. Find the maximum value of the objective function $Z=5 x+10 y$ subject to the constraints $x+2 y \leq 120, x+y \geq 60, x-2 y \geq 0, x \geq 0, y \geq 0$.
2. Find the maximum value of $Z=3 x+4 y$ subjected to constraints $x+y \leq 40, x+2 y \leq 60$, $x$ $\geq 0$ and $y \geq 0$
3. Find the points where the minimum value of $Z$ occurs:

$$
Z=6 x+21 y \text {, subject to } x+2 y \geq 3, x+4 y \geq 4,3 x+y \geq 3, x \geq 0, y \geq 0
$$

4. For the following feasible region, write the linear constraints.

5. The feasible region for LPP is shown shaded in the figure.

Let $Z=3 x-4 y$ be the objective function, then write the maximumvalue of $Z$.

6. Feasible region for an LPP is shown shaded in the following figure. Find the point where minimum of $Z=4 x+3 y$ occurs.

7. Write the linear inequations for which the shaded area in the following figure is the solution set.

8. Write the linear inequations for which the shaded area in the following figure is the solution set.

9. Write the linear inequations for which the shaded area in the following figure is the solution set.

10. Solve the following Linear Programming Problems graphically: Maximise $Z=5 x+3 y$ subject to $3 x+5 y \leq 15,5 x+2 y \leq 10, x \geq 0, y \geq 0$.

ANSWERS

1. 600
2. $\left(2, \frac{1}{2}\right)$
3. 0
4. $\mathrm{x}+2 \mathrm{y} \leq 10, \mathrm{x}+\mathrm{y} \geq 1, \mathrm{x}-\mathrm{y} \leq 0, \mathrm{x}, \mathrm{y} \geq 0$
5. $5 x+4 y \leq 20, x \geq 1, y \geq 2$
6. Maximum $Z=\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$
2.140
7. $x \geq 0, y \geq 0,3 x+2 y \geq 12, x+3 y \geq 11$
8. $(2,5)$
9. $3 \mathrm{x}+4 \mathrm{y} \leq 60, \mathrm{x}+3 \mathrm{y} \leq 30, \mathrm{x} \geq 0, \mathrm{y} \geq 0$

Test -01
M.M. : 20

| 1 | The set of all feasible solutions of a LPP is a $\qquad$ set. <br> (a) Concave <br> (b) Convex <br> (c) Feasible <br> (d) None of these | 1 |
| :---: | :---: | :---: |
| 2 | In a LPP, if the objective function $\mathrm{Z}=a x+b y$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same..........value. <br> (a) minimum <br> (b) maximum <br> (c) zero <br> (d) none of these | 1 |
| 3 | In the feasible region for a LPP is ........., then the optimal value of the objective function $\mathrm{Z}=a x+b y$ may or may not exist. <br> (a) bounded <br> (b) unbounded <br> (c) in circled form <br> (d) in squared form | 1 |
| 4 | Region represented by $x \geq 0, y \geq 0$ is: <br> (a) First quadrant <br> (b) Second quadrant <br> (c) Third quadrant <br> (d) Fourth quadrant | 1 |
| 5 | The feasible region for an LPP is shown shaded in the figure. Let $\mathrm{Z}=3 \mathrm{x}-4 \mathrm{y}$ be objective function. Maximum value of Z is | 1 |


|  | (a) <br> 0 <br> (b) 8 <br> (c) 12 <br> (d) -18 |  |
| :---: | :---: | :---: |
| 6 | The maximum value of $Z=4 x+3 y$, if the feasible region for an LPP is as shown below, is <br> (a) 112 <br> (b) 100 <br> (c) 72 <br> (d) 110 | 1 |
| 7 | In the given figure, the feasible region for a LPP is shown. Find the maximum and minimum value of $Z=x+2 y$ <br> (a) $8,3.2$ <br> (b) $9,3.14$ <br> (c) 9,4 <br> (d) None | 2 |
| 8 | The linear programming problem minimize $\mathrm{Z}=3 \mathrm{x}+2 \mathrm{y}$, subject to constraints $x+y \leq 8,3 x+5 y \leq 15, x, y \geq 0$, has <br> (a) One solution <br> (b) No feasible solution <br> (c) Two solutions <br> (d) Infinitely many solutions | 2 |
| 9 | Corner points of the feasible region determined by the system of linear constraints are $(0,3),(1,1)$ and $(3,0)$. Let $Z=p x+q y$, where $p, q>0$. Condition on $p$ and $q$ so that the minimum of $Z$ occurs at $(3,0)$ and $(1,1)$ is <br> (a) $\mathrm{p}=2 \mathrm{q}$ <br> (b) $\mathrm{p}=\frac{\mathrm{q}}{2}$ <br> (c) $\mathrm{p}=3 \mathrm{q}$ <br> (d) $\mathrm{p}=\mathrm{q}$ | 2 |
| 10 | Solve the following Linear Programming Problem : <br> Max. $Z=x+2 y$ <br> Subject to the constraints : $x+y \leq 50,3 x+y \leq 90, x \geq 0, y \geq 0$ | 3 |
| 11 | Solve the following Linear Programming Problem : <br> $\operatorname{Min} \mathrm{Z}=\mathrm{x}+2 \mathrm{y}$ <br> Subject to the constraints : $x+2 y \geq 100,2 x-y \leq 0,2 x+y \leq 200, x \geq 0, y \geq 0$ | 5 |

## ANSWER :

```
1(b) 2 (b) 3 (b) 4 (a) 5 (a) 6 (a) 7 (b) 8.(a) 9. (b) 10. Max. z =100 11.mmin z= 100
```

Test : 02
M. M. -30

| 1 | The common region determined by all the constraints including non-negative constraints |  |
| :--- | :--- | :--- |
| x, y $\geq 0$ of a linear programming problem is called |  |  |
| (a) Feasible region (b) Feasible solution <br> (c) Optimal solution (d) Constraints | 1 |  |
| 2 | In the feasible region for a LPP is ........, then the optimal value of the objective function | 1 |


|  | $\mathrm{Z}=\mathrm{ax}+$ by may or may not exist. <br> (a) bounded <br> (b) unbounded <br> (c) in circled form <br> (d) in squared form |  |
| :---: | :---: | :---: |
| 3 | $\mathrm{Z}=250 \mathrm{x}+75 \mathrm{y}$ is a linear objective function. Variables x and y are called $\ldots .$. <br> (a) Decision variables <br> (b) Constraints <br> (c) Constant <br> (d) Objective function | 1 |
| 4 | Points within and on the boundary of the feasible region represent ...... <br> (a) Infeasible solution <br> (b) Feasible solution <br> (c) Objective solution <br> (d) None | 1 |
| 5 | The maximum value of $Z=4 x+3 y$, if the feasible region for an LPP is as shown below, is <br> (a) 112 <br> (b) 72 <br> (c) 110 <br> (d) 100 | 1 |
| 6 | Corner points of the feasible region determined by the system of linear constraints are $(0,3),(1,1)$ and (3,0). Let $\mathrm{Z}=\mathrm{px}+\mathrm{qy}$, where $\mathrm{p}, \mathrm{q}>0$. Condition on p and q so that the minimum of Z occurs at $(3,0)$ and $(1,1)$ is <br> (a) $p=2 q$ <br> (b) $p=q / 2$ <br> (c) $p=3 q$ <br> (d) $\mathrm{p}=\mathrm{q}$ | 2 |
| 7 | Write the linear inequations for which the shaded area in the following figure is the solution set <br> (a) $5 \mathrm{x}+\mathrm{y} \leq 100, \mathrm{x}+\mathrm{y} \geq 60$ <br> (c) $5 x+y \geq 100, x+y \leq 60$ <br> (b) $\quad 5 x+y \geq 60, x+y \leq 100$ <br> (d) $5 x+y \leq 100, x+y \leq 60$ | 2 |
| 8 | In figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z=x+2 y$. <br> (a) $8,3.2$ <br> (b) $9,3.14$ <br> (c) 9,4 <br> (d) None | 2 |
| 9 | Solve the following Linear Programming Problem : <br> Max. $Z=x+2 y$ <br> Subject to the constraints : $x+y \leq 50,3 x+y \leq 90, x \geq 0, y \geq 0$ | 3 |
| 10 | Solve the following linear programming problem graphically: Maximise $Z=3 x+4 y$ | 3 |


|  | subject to the constraints : $\mathrm{x}+\mathrm{y} \leq 4, \mathrm{x} \geq 0, \mathrm{y} \geq 0$. | 3 |
| :--- | :--- | :--- |
| 11 | Solve the following linear programming problem graphically: <br> Maximise $\mathrm{Z}=4 \mathrm{x}+\mathrm{y}$ <br> subject to the constraints $: \mathrm{x}+\mathrm{y} \leq 50,3 \mathrm{x}+\mathrm{y} \leq 90, \mathrm{x} \geq 0, \mathrm{y} \geq 0$ | 5 |
| 12 | Find the minimum value of $\mathrm{Z}=11 \mathrm{x}+7 \mathrm{y}$ <br> Subject to $\mathrm{x}+3 \mathrm{y} \leq 9, \mathrm{x}+\mathrm{y} \leq 5, \mathrm{x} \geq 0, \mathrm{y} \geq 0$ | 5 |
| 13 | Solve the Linear Programming graphically: <br> Maximize $\mathrm{Z}=9 \mathrm{x}+3 \mathrm{y}$ subject to2x $+3 \mathrm{y} \leq 13,3 \mathrm{x}+\mathrm{y} \leq 5, \mathrm{x} \geq 0, \mathrm{y} \geq 0$ | 5 |

## ANSWER :

1 (a) 2 (b) 3 (a)
4 (b)
5 (a) 6 (b)
(d) 8.(b) 9. Max. $\mathrm{z}=100$
10. Max. $z=16$
11 Max Z = 110
12. $\operatorname{Min} Z=21$
13. $\operatorname{Max} Z=15$

## PROBABILITY SOME IMPORTANT RESULTS/CONCEPTS

## ** Sample Space and Events :

The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by $S$. The elements of $S$ are called events and a subset of $S$ is called an event.
$\phi(\subset S)$ is called an impossible event and
$S(\subset S)$ is called a sure event.
** Probability of an Event.
(i) If $\mathbf{E}$ be the event associated with an experiment, then probability of $\mathbf{E}$, denoted by $P(E)$ is
defined as $\mathbf{P}(\mathbf{E})=\frac{\text { number of outcomes in } E}{\text { number of total outcomes in sample space } S}$
it being assumed that the outcomes of the experiment in reference are equally likely.
(ii) $\mathrm{P}($ sure event or sample space $)=\mathrm{P}(\mathrm{S})=1$ and $\mathrm{P}($ impossible event $)=\mathrm{P}(\phi)=0$.
(iii) If $\mathrm{E}_{1}, \mathrm{E}_{2}, \mathrm{E}_{3}, \ldots, \mathrm{E}_{\mathrm{k}}$ are mutually exclusive and exhaustive events associated with an experiment
(i.e. if $\left.E_{1} \cup E_{2} \cup E_{3} \cup \ldots . \cup E_{k}\right)=S$ and $E_{i} \cap E_{j}=\phi$ for $i, j \in\{1,2,3, \ldots \ldots, k\} i \neq j$ ), then $\mathrm{P}\left(\mathrm{E}_{1}\right)+\mathrm{P}\left(\mathrm{E}_{2}\right)+\mathrm{P}\left(\mathrm{E}_{3}\right)+\ldots+\mathrm{P}\left(\mathrm{E}_{\mathrm{k}}\right)=1$.
(iv) $\mathrm{P}(\mathrm{E})+\mathrm{P}\left(\mathrm{E}^{\mathrm{C}}\right)=1$
** If E and F are two events associated with the same sample space of a random experiment, the
Conditional probability of the event $E$ given that $F$ has occurred, i.e. $P(E \mid F)$ is given by $\mathbf{P}(E \mid F)=\frac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$
** Multiplication rule of probability: $P(E \cap F)=P(E) P(F \mid E)$
$=\mathrm{P}(\mathrm{F}) \mathrm{P}(\mathrm{E} \mid \mathrm{F})$ provided $\mathrm{P}(\mathrm{E}) \neq 0$ and $\mathrm{P}(\mathrm{F}) \neq 0$.
** Independent Events: E and F are two events such that the probability of occurrence of one of
them is not affected by occurrence of the other.
Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F)=P(E) . P(F)$.

## ** Bayes' Theorem :If $\mathbf{E}_{1}, \mathbf{E}_{2}, \ldots, \mathbf{E}_{\mathrm{n}}$ are $\mathbf{n}$ non empty events which constitute a partition of sample

space $S$, i.e. $E_{1}, E_{2}, \ldots, E_{n}$ are pairwise disjoint and $E_{1} \cup E_{2} \cup \ldots \cup E_{n}=S$ andA is any event of
nonzero probability, then
$\mathbf{P}(E i \mid A)=\frac{P\left(E_{i}\right) \cdot P\left(A \mid E_{i}\right)}{\sum_{j=1}^{n} P\left(E_{j}\right) \cdot P\left(A \mid E_{j}\right)}$ for any $\mathbf{i}=\mathbf{1 , 2 , 3 , \ldots , n}$
** The probability distribution of a random variable X is the system of numbers
$\mathrm{X}: \quad \mathrm{x}_{1} \quad \mathrm{x}_{2} \quad \ldots \quad \mathrm{x}_{\mathrm{n}}$
$\mathrm{P}(\mathrm{X}): \quad \mathrm{p}_{1} \quad \mathrm{p}_{2} \quad \ldots \quad \mathrm{p}_{\mathrm{n}}$
where, $\mathrm{p}_{\mathrm{i}}>0, \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{p}_{\mathrm{i}}=1, \mathrm{i}=1,1,2, \ldots$,
** Binomial distribution: The probability of x successes $\mathrm{P}(\mathrm{X}=\mathrm{x})$ is also denoted by $\mathrm{P}(\mathrm{x})$ and is
given by $P(x)={ }^{n} C_{x} q^{n-x} p^{x}, \quad x=0,1, \ldots, n .(q=1-p)$

## Illustrations

1.If $P(A)=0.8, P(B)=0.5$ and $P(B \mid A)=0.4$, what is the value of $P(A \cap B)$ ?
A. 0.32
B. 0.25
C. 0.1
D. 0.5

Answer: A. 0.32
Explanation: Given, $\mathrm{P}(\mathrm{A})=0.8, \mathrm{P}(\mathrm{B})=0.5$ and $\mathrm{P}(\mathrm{B} \mid \mathrm{A})=0.4$
By conditional probability, we have;
$\mathrm{P}(\mathrm{B} \mid \mathrm{A})=\mathrm{P}(\mathrm{A} \cap \mathrm{B}) / \mathrm{P}(\mathrm{A}) \quad \Rightarrow \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\mathrm{P}(\mathrm{B} \mid \mathrm{A}) . \mathrm{P}(\mathrm{A})=0.4 \times 0.8=0.32$
2.If $P(A)=6 / 11, P(B)=5 / 11$ and $P(A \cup B)=7 / 11$, what is the value of $P(B \mid A)$ ?
A. $1 / 3$
B. $2 / 3$
C. 1
D. None of the
above

Answer: B. 2/3
Explanation: By definition of conditional probability we know;
$\mathrm{P}(\mathrm{B} \mid \mathrm{A})=\mathrm{P}(\mathrm{A} \cap \mathrm{B}) / \mathrm{P}(\mathrm{A}) \ldots(\mathrm{i})$
Also,
$\mathrm{P}(\mathrm{A} \cap \mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A} \mathrm{U} B)$
$=6 / 11+5 / 11-7 / 11=4 / 11$
Now putting the value of $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ in eq.(i), we get;
$\mathrm{P}(\mathrm{B} \mid \mathrm{A})=(4 / 11) /(6 / 11)=4 / 6=2 / 3$
3.Find $P(E \mid F)$, where $E$ : no tail appears, $F$ : no head appears, when two coins are tossed in the air.
A. 0
B. $1 / 2$
C. 1
D. None of the
above
Answer: A. 0
Explanation: Given,
E: no tail appears
And F: no head appears
$\Rightarrow \mathrm{E}=\{\mathrm{HH}\}$ and $\mathrm{F}=\{\mathrm{TT}\}$
$\Rightarrow \mathrm{E} \cap \mathrm{F}=\phi$
As we know, two coins were tossed;
$P(E)=1 / 4$
$\mathrm{P}(\mathrm{F})=1 / 4$
$\mathrm{P}(\mathrm{E} \cap \mathrm{F})=0 / 4=0$
Thus, by conditional probability, we know that;
$\mathrm{P}(\mathrm{E} \mid \mathrm{F})=\mathrm{P}(\mathrm{E} \cap \mathrm{F}) / \mathrm{P}(\mathrm{F})$
$=0 /(1 / 4)$
$=0$
4.If $P(A \cap B)=70 \%$ and $P(B)=85 \%$, then $P(A / B)$ is equal to:
A. $17 / 14$
B. $14 / 17$
C. $7 / 8$
D. $1 / 8$

Answer: B.14/17
Explanation: By conditional probability, we know;
$\mathrm{P}(\mathrm{A} \mid \mathrm{B})=\mathrm{P}(\mathrm{A} \cap \mathrm{B}) / \mathrm{P}(\mathrm{B})$
$=(70 / 100) \times(100 / 85)$
$=14 / 17$
5. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls? Given that
(i) the youngest is a girl?
(ii) atleast one is a girl?

Answer:
Let B and b represent elder and younger boy child. Also, $G$ and $g$ represent elder and younger girl child. If a family has two children, then all possible cases are
$S=\{\mathrm{Bb}, \mathrm{Bg}, \mathrm{Gg}, \mathrm{Gb}\}$
$\therefore \mathrm{n}(\mathrm{S})=4$
Let us define event $A$ : Both children are girls, then $A=\{G g\} \Rightarrow n(A)=1$
(i) Let $\mathrm{E}_{1}$ : The event that youngest child is a girl.

Then, $\mathrm{E}_{1}=\{\mathrm{Bg}, \mathrm{Gg}\}$ and $\mathrm{n}\left(\mathrm{E}_{1}\right)=2$

$$
\begin{aligned}
& \text { so } \quad P\left(E_{1}\right)=\frac{n\left(E_{1}\right)}{n(S)}=\frac{2}{4}=\frac{1}{2} \\
& \text { and } A \cap E_{1}=\{G g\} \Rightarrow n\left(A \cap E_{1}\right)=1 \\
& \text { so } P\left(A \cap E_{1}\right)=\frac{n\left(A \cap E_{1}\right)}{n(S)}=\frac{1}{4}
\end{aligned}
$$

Now, $P\left(\frac{A}{E_{1}}\right)=\frac{P\left(A \cap E_{1}\right)}{P\left(E_{1}\right)}=\frac{1 / 4}{1 / 2}=\frac{1}{2}$
$\therefore$ Required probability $=\frac{1}{2}$
(ii) Let $\mathrm{E}_{2}$ : The event that atleast one is girl.

Then, $\mathrm{E}_{2}=\{\mathrm{Eg}, \mathrm{Gg}, \mathrm{Gb}\} \Rightarrow \mathrm{n}\left(\mathrm{E}_{2}\right)=3$,
6.If $\mathrm{P}($ not A$)=0.7, \mathrm{P}(\mathrm{B})=0.7$ and $\mathrm{P}(\mathrm{B} / \mathrm{A})=0.5$, then find $\mathrm{P}(\mathrm{A} / \mathrm{B})$.

Answer:
Given, $P\left(A^{\prime}\right)=0.7, P(B)=0.7$ and $P\left(\frac{B}{A}\right)=0.5$
Clearly, $P(A)=1-P\left(A^{\prime}\right)=1-0.7=0.3$
Now, $P\left(\frac{B}{A}\right)=\frac{P(A \cap B)}{P(A)}$
$\Rightarrow \quad 0.5=\frac{P(A \cap B)}{0.3}$
$\Rightarrow \quad P(A \cap B)=0.15$
$\therefore \quad P\left(\frac{A}{B}\right)=\frac{P(A \cap B)}{P(B)}=\frac{0.15}{0.7} \Rightarrow P\left(\frac{A}{B}\right)=\frac{3}{14}$
7. A die marked $1,2,3$ in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is marked red'. Find whether the events A and B are independent or not.
Or
A die, whose faces are marked 1,2,3 in red and 4,5,6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.
Answer:
When a die is thrown, the sample space is
$\mathrm{S}=\{1,2,3,4,5,6\}$
$\Rightarrow \mathrm{n}(\mathrm{S})=6$
Also, A : number is even and B : number is red.
$\therefore \mathrm{A}=\{2,4,6\}$ and $\mathrm{B}=\{1,2,3\}$ and $\mathrm{A} \cap \mathrm{B}=\{2\}$
$\Rightarrow \mathrm{n}(\mathrm{A})=3, \mathrm{n}(\mathrm{B})=3$ and $\mathrm{n}(\mathrm{A} \cap \mathrm{B})=1$
Now,

$$
\begin{aligned}
& P(A)=\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2} \\
& P(B)=\frac{n(B)}{n(S)}=\frac{3}{6}=\frac{1}{2}
\end{aligned}
$$

and

$$
P(A \cap B)=\frac{n(A \cap B)}{n(S)}=\frac{1}{6}
$$

Now, $\quad P(A) \times P(B)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4} \neq \frac{1}{6}=P(A \cap B)$

$$
\therefore \quad P(A \cap B) \neq P(A) \times P(B)
$$

Thus, A and B are not independent events.
8. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8 , given that the red die resulted in a number less than 4 .
Answer:
Let us denote the numbers on black die by $\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots \ldots, \mathrm{~B}_{6}$ and the numbers on red die by $\mathrm{R}_{1}, \mathrm{R}_{2}, \ldots \ldots, \mathrm{R}_{6}$.
Then, we get the following sample space.
$s=\left\{\left(B_{1}, R_{1}\right),\left(B_{1}, R_{2}\right), \ldots \ldots,\left(B_{1}, R_{6}\right),\left(B_{2}, R_{2}\right), \ldots \ldots .,\left(B_{6}, B_{1}\right),\left(B_{6}, B_{2}\right), \ldots \ldots,\left(B_{6}, R_{6}\right)\right.$
Clearly, $\mathrm{n}(\mathrm{S})=36$
Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a number less than 4.
Then, $A=\left\{\left(B_{2}, R_{6}\right),\left(B_{6}, R_{2}\right),\left(B_{3}, R_{5}\right),\left(B_{5}, R_{3}\right),\left(B_{4}, R_{4}\right)\right\}$
and $B=\left\{\left(B_{1}, R_{1}\right),\left(B_{1}, R_{2}\right),\left(B_{1}, R_{3}\right),\left(B_{2}, R_{1}\right),\left(B_{2}, R_{2}\right),\left(B_{2}, R_{3}\right), \ldots \ldots \ldots,\left(B_{6}, R_{1}\right),\left(B_{6}, R_{2}\right)\right.$,
$\left.\left(\mathrm{B}_{6}, \mathrm{R}_{3}\right)\right\}$
$\Rightarrow \mathrm{A} \cap \mathrm{B}=\left\{\left(\mathrm{B}_{6}, \mathrm{R}_{2}\right),\left(\mathrm{B}_{5}, \mathrm{R}_{3}\right)\right\}$
Now, required probability,
$\mathrm{p}(\mathrm{AB})=\mathrm{P}(\mathrm{A} \cap \mathrm{B}) \mathrm{P}(\mathrm{B})=2361836=218=19$
9. Evaluate $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$, if $2 \mathrm{P}(\mathrm{A})=\mathrm{P}(\mathrm{B})=513$ and $\mathrm{P}(\mathrm{A} / \mathrm{B})=25$.

Answer:
We have, $2 P(A)=P(B)=\frac{5}{13}$

$$
\begin{aligned}
& \Rightarrow P(A)=\frac{5}{26} \text { and } P(B)=\frac{5}{13} \text { and } P(A / B)=\frac{2}{5} \\
& \because P\left(\frac{A}{B}\right)=\frac{P(A \cap B)}{P(B)} \\
& \begin{aligned}
& \therefore \quad \frac{2}{5}=\frac{P(A \cap B)}{5 / 13} \\
& \Rightarrow \quad P(A \cap B)=\frac{2}{5} \times \frac{5}{13}=\frac{2}{13} \\
& \because P(A \cup B)=P(A)+P(B)-P(A \cap B) \\
&=\frac{5}{26}+\frac{5}{13}-\frac{2}{13} \\
&=\frac{5+10-4}{26}=\frac{11}{26}
\end{aligned}
\end{aligned}
$$

10. Prove that if E and F are independent events, then the events E and $\mathrm{F}^{\prime}$ are also independent. (Delhi 2017)
Answer:
Given, E and F are independent events, therefore
$\Rightarrow \mathrm{PE}(\cap \mathrm{F})=\mathrm{P}(\mathrm{E}) \mathrm{P}(\mathrm{F})$
Now, we have,
$\mathrm{P}\left(\mathrm{E} \cap \mathrm{F}^{\prime}\right)+\mathrm{P}(\mathrm{E} \cap \mathrm{F})=\mathrm{P}(\mathrm{E})$
$\mathrm{P}\left(\mathrm{E} \cap \mathrm{F}^{\prime}\right)=\mathrm{P}(\mathrm{E})-\mathrm{P}(\mathrm{E} \cap \mathrm{F})$
$P\left(E \cap F^{\prime}\right)=P(E)-P(E) P(F)$ [using Eq. (i))
$\mathrm{P}\left(\mathrm{E} \cap \mathrm{F}^{\prime}\right)=\mathrm{P}(\mathrm{E})[1-\mathrm{P}(\mathrm{F})]$
$\mathrm{P}\left(\mathrm{E} \cap \mathrm{F}^{\prime}\right)=\mathrm{P}(\mathrm{E}) \mathrm{P}\left(\mathrm{F}^{\prime}\right)$
$\therefore \mathrm{E}$ and F 'are also independent events.
Hence proved.

## III : Problems for Practice:

1.A bag contains 5 red and 3 blue balls.If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is (a)45/196 $\quad$ (b)135/392 $\quad$ (c)15/56 $\quad$ (d) $15 / 29$ 2. A flashlight has 8 batteries out of which 3 are dead.If two batteries are selected without replacement and tested, the probability that both are dead is
(a)33/56
(b)9/64
(c) $1 / 14$
(d) $3 / 28$
3. Probability that A speaks truth is $4 / 5$.A coin is tossed .A reports that a head appears.The probability that actually there was a head is
(a) $4 / 5$
(b) $1 / 2$
(c) $1 / 5$
(d) $2 / 5$
4. A and B are two students. Their chances of solving a problem correctly are $1 / 3$ and $1 / 4$ respectively. If the probability of their making a common error is $1 / 20$ and they obtain the same number, then the probability of their answer to be correct is
(a) $1 / 12$
(b) $1 / 40$
(c) $13 / 120$
(d) $10 / 13$
5. mark the correct choice
(a) Statement-1 and statement-2 are true ; statement -2 is a correct explanation for statement -1
(b) Statement-1 and statement-2 are true ; statement -2 is not a correct explanation for statement -1
(c) Statement- 1 is true , statement- 2 is false
(d) Statement- 1 is false , statement- 2 is true

Statement-1 (assertion) 20 persons are sitting in a row. Two of these persons are selected at random. The probability that the two selected persons are not together is 0.9 .
Statement-2 (Reason) If $\bar{A}$ denotes the negation of an event $A$, then $P(\bar{A})=1-P(A)$.
6. In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B, 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shops randomly and it is found
to be adulterated. Find the probability (i) Getting adulterated ghee (ii) it is getting from shop B.
7.Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society?
8. In a game of Archery, each ring of the Archery target is valued. The centre most ring is worth 10 point and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9 . Based on the above information answer the following questions. If both of them hit the Archery target, then find the probability that
(i) exactly one of them earns 10 points (ii) both of them earn 10 points.
9. If $\mathrm{P}(\mathrm{A})=4 / 7, \mathrm{P}(\mathrm{B})=0$, then find $\mathrm{P}(\mathrm{A} / \mathrm{B})$.
10. Write the probability of an even prime number on each die, when a pair of dice is rolled.
11. Two independent events $A$ and $B$ are given such that $P(A)=0.3$ and $P(B)=0.6$ find $P(A$ and not $B$ ).
12. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and $B$ be the event 3 on the die. Check whether $A$ and $B$ are independent events or not.
13. Let $A$ and $B$ be two events. If $P(A)=0.2, P(B)=0.4, P(A \cap B)=0.6$, then find $P(A \cup B)$.
14. How many times must a man toss a fair coin, so that the probability of having at least one head is more than $80 \%$ ?
15. A problem in mathematics is given to 3 students whose chances of solving it are $1 / 2,1 / 3$ and $1 / 4$, . What is the probability that The (i) problem is solved (ii) exactly one of them will solve it?
16. X is taking up subjects, Mathematics, Physics and Chemistry in the examination. His probabilities of getting Grade A in these subjects are $0.2,0.3$ and 0.5 respectively.
Find the probability that he gets (i) Grade A in all subjects. (ii) Grade A in no subject
(iii) Grade A in two subjects.
17. A speak truth in $60 \%$ of the case, while B in $90 \%$ of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?
18. There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?
19.(a) 12 cards numbered 1 to 12 , are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3 ,find the probability that it is an even number.
(b) 12 cards, numbered 1 to 12 , are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 5 , find the probability that it is an odd number.
20. $10 \%$ of the bulbs produced in a factory are of red colour and $2 \%$ are red and defective. If one bulb is picked up at random , determine the probability of its being defective if it is red. 21. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $3 / 10,1 / 5,1 / 10$ and $2 / 5$. The probabilities that he will be late are $1 / 4,1 / 3$, and $1 / 12$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late.What is the probability that he comes by train? 22. Suppose that the reliability of a HIV test is specified as follows: of people having HIV, $90 \%$ of the test detects the disease but $10 \%$ go undetected. Of the people free of HIV, $99 \%$ of the tests are judged HIV-ve but $1 \%$ are diagnosed as showing HIV +ve. From a large population of which only $0.1 \%$ have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV?
23. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letters came from TATA NAGAR?

IV: Answers: 1.(c) 2.(d) 3.(a) 4.(d) 5. (a) 6. (i)43/77 (ii) 21/43 7. 4/9 8.(i)0.26 (ii) 0.72 , 9.does not exist. 10.5/18 11.0.12 12. Yes 13. 0 14.3 15.(i) $3 / 4$ (ii) $11 / 24$ 16. (i) 0.03 (ii) 0.28 (iii) $0.2217 .42 \%$, yes $18.64 / 189$ 19. (a) $5 / 9$ (b) $3 / 7 \quad 20.1 / 5 \quad 21.1 / 2 \quad 22.0 .083$ approx. 23. 7/11.

## TEST-1 (20 Marks)

SECTION A (1MARK)

1. A bag contains 5 white and 4 red balls. 2 balls are drawn from the bag. Find the probability that both balls are white.
2 find the probability of a red king in a pack of 52 cards.
SECTION B(3MARKS)
3 The probability that a bulb produced by a factory will fuse after 100 days of use is 0.05 . Find the probability that out of such five such bulbs
(i) Not more than one
(ii) More than one

Will fuse after 100 days of use. 4 In a bolt factory, 3 machines A, B and C manufacture 25, 35 and 40 per cent of the total bolts manufactured.
Of these output, 5, 4 and 2 per cent are defective respectively. A bolt is drawn at random and is found to be defective. Find the probability that it was manufactured by either machine A. 4 5 An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean and variance of $X$

SECTION C(5MARKS)
6 A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will
come by train, bus, scooter or by other means of transport are respectively $3 / 10,1 / 5,1 / 10$ and $2 / 5$. The probabilities
that he will be late are $1 / 4,1 / 3$ and $1 / 12$, if he comes by train, bus and scooter respectively, but if he comes by other
means of transport, then he will not be late. When he arrives, he is late. What is the
probability that he
comes by train?

## TEST-2 (30 Marks)

SECTION A (1MARK)

1. Two coins are tossed. What is the probability of coming up two heads if it is known that at least one head comes up.
2. Four cards are drawn from 52 cards with replacement. Find the probability of getting at least 3 aces.

SECTION B(3MARKS)
3. A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of
getting
(i) Exactly 6 successes
(ii) At most 6 successes. 4
4. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering from it is 0.99 . The probability that the doctor diagnosis correctly that a person has T.B. on the basis of X-ray is 0.001 . In a certain city, 1 in 1,000 persons suffers from T.B. A person selected at random is diagnosed to have T.B. What is the chance that the person has actually T.B.?
5.A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of the three Marbles drawn be black if the first marble is red?

## SECTION C(5MARKS)

1. Suppose that the reliability of a HIV test is specified as follows: of people having HIV, $90 \%$ of the test detects the disease but $10 \%$ go undetected. Of the people free of HIV, $99 \%$ of the tests are judged HIV-ve but $1 \%$ are diagnosed as showing HIV +ve. From a large population of which only $0.1 \%$ have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV ?
2. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $1 / 3$, and probability that he copies the answer is $1 / 6$. The probability that his answer is correct, given that he copied it, is $1 / 8$. Find the probability that he knew the answer to the question, given that he correctly answered it.

## SECTION D(CASE STUDY BASEDQUESTIONS)

1. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed oneyear period with probability 0.6 , whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

## Based on the given information, answer the following questions.(2 MARKS EACH)

(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?
(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?
2. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers.

The probabilities of accidents are $0.01,0.03$ and 0.15 respectively. One of the insured person meets with an accident.
Based on the above information answer ANY OF THE FOUR of the following:
(i) The probability that the person is a car driver is
a) $1 / 3$
b) $1 / 6$
c) $1 / 2$
d) $1 / 4$
(ii) The probability that the truck driver met with an accident is
a) 0.01
b) 0.02
c) 0.03
d) 0.15
(iii) The total probability of committing an accident is
a) $13 / 15$
b) $13 / 150$
c) $6 / 52$
d) $13 / 25$
(iv) One of the insured person meets with an accident. Then the probability that he is a scooter Driver is
a) $1 / 52$
b) $3 / 52$
c) $15 / 52$
d) $6 / 125$
(v) One of the insured person meets with an accident. Then the probability that he is not a scooter Driver is
a) $50 / 52$
b) $53 / 52$
c) $51 / 52$
d) $15 / 52$

## KENDRIYA VIDYALAYA SANGATHAN PATNA REGION

CLASS - XII (23-24)
SAMPLE PAPER
MATHEMATICS (041)
BLUE PRINT

Time : 3Hours
Max. Marks: 80

## CHAPTERWISE DISTRIBUTUION

| Sr. no. | Chapter Name | $\begin{aligned} & \text { MCQ' s } \\ & \text { and AR (1 } \\ & \text { Mark) } \end{aligned}$ | Very Short <br> Answer <br> (VSA) (2 <br> Marks) | Short <br> Answer- <br> (SA) <br> (3Marks) | Long <br> Answer <br> (LA) <br> (5Marks) | Case <br> study (4 <br> Marks) | Total <br> Marks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Relation and function | 3 | 1 |  |  |  | 5 |
| 2. | Inverse Trigonometric Function | 1 | 1 | - |  | - | 3 |
| 3. | Matrices | 2 |  | 1 |  | - | 5 |
| 4. | Determinants | 1 | - | - |  | 1 | 5 |
| 5. | Continuity and Differentiability | 1 | 1 | 1 |  | - | 6 |
| 6. | Application of Derivative |  |  | 1 | 1 | 1 | 12 |
| 7. | Integrals | 2 |  | 2 |  | - | 8 |
| 8. | Application of Integrals | 1 |  |  | 1 | - | 6 |
| 9. | Differential Equation | 1 | 1 |  |  | - | 3 |
| 10. | Vector Algebra | 3 |  | - | 1(2 marks) |  | 5 |
| 11. | Three Dimensional Geometry | 2 |  | - | 1(3 marks) | 1 | 9 |
| 12. | Linear Programming | - | - | - | 1 |  | 5 |
| 13. | Probability | 3 | 1 | 1 |  | - | 8 |
|  | Total | $20 \times 1=20$ | $5 \times 2=10$ | $6 \times 3=18$ | $4 \times 5=20$ | $3 \times 4=12$ | 80 |

## UNIT WISE DISTRIBUTUION

| No. | Units | Marks |
| :---: | :---: | :---: |
| i. | Relation and Functions | 08 |
| ii. | Algebra | 10 |
| iii. | Calculus | 35 |
| iv. | Vector and Three- Dimensional Geometry | 14 |
| v. | Linear Programming | 05 |
| vi. | Probability | 08 |
|  | Total | $\mathbf{8 0}$ |

# KENDRIYA VIDYALAYA SANGATHAN PATNA REGION <br> SAMPLE PAPER <br> Class XII <br> Session 2023-24 <br> Mathematics 

Time Allowed: 3 Hours
Maximum Marks: 80

## General Instructions:

1. This question paper contains 5 sections A,B,C,D and E .Each section is compulsory. However ,there are internal choices in some questions.
2. Section A has 18 MCQ's and 2 Assertion -Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based /case /passage based/integrated units of assessment (4 marks each) with sub-parts.

| Sr. <br> Sr. | Section - A | Marks |
| :---: | :---: | :---: |
|  | Section I <br> All questions are compulsory. In case of internal choices attempt any one. |  |
| 1 | Let $R$ be the relation on $Z$ as $R=\left\{(a, b): a^{2}+b^{2}=25\right\}$, then domain of R is <br> (a) $\{3,4,5\}$ <br> (b) $\{0,3,4,5\}$ <br> (c) $\{-5,-4,-3,0,3,4,5\}$ <br> (d) $\{1,3,5\}$ | 1 |
| 2. | Set A has 3 elements and the set B has 4 elements. The number of injective mapping that can be defined from A to B. <br> (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) <br> 12 | 1 |
| 3. | The function $\mathrm{f}: \mathrm{N} \rightarrow \mathrm{N}$ given by $\mathrm{f}(\mathrm{x})=\mathrm{x}^{3} \quad$, Is <br> (a) One-One function <br> (b) Many-one function <br> (c) Onto function <br> (d) one-one onto function | 1 |
| 4. | The number of possible matrices of order $2 \times 2$ with each entry 2 or 0 . <br> (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) 12 | 1 |
| 5. | If A be the square matrix of order $2 \times 2$, and $\|A\|=3$, then the value of $\|2 \mathrm{~A}\|$ is | 1 |


|  | (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) <br> 12 |  |
| :---: | :---: | :---: |
| 6. | The Integral of $\int \frac{1-\sin x}{\cos ^{2} x} d x$, is <br> (a) $\tan x-\sec x+c$ <br> (b) $\tan x+\sec x+c$ <br> (c) $\tan x \cdot \sec x+c$ <br> d) $\sec x+c$ |  |
| 7. | The value of $\int_{-2}^{2} f(x) d x$, if $f$ is a odd function. <br> (a) -2 <br> (b) 0 <br> (c) 2 <br> (d) 4 | 1 |
| 8. | The area bounded by the curve $y=\operatorname{Sinx}$, between 0 to $\pi$ Is <br> (a) 0 sq.unit <br> (b) 1 sq. unit <br> (c) 2 sq.unit <br> (d) 4 sq. unit | 1 |
| 9. | The order and degree of differential equation $\frac{d^{4} y}{d x^{4}}+\sin \left(\frac{d^{3} y}{d x^{3}}\right)=0$, is <br> (a) Order 4,degree 3 <br> (b) order 3,degree 4 <br> (c) order 3 ,degree not defined <br> (d) order 4, degree not defined | 1 |
| 10. | Let $\vec{a}=\hat{\imath}+4 \hat{\jmath}+2 \hat{k}$ and $\vec{b}=3 \hat{\imath}-2 \hat{\jmath}+7 \hat{k}$, then a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ is <br> a) $\vec{d}=32 \hat{\imath}-\hat{\jmath}-14 \hat{k}$ <br> b) $\vec{d}=32 \hat{\imath}+\hat{\jmath}-14 \hat{\jmath}$ <br> c) $\vec{d}=32 \hat{\imath}-\hat{\jmath}+14 \hat{k}$ <br> d) $\vec{d}=32 \hat{\imath}+\hat{\jmath}+14 \hat{k}$ | 1 |
| 11. | If $\|\vec{a}\|=10,\|\vec{b}\|=2$ and $\vec{a} \cdot \vec{b}=12$, the value of $\|\vec{a} \times \vec{b}\|$, is <br> (a) 12 <br> (b) 16 <br> (c) 20 <br> (d) | 1 |
| 12. | If $\vec{a}$ and $\vec{b}$ are unit vector then the angle between $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is: <br> a) $\frac{\pi}{2}$ <br> b) $\frac{\pi}{12}$ <br> c) $\frac{\pi}{4}$ <br> d) $\frac{\pi}{3}$ | 1 |
| 13. | The angle between the pair of lines given by $\vec{r}=3 \vec{\imath}+2 \vec{\jmath}-4 \vec{k}+\varphi(\vec{\imath}+2 \vec{\jmath}+$ $2 \vec{k})$ and $\vec{r}=5 \vec{\imath}-2 \vec{\jmath}+\mu(3 \vec{\imath}+2 \vec{\jmath}+6 \vec{k})$ is, <br> (a) $\cos ^{-1} \frac{19}{21}$ <br> (b) $\sin ^{-1} \frac{19}{21}$ <br> (c) $\tan ^{-1} \frac{19}{21}$ <br> (d) $\cos ^{-1} \frac{21}{19}$ | 1 |
| 14. | The value of k for which the function $f(x)=\left\{\begin{array}{ll}\frac{1-\cos 2 x}{2 x^{2}} & \text { if } x \neq 0 \\ k & \text { if } x=0\end{array}\right.$ is continuous at $\mathrm{x}=0$ is <br> a) 0 <br> b) -1 <br> c) 1 <br> d) 2 | 1 |
| 15. | The value of $\sin ^{-1}\left(\sin \frac{2 \pi}{3}\right)$ is <br> (a) $\frac{2 \pi}{3}$ <br> (b) $\frac{\pi}{3}$ <br> (c) $-\frac{2 \pi}{3}$ <br> (d) $-\frac{\pi}{3}$ | 1 |


| 16. | An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is <br> (a). $\frac{6}{15}$ <br> (b). $\frac{4}{15}$ <br> (c) $\frac{8}{15}$ <br> (d) $\frac{1}{15}$ | 1 |
| :---: | :---: | :---: |
| 17. | Three coins are tossed once. Find the probability of getting at least one head. <br> a) $\frac{1}{8}$ <br> b) $\frac{1}{4}$ <br> c) $\frac{7}{4}$ <br> d) $\frac{7}{8}$ | 1 |
| 18. | Find $P(A / B)$ if $P(A)=0.4, P(B)=0.8$ and $P(B / A)=0.6$ <br> a) 0.3 <br> b) 0.4 <br> c) 0.5 <br> d) 0.6 |  |
|  | ASSERTION -REASON BASED QUESTIONS <br> In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. <br> (a) Both $(A)$ and $(R)$ are true and $(R)$ is the correct explanation of $(A)$. <br> (b) Both (A) and (R) are true but (R) is not the correct explanation of $(A)$. <br> (c) (A) is true but (R) is false. <br> (d) (A) is false but (R) is true. |  |
| 19. | Assertion: If $A=\left[\begin{array}{ll}2 & 4 \\ 6 & 0\end{array}\right]$, then $A=\left[\begin{array}{ll}2 & 5 \\ 5 & 0\end{array}\right]+\left[\begin{array}{cc}0 & -1 \\ 1 & 0\end{array}\right]$ is a unique way of expressing A <br> Reason: Every matrix can be represented as a sum of symmetric and skew symmetric matrix. |  |
| 20. | Assertion (A) : The angle between the straight line $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=$ $\frac{y+2}{2}=\frac{z-3}{-3}$ is $90^{0}$ <br> Reason (R) : Skew lines are lines in different planes which are parallel and intersecting |  |
|  | Section B <br> (This section comprises of very short answer type questions(VSA) of 2 marks each) |  |
| 21. | Find the value of $\tan ^{-1}(1)+\cos ^{-1}\left(-\frac{1}{2}\right)+\sin ^{-1}\left(-\frac{1}{2}\right)$ <br> OR <br> Draw the graph of $\sin ^{-1} x, \mathrm{x} \in\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ | 2 |
| 22. | For what value of k is the function | 2 |


|  | $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{l} \frac{\sin 5 x}{3 x}+\cos x, \text { if } x \neq 0 \\ k, \quad \text { if } \quad x=0 \end{array} \quad \text { continuous at } \mathrm{x}=0\right.$ |  |
| :---: | :---: | :---: |
| 23. | Solve the differential equation, $\frac{d y}{d x}=\left(1+\mathrm{x}^{2}\right)\left(1+\mathrm{y}^{2}\right)$ | 2 |
| 24. | If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x)=\sin x$ and $g(x)=5 x^{2}$, then find fog $(x)$ and $\operatorname{gof}(\mathrm{x})$ | 2 |
| 25. | If $P(A)=0.8, P(B)=0.5$ and $P(B / A)=0.4$ find $P(A U B)$ and $P(B / A)$. <br> OR <br> A speaks truth in $80 \%$ cases and B speaks truth in $90 \%$ cases. In what percentage of cases are they likely to disagree with each other in stating the same fact? | 2 |
|  | Section C (This section comprises of short answer type questions(SA) of $\mathbf{3}$ marks each) |  |
| 26. | Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, $90 \%$ of the test detect the disease but $10 \%$ go undetected. Of people free of HIV, $99 \%$ of the test are judged HIV-ive but $1 \%$ are diagnosed as showing HIV+ive. From a large population of which only $0.1 \%$ have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV? | 3 |
| 27. | Find the intervals in which the function $f(x)=20-9 x+6 x^{2}-x^{3}$ is <br> (i) strictly increasing. <br> (ii) strictly decreasing. | 3 |
| 28. | If $\mathrm{x}=\mathrm{a}[\cos \theta+\log \tan (\theta / 2)]$ and $\mathrm{y}=\operatorname{asin} \theta$, find $\frac{d y}{d x}$ at $\theta=\frac{\pi}{4}$ <br> OR <br> If $\mathrm{y}=\mathrm{e}^{\mathrm{x}} \sin \mathrm{x}$, prove that $\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+2 y=0$ | 3 |
| 29. | Find $\int e^{x} \frac{x-3}{(x-1)^{3}} d x$ <br> Evaluate $\int \frac{3 x-2}{(x+1)^{2}(x+3)} d x$ | 3 |


| 30. | Evaluate $\int_{0}^{2 \pi} \frac{1}{1+e^{\text {sinx }}} d x$ | 3 |
| :---: | :---: | :---: |
| 31. | Find the matrix $A$ for which $A\left[\begin{array}{cc}5 & 3 \\ -1 & -2\end{array}\right]=\left[\begin{array}{cc}14 & 7 \\ 7 & 7\end{array}\right]$. <br> OR $A=\left[\begin{array}{cc} 1 & \tan x \\ -\tan x & 1 \end{array}\right] \text {, show that } A^{T} A^{-1}=\left[\begin{array}{cc} \cos 2 x & -\sin 2 x \\ \sin 2 x & \cos 2 x \end{array}\right]$ | 3 |
|  | Section -D <br> (This section comprises of long answer type questions(LA) of 5 marks each) |  |
| 32. | Sketch the graph of $y=\|x+3\|$ and evaluate $\int_{-6}^{0}\|x+3\| d x$. <br> OR <br> Using integration find the area of region bounded by the triangle whose vertices are $\mathrm{A}(1,0), \mathrm{B}(2,2)$ and $\mathrm{C}(3,1)$. | 5 |
| 33. | (a)Find the shortest distance between the following lines: $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text { and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ <br> (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b}=\vec{c} \cdot \vec{a}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}= \pm 2(\vec{b} \times \vec{c})$ | $3+2$ |
| 34. | Solve the following Linear Programming Problem graphically : Maximise $Z=3 x+9 y$ subject to the constraints: $x+3 y \leq 60$ $x+y \geq 10$ $x \leq y, x \geq 0, y \geq 0$ | 5 |
| 35. | Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos ^{-1} 1 / \sqrt{3}$. <br> OR <br> Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot ^{-1} \sqrt{ } 2$. |  |


|  |  |  |
| :--- | :--- | :--- | :--- |
| 36. | A water tank has a shape of an inverted right circular cone with its axis vertical and <br> (Thistex lowermost. Its semi-vertical angle is tan ${ }^{-1}$ (0.5). If water is poured into a <br> constant rate of 5 cubic meter per hour. |  |



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KENDRIYA VIDYALAYA SANGATHAN
PATNA REGION
SAMPLE PAPER
Class XII
Session 2023-24
Mathematics
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## Time Allowed: 3 Hours

Maximum Marks: 80

## General Instructions:

1. This question paper contains 5 sections $A, B, C, D$ and $E$. Each section is compulsory. However ,there are internal choices in some questions.
2. Section A has 18 MCQ's and 2 Assertion -Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based /case /passage based/integrated units of assessment (4 marks each) with sub-parts.

| Sr. <br> No. | Section-A | Marks |
| :---: | :---: | :---: |
|  | Section I <br> All questions are compulsory. In case of internal choices attempt any one. |  |
| 1 | Let $R$ be the relation on $Z$ as $R=\left\{(a, b): a^{2}+b^{2}=25\right\}$ ,then domain of $R$ is <br> (a) $\{3,4,5\}$ <br> (b) $\{0,3,4,5\}$ <br> (c) $\{-5,-4,-3,0,3,4,5\}$ <br> (d) $\{1,3,5\}$ <br> (c) (c) $\{-5,-4,-3,0,3,4,5\}$ | 1 |
| 2. | Set A has 3 elements and the set B has 4 elements. The number of injective mapping that can be defined from $A$ to $B$. <br> (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) 12 <br> (a) 24 | 1 |
| 3. | The function $\mathrm{f}: \mathrm{N} \rightarrow \mathrm{N}$ given by $\mathrm{f}(\mathrm{x})=\mathrm{x}^{3}$, Is <br> (a) One-One function <br> (b) Many-one function <br> (c) Onto function <br> (d) one-one onto function <br> (a) One-One function | 1 |


| 4. | The number of possible matrices of order $2 \times 2$ with each entry 2 or 0 . <br> (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) 12 <br> (c) 16 | 1 |
| :---: | :---: | :---: |
| 5. | If $A$ be the square matrix of order $2 \times 2$, and $\|A\|=3$, then the value of $\|2 A\|$ is <br> (a) 24 <br> (b) 20 <br> (c) 16 <br> (d) 12 <br> (d) 12 | 1 |
| 6. | The Integral of $\int \frac{1-\sin x}{\cos ^{2} x} d x$, is <br> (a) $\tan x-\sec x+c$ <br> (b) $\tan x+\sec x+c$ <br> (c) $\tan x \cdot \sec x+c$ <br> d) $\sec x+c$ <br> (a) $\tan x-\sec x+c$ |  |
| 7. | The value of $\int_{-2}^{2} f(x) d x$, if $f$ is a odd function. <br> (a) -2 <br> (b) 0 <br> (c) 2 <br> (d) 4 <br> (b) 0 | 1 |
| 8. | The area bounded by the curve $y=\operatorname{Sin} x$, between 0 to $\pi$ Is <br> (a) 0 sq.unit <br> (b) 1 sq. unit <br> (c) 2 sq.unit <br> (d) 4 sq. unit <br> (c) 2 sq.unit | 1 |
| 9. | The order and degree of differential equation $\frac{d^{4} y}{d x^{4}}+\sin \left(\frac{d^{3} y}{d x^{3}}\right)=0$, is <br> (a) Order 4,degree 3 <br> (b) order 3,degree 4 <br> (c) order 3 ,degree not defined <br> (d) order 4,degree not defined <br> (d) order 4, degree not defined | 1 |
| 10. | Let $\vec{a}=\hat{\imath}+4 \hat{\jmath}+2 \hat{k}$ and $\vec{b}=3 \hat{\imath}-2 \hat{\jmath}+7 \hat{k}$, then a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ is <br> a) $\vec{d}=32 \hat{\imath}-\hat{\jmath}-14 \hat{k}$ <br> b) $\vec{d}=32 \hat{\imath}+\hat{\jmath}-14 \hat{\jmath}$ <br> c) $\vec{d}=32 \hat{\imath}-\hat{\jmath}+14 \hat{k}$ <br> d) $\vec{d}=32 \hat{\imath}+\hat{\jmath}+14 \hat{k}$ <br> a) $\vec{d}=32 \hat{\imath}-\hat{\jmath}-14 \hat{k}$ | 1 |
| 11. | If $\|\vec{a}\|=10,\|\vec{b}\|=2$ and $\vec{a} \cdot \vec{b}=12$, the value of $\|\vec{a} \times \vec{b}\|$, is <br> (a) 12 <br> (b) 16 <br> (c) 20 <br> (d) 14 <br> (b) 16 | 1 |
| 12. | If $\vec{a}$ and $\vec{b}$ are unit vector then the angle between $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is: <br> a) $\frac{\pi}{2}$ <br> b) $\frac{\pi}{12}$ <br> c) $\frac{\pi}{4}$ <br> d) $\frac{\pi}{3}$ | 1 |


|  | a) $\frac{\pi}{2}$ |  |
| :---: | :---: | :---: |
| 13. | The angle between the pair of lines given by $\vec{r}=3 \vec{\imath}+2 \vec{\jmath}-4 \vec{k}+\varphi(\vec{\imath}+2 \vec{\jmath}+$ $2 \vec{k})$ and $\vec{r}=5 \vec{\imath}-2 \vec{\jmath}+\mu(3 \vec{\imath}+2 \vec{\jmath}+6 \vec{k})$ is, <br> (a) $\cos ^{-1} \frac{19}{21}$ <br> (b) $\sin ^{-1} \frac{19}{21}$ <br> (c) $\tan ^{-1} \frac{19}{21}$ <br> (d) $\cos ^{-1} \frac{21}{19}$ <br> (a) $\cos ^{-1} \frac{19}{21}$ | 1 |
| 14. | The value of k for which the function $f(x)=\left\{\begin{array}{ll}\frac{1-\cos 2 x}{2 x^{2}} & \text { if } x \neq 0 \\ k & \text { if } x=0\end{array}\right.$ is continuous at $\mathrm{x}=0$ is <br> a) 0 <br> b) -1 <br> c) 1 <br> d) 2 <br> c) 1 | 1 |
| 15. | The value of $\sin ^{-1}\left(\sin \frac{2 \pi}{3}\right)$ is <br> (a) $\frac{2 \pi}{3}$ <br> (b) $\frac{\pi}{3}$ <br> (c) $-\frac{2 \pi}{3}$ <br> (d) $-\frac{\pi}{3}$ <br> (b) $\frac{\pi}{3}$ | 1 |
| 16. | An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is <br> (a). $\frac{6}{15}$ <br> (b). $\frac{4}{15}$ <br> (c) $\frac{8}{15}$ <br> (d) $\frac{1}{15}$ <br> (c) $\frac{8}{15}$ | 1 |
| 17. | Three coins are tossed once. Find the probability of getting at least one head. <br> a) $\frac{1}{8}$ <br> b) $\frac{1}{4}$ <br> C) $\frac{7}{4}$ <br> d) $\frac{7}{8}$ <br> d) $\frac{7}{8}$ | 1 |

\begin{tabular}{|c|c|c|}
\hline 18. \& \begin{tabular}{l}
Find \(P(A / B)\) if \(P(A)=0.4, P(B)=0.8\) and \(P(B / A)=0.6\) \\
a) 0.3 \\
b) 0.4 \\
c) 0.5 \\
d) 0.6 \\
a) 0.3
\end{tabular} \& \\
\hline \& \begin{tabular}{l}
ASSERTION -REASON BASED QUESTIONS \\
In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. \\
(a) Both \((A)\) and \((R)\) are true and \((R)\) is the correct explanation of \((A)\). \\
(b) Both (A) and (R) are true but (R) is not the correct explanation of \((A)\). \\
(c) (A) is true but (R) is false. \\
(d) (A) is false but (R) is true.
\end{tabular} \& \\
\hline 19. \& \begin{tabular}{l}
Assertion: If \(\mathrm{A}=\left[\begin{array}{ll}2 \& 4 \\ 6 \& 0\end{array}\right]\), then \(\mathrm{A}=\left[\begin{array}{ll}2 \& 5 \\ 5 \& 0\end{array}\right]+\left[\begin{array}{cc}0 \& -1 \\ 1 \& 0\end{array}\right]\) is a unique way of expressing A \\
Reason: Every matrix can be represented as a sum of symmetric and skew symmetric matrix. \\
(a)
\end{tabular} \& \\
\hline 20. \& \begin{tabular}{l}
Assertion (A) : The angle between the straight line \(\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}\) and \(\frac{x-1}{1}=\) \(\frac{y+2}{2}=\frac{z-3}{-3}\) is \(90^{\circ}\) \\
Reason (R) : Skew lines are lines in different planes which are parallel and intersecting \\
(c)
\end{tabular} \& \\
\hline \& \begin{tabular}{l}
Section B \\
(This section comprises of very short answer type questions(VSA) of 2 marks each)
\end{tabular} \& \\
\hline 21. \& \begin{tabular}{l}
Find the value of \(\tan ^{-1}(1)+\cos ^{-1}\left(-\frac{1}{2}\right)+\sin ^{-1}\left(-\frac{1}{2}\right)\) \\
OR \\
Draw the graph of \(\sin ^{-1} x, \mathrm{x} \in\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]\) \\
Solution:
\[
\begin{aligned}
\& \tan ^{-1}(1)+\cos ^{-1}\left(-\frac{1}{2}\right)+\sin ^{-1}\left(-\frac{1}{2}\right) \\
\& =\frac{\pi}{4}+\frac{2 \pi}{3}+\left(-\frac{\pi}{6}\right)
\end{aligned}
\]
\end{tabular} \& 2

2
1
.5
.5 <br>
\hline
\end{tabular}

|  | $=\frac{3 \pi}{4}$ <br> OR <br> For correct graph | 2 |
| :---: | :---: | :---: |
| 22. | For what value of $k$ is the function $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{l} \frac{\sin 5 x}{3 x}+\cos x, \text { if } x \neq 0 \\ k, \quad \text { if } \quad x=0 \end{array} \quad \text { continuous at } \mathrm{x}=0\right.$ <br> Solution: <br> Since , $\mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=0$ $\operatorname{Lim}_{x \rightarrow 0} f(x)=f(0)$ <br> Or $\operatorname{Lim}_{x \rightarrow 0} \frac{\sin 5 x}{3 x}+\cos x=k$ <br> Or $\operatorname{Lim}_{x \rightarrow 0} \frac{\sin 5 x}{5 x} \times \frac{5}{3}+\cos x=k$ <br> Or $1 \times \frac{5}{3}+1=k$ $\mathrm{K}=\frac{8}{3}$ | . 5 <br> . 5 <br> . 5 <br> . 5 |
| 23. | Solve the differential equation, $\frac{d y}{d x}=\left(1+\mathrm{x}^{2}\right)\left(1+\mathrm{y}^{2}\right)$ <br> Solution: $\quad \frac{d y}{1+y^{2}}=\left(1+x^{2}\right) d x$ (applying variable separable) <br> Integrating both sides $\tan ^{-1} y=x+x^{3} / \mathrm{C}+\mathrm{C}$ | 2 1 1 |
| 24. | If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x)=\sin x$ and $g(x)=5 x^{2}$, then find fog(x) and gof( $x$ ) <br> Solution:- for $\operatorname{fog}(\mathrm{x})=\mathrm{f}(\mathrm{g}(\mathrm{x}))=\mathrm{f}\left(5 \mathrm{x}^{2}\right)=\sin \left(5 \mathrm{x}^{2}\right)$ $\operatorname{gof}(x)=g(f(x)))=g(\sin x)=5 \sin ^{2} x$ | $\begin{array}{ll}2 \\ \\ 1 \\ 1 & \\ 1\end{array}$ |
| 25. | If $P(A)=0.8, P(B)=0.5$ and $P(B / A)=0.4$ find $P(A U B)$ and $P(B / A)$. <br> OR <br> A speaks truth in $80 \%$ cases and $B$ speaks truth in $90 \%$ cases. In what percentage of cases are they likely to disagree with each other in stating the same fact? <br> Solution:- <br> For finding $\mathrm{P}(\mathrm{AUB})=0.98$ <br> For finding $\mathrm{P}(\mathrm{B} / \mathrm{A})=0.64$ <br> OR $P(A)=0.8, P(B)=0.9$ | 2 <br>  <br>  <br>  <br> 1.5 <br>  <br> . 5 <br> .5 |

\begin{tabular}{|c|c|c|}
\hline \& \[
\begin{aligned}
\& \mathrm{P}(\bar{A})=0.2 \quad \mathrm{P}(\bar{B})=0.1 \\
\& \mathrm{P}(\mathrm{~A} \bar{B} \text { or } \bar{A} \mathrm{~B})=0.8 \times 0.1+0.2 \times 0.9=0.26
\end{aligned}
\] \& \[
\begin{aligned}
\& .5 \\
\& 1
\end{aligned}
\] \\
\hline \& Section C
(This section comprises of short answer type questions(SA) of 3
marks each) \& \\
\hline 26. \& \begin{tabular}{l}
Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, \(90 \%\) of the test detect the disease but \(10 \%\) go undetected. Of people free of HIV, \(99 \%\) of the test are judged HIV-ive but \(1 \%\) are diagnosed as showing HIV+ive. From a large population of which only \(0.1 \%\) have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV? \\
Solution:- Let E be the event that a person selected is actually having HIV And E' be the event that a person selected is not actually having HIV. \\
A be the event of selected person diagnosed as HIV +ve \\
\(P(E)=0.001\) \\
\(P\left(E^{\prime}\right)=1-0.001=0.999\) \\
\(\mathrm{P}(\mathrm{A} / \mathrm{E})=0.9 \quad \mathrm{P}\left(\mathrm{A} / \mathrm{E}^{\prime}\right)=0.01\) \\
By Baye's theorem, Required Probability \(=\mathrm{P}(\mathrm{E} / \mathrm{A})=\frac{P(E) * P\left(\frac{A}{E}\right)}{P(E) * P\left(\frac{A}{E}\right)+P\left(E^{\prime}\right) * P\left(\frac{A}{E^{\prime}}\right)}=0.083\)
\end{tabular} \& \begin{tabular}{l}
\[
3
\] \\
0.5 \\
1 \\
1
\[
0.5
\]
\end{tabular} \\
\hline 27. \& \begin{tabular}{l}
Find the intervals in which the function \(f(x)=20-9 x+6 x^{2}-x^{3}\) is \\
(i) strictly increasing. \\
(ii) strictly decreasing. \\
Solution:- \(f^{\prime}(x)=-9+12 x-3 x^{2}\) \\
Putting \(f^{\prime}(x)=0\) \\
Or, \(-9+12 x-3 x^{2}=0\)
\[
(x-3)(x-1)=0
\]
\[
X=1,3
\] \\
\(f(x)\) is strictly increasing in \((1,3)\) and strictly decreasing in \((-\infty, 1) \cup(3, \infty)\)
\end{tabular} \& \begin{tabular}{l}
0.5 \\
1 \\
1 \\
0.5
\end{tabular} \\
\hline 28. \& \begin{tabular}{l}
If \(\mathrm{x}=\mathrm{a}[\cos \theta+\log \tan (\theta / 2)]\) and \(\mathrm{y}=\operatorname{asin} \theta\), find \(\frac{d y}{d x}\) at \(\theta=\frac{\pi}{4}\) \\
OR \\
If \(\mathrm{y}=\mathrm{e}^{\mathrm{x}} \sin \mathrm{x}, \mathrm{prove}\) that \(\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+2 y=0\) \\
Solution:- If \(\mathrm{x}=\mathrm{a}[\cos \theta+\log \tan (\theta / 2)]\)
\[
\begin{aligned}
\& \frac{d x}{d \theta}=a\left(-\sin \theta+\frac{1}{\tan \theta / 2} \times \frac{\frac{\sec ^{2} \theta}{2}}{2}\right)\left(-\sin \theta+\frac{1}{2 \sin \theta / 2 \cdot \cos \theta / 2}\right)= \\
\& a\left(-\sin \theta+\frac{1}{\sin \theta}\right)=\mathrm{a}\left(\frac{\cos ^{2} \theta}{\sin \theta}\right) \\
\& \quad \operatorname{And~} \mathrm{y}=\operatorname{asin} \theta \\
\& \frac{d y}{d \theta}=\mathrm{a} \cos \theta \\
\& \frac{d y}{d x}=\frac{d y}{d \theta} \\
\& \frac{d x}{d \theta} \\
\& \frac{d y}{d x} \mathrm{tan} \theta \\
\& \mathrm{at} \mathrm{x}=\frac{\pi}{4}=1
\end{aligned}
\]
\end{tabular} \& 3

1
0.5
1
0.5 <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline 29. \& \begin{tabular}{l}
Find \(\int e^{x} \frac{x-3}{(x-1)^{3}} d x\) \\
OR \\
Evaluate
\[
\int \frac{3 x-2}{(x+1)^{2}(x+3)} d x
\] \\
Solution:- \(\int e^{x} \frac{x-3}{(x-1)^{3}} d x=\int e^{x}\left[\frac{x-1-2}{(x-1)^{3}}\right] d x=\int e^{x}\left[\frac{1}{(x-1)^{3}}-\frac{2}{(x-1)^{3}}\right] d x\)
\[
=\int \mathrm{e}^{\mathrm{x}}\left[\mathrm{f}(\mathrm{x})+f^{\prime}(x)\right] d x=\mathrm{e}^{\mathrm{x}} f(x)+c=\mathrm{e}^{\mathrm{x}} \frac{1}{(\mathrm{x}-1)^{3}}+\mathrm{c}
\] \\
OR \\
Let \(\frac{3 x-2}{(x+1)^{2}(x+3)}=\frac{A}{(x+1)}+\frac{B}{(\mathrm{x}+1)^{2}}+\frac{C}{(x+3)}\) \\
By equating coefficients , \(\mathrm{A}=11 / 4 \quad, \mathrm{~B}=-5 / 2 \quad, \mathrm{C}=-11 / 4\) \\
Now, \(\int \frac{3 x-2}{(x+1)^{2}(x+3)} d x\)
\[
=\int\left(\frac{11 / 4}{(x+1)}+\frac{-5 / 2}{(\mathrm{x}+1)^{2}}+\frac{-11 / 4}{(x+3)}\right) d x=11 / 4 \log (x+1)+5 / 2 \frac{1}{(x+1)}-11 / 4 \log (\mathrm{x}+3)+\mathrm{c}
\]
\end{tabular} \& 3

2
2

1

2
2 <br>

\hline 30. \& | Evaluate $\int_{0}^{2 \pi} \frac{1}{1+e^{\operatorname{sinx}}} d x$ $\begin{align*} & \text { Solution:- Let } I=\int_{0}^{2 \pi} \frac{1}{1+e^{\sin x}} d x-  \tag{i}\\ & =\int_{0}^{2 \pi} \frac{1}{1+e^{\sin (2 \pi-x)}} d x \\ & \mathrm{I}=\int_{0}^{2 \pi} \frac{1}{1+e^{-\sin x}} d x \\ & \quad=\int_{0}^{2 \pi} \frac{e^{\sin x}}{1+e^{\sin x}} d x \tag{ii} \end{align*}$ |
| :--- |
| [by property $\int_{0}^{a} \mathrm{f}(\mathrm{x}) d x=\int_{0}^{a} \mathrm{f}(\mathrm{a}-\mathrm{x}) d x$ |
| By adding equations (i) and (ii) $\begin{aligned} & 2 \mathrm{I}=\int_{0}^{2 \pi} d x \quad=2 \pi \\ & I=\pi \end{aligned}$ | \& 3

2

1 <br>

\hline 31. \& | Find the matrix $A$ for which $A\left[\begin{array}{cc}5 & 3 \\ -1 & -2\end{array}\right]=\left[\begin{array}{cc}14 & 7 \\ 7 & 7\end{array}\right]$. OR $A=\left[\begin{array}{cc} 1 & \tan x \\ -\tan x & 1 \end{array}\right] \text {, show that } A^{T} A^{-1}=\left[\begin{array}{cc} \cos 2 x & -\sin 2 x \\ \sin 2 x & \cos 2 x \end{array}\right]$ |
| :--- |
| Solution :- As per multiplication rule order of matrix A would be $2 \times 2$ Consider $\mathrm{A}=\left[\begin{array}{ll}x & y \\ z & t\end{array}\right]$. | \& 3

0.5 <br>
\hline
\end{tabular}

|  | $\left[\begin{array}{ll}x & y \\ z & t\end{array}\right]\left[\begin{array}{cc}5 & 3 \\ -1 & -2\end{array}\right]=\left[\begin{array}{cc}14 & 7 \\ 7 & 7\end{array}\right]$ <br> $\left[\begin{array}{cc}5 x-y & 3 x-2 y \\ 5 z-t & 3 z-2 t\end{array}\right]=\left[\begin{array}{cc}14 & 7 \\ 7 & 7\end{array}\right]$ <br> By equality concept , $5 x-y=14---------(i)$ $3 x-2 y=7------------(i i)$ $\begin{equation*} 5 z-t=7- \tag{iii} \end{equation*}$ $\begin{equation*} 3 \mathrm{z}-2 \mathrm{t}=7 \tag{iv} \end{equation*}$ <br> On solving , $x=3, y=1, z=1$ and $t=-2$ <br> So , $A=\left[\begin{array}{cc}3 & 1 \\ 1 & -2\end{array}\right]$. <br> OR <br> For $\mathrm{A}^{\mathrm{T}}=\left[\begin{array}{cc}1 & -\tan x \\ \tan x & 1\end{array}\right]$, <br> For $A^{-1}=\frac{1}{\sec ^{2} x}\left[\begin{array}{cc}1 & -\tan x \\ \tan x & 1\end{array}\right]=\cos ^{2} x\left[\begin{array}{cc}1 & -\tan x \\ \tan x & 1\end{array}\right]$, <br> For $A^{\top} A^{-1}=\left[\begin{array}{cc}\sec 2 x & -\sin 2 x \\ \sin 2 x & \cos 2 x\end{array}\right]$ | 1 <br> 1 <br> 0.5 <br> 0.5 <br> 1 <br> 1.5 |
| :---: | :---: | :---: |
|  | Section -D <br> (This section comprises of long answer type questions(LA) of 5 marks each) |  |
| 32. | Sketch the graph of $y=\|x+3\|$ and evaluate $\int_{-6}^{0}\|x+3\| d x$. <br> OR <br> Using integration find the area of region bounded by the triangle whose vertices are $\mathrm{A}(1,0), \mathrm{B}(2,2)$ and $\mathrm{C}(3,1)$. <br> Solution:-For correct graph $\mathbf{Y}=\|\mathrm{x}+3\|=\left\{\begin{array}{c} (x+3), x+3 \geq 0 \Rightarrow x \geq-3 \\ -(x+3), x+3<0 \Rightarrow x<-3 \end{array}\right.$  $\begin{aligned} & \text { Now, } \int_{-6}^{0}\|\mathrm{x}+3\| \mathrm{dx}=\int_{-6}^{-3}\|\mathrm{x}+3\| \mathrm{dx}+\int_{-3}^{0}\|\mathrm{x}+3\| \mathrm{dx} \\ & =\int_{-6}^{-3}-(\mathrm{x}+3) \mathrm{dx}+\int_{-3}^{0}(\mathrm{x}+3) \mathrm{dx} \\ & =9 \text { sq. unit } \end{aligned}$ <br> OR <br> Equation of $A B$ is given as $y=2 x-2$ <br> Equation of $B C$ is $y=4-x$ and equation of CA is $y=(x-1) / 2$ | 5 <br> 2 <br> 2 <br>  <br>  <br>  <br>  <br>  <br>  <br>  |

\begin{tabular}{|c|c|c|}
\hline \& \begin{tabular}{l}
 \\
Area of \(\triangle A B C=\int_{1}^{2}(2 x-2) d x+\int_{2}^{3}(4-x) d x-\int_{1}^{3} \frac{(x-1)}{2} d x\) \(=3 / 2\) sq units
\end{tabular} \& 1

2.5 <br>

\hline 33. \& | (a)Find the shortest distance between the following lines: $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text { and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ |
| :--- |
| (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b}=\vec{c} \cdot \vec{a}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}= \pm 2(\vec{b} \times \vec{c})$ Solution:- |
| (a) |
| Given lines are $\vec{r}=(3 \hat{i}+5 \hat{j}+7 \hat{k})+\lambda(\hat{i}-2 \hat{j}+\hat{k})$, and $\begin{aligned} & \vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\mu(7 \hat{i}-6 \hat{j}+\hat{k}) \\ & \begin{aligned} & a_{1}=3 \hat{i}+5 \hat{j}+7 \hat{k}, \quad \overrightarrow{b_{1}}=\hat{i}-2 \hat{j}+\hat{k} ; \quad \overrightarrow{a_{2}}=-\hat{i}-\hat{j}-\hat{k}, \quad \overrightarrow{b_{2}}=7 \hat{i}-6 \hat{j}+\hat{k} \\ & \vec{a}_{2}-\vec{a}_{1}=-4 \hat{i}-6 \hat{j}-8 \hat{k}, \quad \overrightarrow{b_{1}} \times \vec{b}_{2}=\left\|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{array}\right\|=4 \hat{i}+6 \hat{j}+8 \hat{k} \\ & S . D .=\frac{\left\|\left(\vec{a}_{2}-\vec{a}_{1}\right) \cdot\left(\overrightarrow{b_{1}} \times \vec{b}_{2}\right)\right\|}{\left\|\vec{b}_{1} \times \vec{b}_{2}\right\|}=\frac{\|(-4 \hat{i}-6 \hat{j}-8 \hat{k}) \cdot(4 \hat{i}+6 \hat{j}+8 \hat{k})\|}{\|4 \hat{i}+6 \hat{j}+8 \hat{k}\|} \\ &=\frac{\|-16-36-64\|}{\sqrt{16+36+64}}=\frac{116}{\sqrt{116}}=\sqrt{116}=2 \sqrt{29} \end{aligned} \end{aligned}$ |
| (b) $a^{\vec{~}} \cdot b^{\vec{~}}=a^{\vec{~}} \cdot c^{\vec{~}}=0$ it means $a^{\vec{~}}$ is perpendicular to both $b^{\Delta}$ and $c^{\vec{~}}$ vector $\begin{aligned} & \vec{a}=\mu\left(b^{\vec{~}} \times c\right)^{\vec{~}} \\ & \left\lvert\, \overrightarrow{a^{\vec{~}}\|=\mu\| b^{\rightharpoonup}\| \| \vec{c} \left\lvert\, \sin \frac{\pi}{6} \Rightarrow \mu= \pm 2\right.}\right. \end{aligned}$ | \& 3+2 <br>

\hline
\end{tabular}



|  | $h=\frac{l}{\sqrt{3}}$ <br> $\frac{d^{2} v}{d h^{2}}=-2 \pi h<0$ at <br> $V$ is maximum at $h=\frac{l}{\sqrt{3}}$ $h=\frac{l}{\sqrt{3}}$ <br> so, $\frac{h}{l}=\frac{1}{\sqrt{3}}=\cos \alpha$ $\alpha=\cos _{\mathrm{OR}}{ }^{-1} \frac{1}{\sqrt{3}}$ <br> Same as above | 1 |
| :---: | :---: | :---: |
|  | Section -E <br> (This section comprises of long answer type questions(LA) of 5 marks each) |  |
| 36. | A water tank has a shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan ^{-1}(0.5)$. If water is poured into a constant rate of 5 cubic meter per hour. <br> (i) Find the rate at which the level of water is rising at instant when the depth of water of tank is h m . <br> (ii). Find the rate at which the surface of tank covered by water is increasing at an instant when radius $r=2 \sqrt{2} \mathrm{~m}$ <br> Solution:- $\begin{aligned} & \text { (i) } \frac{r}{h}=0.5 \Rightarrow \mathrm{~h}=2 \mathrm{r} \\ & \frac{d v}{d v}=\frac{1}{3} \pi\left(\frac{h}{2}\right)^{2} \mathrm{~h}=\pi \frac{h^{2}}{4} \frac{d h}{d t} \\ & \frac{d h}{d t}=\frac{5 \times 4}{\pi h^{2}}=\frac{20}{\pi h^{2}} \mathrm{~m} / \mathrm{h} \end{aligned}$ <br> (ii). $\mathrm{S}=\pi \mathrm{r} \sqrt{\mathrm{r}^{2}+h^{2}}=\pi \frac{\mathrm{h}}{2} \sqrt{5} \mathrm{~h} / 2$ $\frac{d s}{d t}=\pi \frac{\mathrm{h}}{2} \sqrt{5} \frac{d h}{d t}=\pi \frac{\mathrm{h}}{2} \sqrt{5} \frac{20^{2}}{\pi h^{2}}=5 \sqrt{10} / 4 \mathrm{sq} \mathrm{~m} / \mathrm{h}$ | $1 / 2$ 1 $1 / 2$ 1 1 1 |


|  |  |  |
| :---: | :---: | :---: |
| 37. | Three friends are flying the kites in the sky .At some instant of time the coordinates of kites are $\mathrm{A}(1,1,0), \mathrm{B}(1,2,1)$ and $\mathrm{C}(-2,2,-1)$. <br> (i) Find the direction ratio's of lines $A B$ and $A C$. <br> (ii) What is the angle between the lines $A B$ and $A C$ ? <br> Solution :- (i) D'Ratios of line $A B$ are $0,1,1$ <br> And that of line $A C$ are $-3,1,-1$ <br> (ii)here $, a_{1}=0, b_{1}=1$ and $c_{1}=1$ and $a_{2}=-3, b_{2}=1, c_{2}=-1$ $\begin{array}{r} \cos \theta=\frac{a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}}{\sqrt{\left(a_{1}{ }^{2}\right.}+b_{1}{ }^{2}+c_{1}{ }^{2} \cdot \sqrt{\left(a_{2}{ }^{2}\right.}+b_{2}{ }^{2}+c_{2}{ }^{2}}= \\ \\ \theta=90^{\circ} \end{array}$ | 1 <br> 3 |
| 38. | A total amount of Rs. 7000 is deposited in three different saving bank accounts with annual interest rates of $5 \%, 8 \%$ and $8 \frac{1}{2} \%$ respectively. The total annual interest from these three accounts is Rs. 550.Equal amounts have been deposited in the $5 \%$ and $8 \%$ saving accounts. Using given information answer the following questions. |  |



